Rocket Propulsion

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Lecture 35

Introduction to Hybrid Rockets and a Simple Illustration of Combustion Instability in Liquid Propellant Rockets

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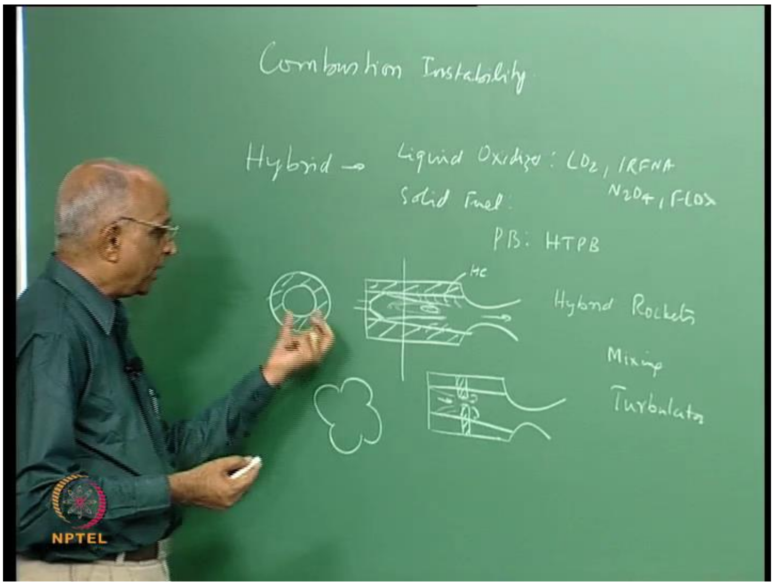

We will talk on combustion instability in chemical propellant rockets. Namely, combustion instability in solid, liquid and other types of rockets, but I thought in the last class we just started with hybrid rockets and therefore, let me spend a couple of minutes on the hybrid rockets. The very name hybrid means, it is a combination of two phases, maybe a liquid and a solid as propellants, or a gas and a liquid and so on.

But what is normally used in hybrid is a liquid as an oxidizer and solid as a fuel, and we know that a solid fuel is like polybutadiene HTPB, PBAN, PVC, etc., like hydroxy-terminated polybutadiene which we used as a binder in solid propellant rockets.

The liquid oxidizers which are used could be anything, could be liquid oxygen, could be nitric acid, inhibited red fuming nitric acid, could be N_2O_4; we talked in terms of FLOX - liquid oxygen to which was added fluorine to make it more powerful and these are the liquid oxidizers. But the solid fuel is essentially polybutadiene HTPB, which has more hydrogen. How will the construction look like? You have a cylinder, a case, you have a nozzle, you have the solid fuel, which is kept over here. It could be any configuration of the solid grain.

Let us take a star configuration or a circular configuration. I take a section over here this is my solid fuel. And what is it I do? I spray the liquid oxidizer onto the fuel, and in case the liquid fuel like let us say RFNA is hyperbolic with respect to the fuel then it begins to react at the surface. Vapor is evolved at the fuel surface from the heat transfer and I have vapor which keeps coming out, and we have the oxidizer adjacent to it. A stoichiometric mixture or a mixture between the oxidizer and the fuel vapor gets formed, it burns and then you get the thrust. This is the principle of a hybrid rocket.

Essentially it is somewhat in between a solid rocket and a liquid rocket, but in this case what happens is provided the fuel surface is hyperbolic with respect to the oxidizer, the moment you inject it chemical reaction begin to occur, the heat feedback to the surface generates the fuel vapor say hydrocarbon vapor, and that hydrocarbon vapor mixes with the oxidizer and you get combustion taking place and the combustion products are exhausted through a nozzle, and this is the principle of hybrid rockets.

In case, the surface is not hypergolic with respect to the oxidizer, then in that case I have to coat on the surface a substance which initially will start combustion when it comes into contact with the oxidizer. It could be a paste containing amines, which is hypergolic with respect to the liquid oxidizer heat say N_2O_4. Once combustion starts, the heat is getting generated and it is the heat transfer namely the convective heat transfer which evaporates the solid fuel. The fuel vapor mixes with the oxidizer vapor formed from its droplets, and combustion progresses.

Therefore, the controlling event in these hybrid rockets is generation of fuel vapor and essentially mixing of it with the oxidizer vapor. The oxidizer vapor is formed by evaporation of the oxidizer droplets formed in the spray. What is mixing? At the surface you have hydrocarbon vapors being formed; it mixes with the free stream of oxidizer vapor, because when I inject the oxidizer as a spray and this also vaporizes. Mixing is sort of a problem in the short length of the combustor. You can recall the stream tube mode of combustion in liquid propellant rockets. There was this researcher by name Professor Dadieu who suggested introducing a turbulator in the hybrid chamber. Since mixing is a problem put something like a turbulator in the hybrid combustion chamber. What is this turbulator? It is used in many engineering situations. Namely, when I look at this particular thing fuel is getting evaporated here oxidizer is coming over here.

We make this constriction over here in the form of let us say petals. We put this configuration over here in this hole or port volume over here. When the gas is flowing through it, eddies of different sizes are formed behind it and it helps to mix the gases. A turbulator would provide for mixing the fuel and oxidizer vapor so that combustion gets completed in the chamber. Therefore, you know in the early stages whenever people were working with hybrid, they never really used turbulator they got very low performance; after incorporating a turbulator mixing is better, but still the performance is not as good as in liquid propellant rockets but it is better than solid fuel rockets. I think this is all about hybrid rockets. But there is one distinct advantage in using hybrid rockets. What is the advantage?

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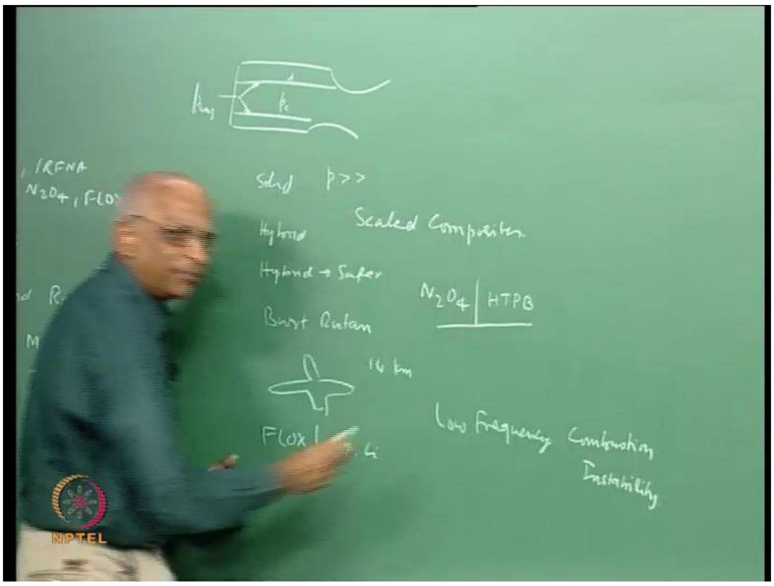

Let us say you have a defective solid fuel; the solid fuel has a crack in it or some opening in it. Therefore, the surface area increases over and above the design value. And you are spraying liquid oxidizer onto the surface. Because the surface area is increasing, more pressure gets generated in the combustion chamber; when pressure gets generated automatically the pressure drop across the injector gets decreased, because pressure here is higher while the injection pressure is a constant pressure, pressure drop across the injector decreases and therefore the flow rate of oxidizer decreases. This decreases the chamber pressure and therefore the chamber pressure is self regulating.

In solid propellant rockets if there is a crack, if surface area increases pressure continues to increase as the burning rate increases with pressure. In the case of hybrid if there is some surface defect, what happens is pressure gets generated, once pressure gets generated automatically the pressure difference at injection decreases, and pressure in the combustion chamber being higher decreases the flow rate, the flow rate of oxidizer decreases automatically and the thrust gets regulated. This makes the hybrid rocket much safer than a solid propellant rocket as it can accommodate certain amount of surface defects.

A British millionaire by name Burt Rutan formed a company known as Scaled composites, and he used hybrid rockets for ferrying people to space, maybe he says space can be used for tourists who would like a voyage in space and view the Earth from space. He used an aircraft White Knight, which I showed you as a slide the last time. In the belly of the aircraft he mounts a space capsule powered by a hybrid rocket. The White Knight aircraft goes up to 15 kilometers height, from there the hybrid rocket takes the space capsule into a sub-orbital flight in space. The purpose of the hybrid rocket is for ferrying people from 14 or 15 kilometers to space and return to the ground. This is how the hybrid rocket is used.

The company which does it is a private company known as Scaled Composites. Last week the launch pad was officially inaugurated in the desert in California. I think we should keep such developments in mind. Therefore, I would conclude by saying that a hybrid rocket has not been very much used in practice. It uses convective heat transfer for vaporizing the solid fuel, the fuel vapor mixes with oxidizer vapor and burns. It has lower performance than liquid propellant rocket; however, it is safer than many of the other types of rockets, because it is self regulating and it is finding increased applications. I do foresee much more applications for hybrid rockets in the years to come.

We can even think in terms of other propellant may be FLOX if you want very high performance, maybe with metal embedded in the fuel, like maybe you could have some light metals say magnesium or lithium. What is used by Scaled composites is N_2O_4 as oxidizer and HTPB as fuel. I think this is all about hybrid rockets, but there is one problem with hybrid rockets because of the slow rate of reactions it is very susceptible to combustion instability in the low frequency mode, and I am going to talk about it in the following.

The subject of combustion instability is new for us.

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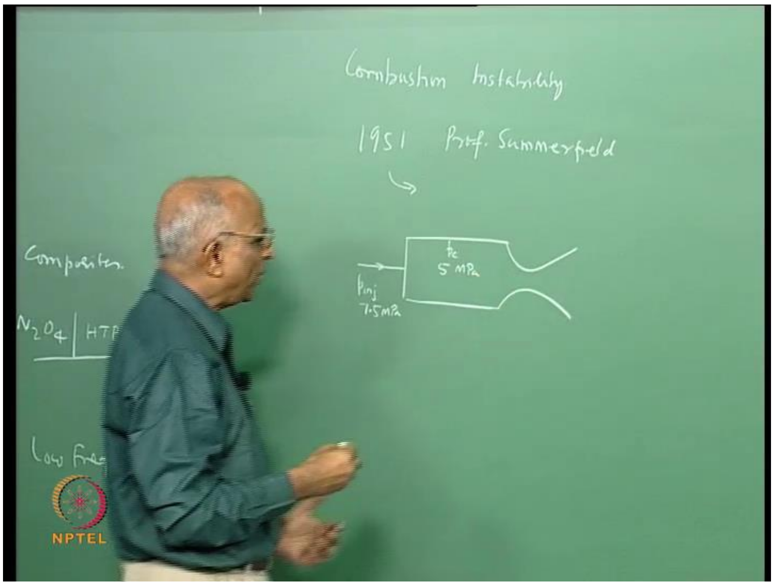

I will start with a very illustrative example and this example is not mine. This example was given in 1951 by Professor Summerfield; he was at Princeton and he published a paper in 1951. I think at that time in Rocket Society Journal; AIAA journal was not there and the American Rocket Society Journal preceded the AIAA Journal. In this paper he gives an example, I modify the numbers because his numbers were in foot pound system of units - FPS system and not that that easy to work on the board. I take a typical liquid propellant rocket and follow his line of arguments in his particular paper. Let the chamber pressure of a liquid propellant rocket be 5 MPa; 5 mega Pascal. 5 mega Pascal is something like 50 bar pressure.

Let the fuel and oxidizer both of them be injected into the chamber at a pressure of 7.5 MPa. I just choose the numbers to illustrate some phenomenon; that means, propellant is injected at a pressure of 75 atmospheres, and the chamber pressure pc is equal to 5 MPa or 50 atmospheres.

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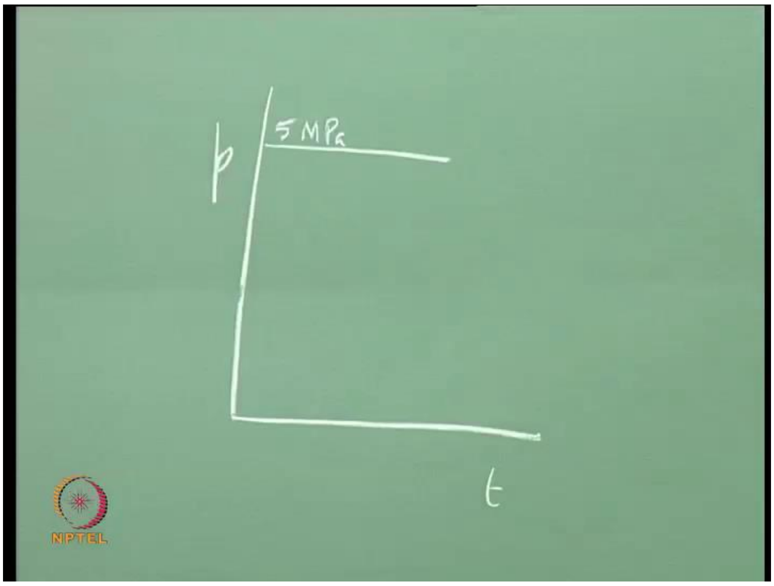

Now, let us put it down on the board. Well, we expect the chamber pressure with respect to time if it is burning steadily to be always 5 MPa and this is what I show here. I start the burning by injecting propellant at 7.5 MPa and hot gases steadily leave through the nozzle.

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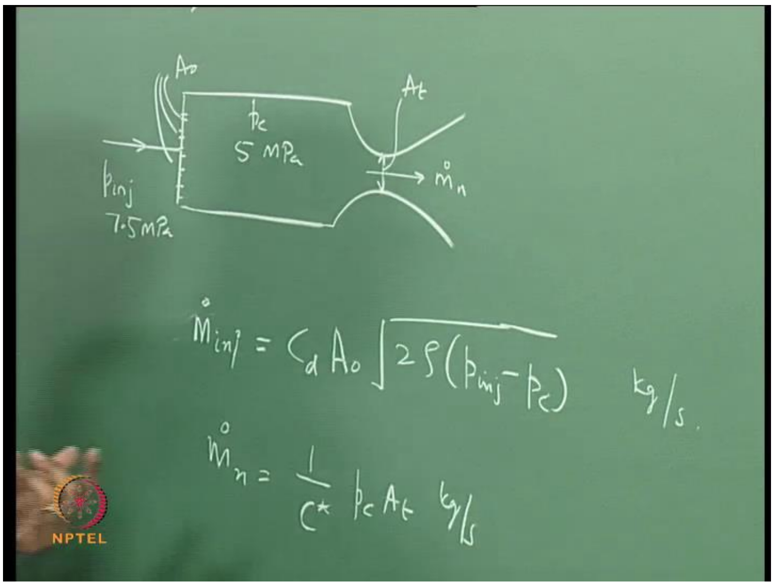

And why do you get a chamber pressure? After all, you add some mass let us say a mass of propellant is injected, and how do you calculate the mass of propellant which is injected? You inject something; that means, you have injector orifices having total area A_0, let the discharge coefficient Cd of the oxidizer and fuel orifice be the same; Cd into A_0 into under root of 2 rho into you have p injected minus p chamber is the rate at which mass is injected so many kilograms per second. Under steady conditions, and what we said is Cd A_0 into under root 2 into delta p by rho is the volume flow rate and multiplied by density gives the mass flow rate. And what is the rate at which gases are leaving m dot through the nozzle? It is leaving at the rate pc into A_T divided by C star. This is the rate at which gases are leaving: so much kilograms per second.

Therefore, we now say this is the value of A_T, and the number of holes for the fuel and oxidizer, total area flow of injection is A_0; all put together. Now, I ask myself one question. What will be the equilibrium pressure i.e., mass injected and burnt is equal to the mass which is leaving. But, I have a strange problem. The fuel or oxidizer viz., the propellant which is injected takes some time to burn. Suppose, I inject a parcel of propellants at this rate here, it is going to start burning after sometime because there has to be a delay, there has to be something like a combustion delay. Let me go back to the previous discussions again, because this point is important and this will be central to all our discussions.

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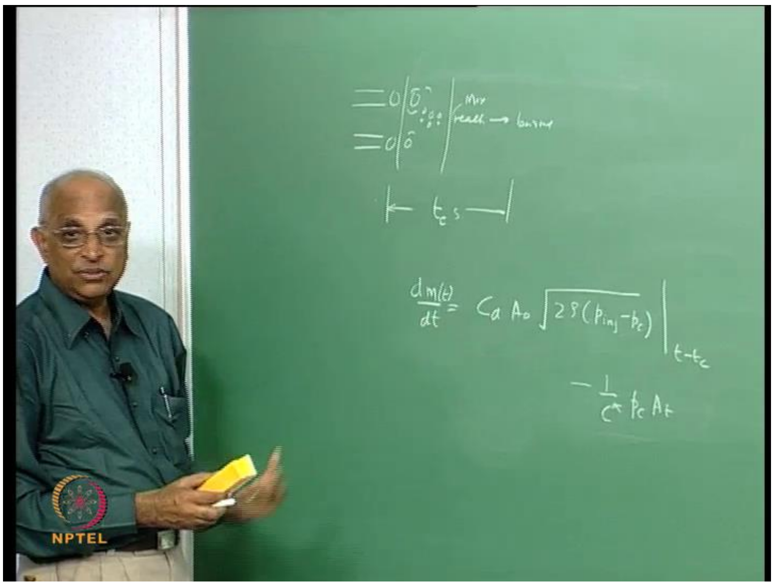

All what we are saying is there is a certain delay, because whenever I inject something maybe I inject it as a liquid, it breaks into droplets maybe fuel breaks into droplets, then it evaporates, the fuel and vapor mix together, then they react to form the burnt gases. That means, the process of combustion takes a finite amount of time, the vaporization takes time, atomization is fast, reaction maybe fast, mixing before the chemical reaction also takes sometime. Therefore I conclude that the entire process of injection to burning takes some time of let us say t delay, t due to combustion delay and is t_c seconds.

Then I need to make some changes in this equation for the equilibrium pressure. What is that I am telling? Something is entering, it has to form gases before it leaves the nozzle and therefore, if I were to write an equation which takes care of the rate of mass variations, what is it we would be writing? I would be writing under normal circumstances dm by dt the rate at which mass is getting accumulated in the chamber is equal to Cd A_0 under root 2 into density into p injected minus pc, this the mass which is coming in, and what is going out? Minus 1 over C star into pc into A_t.

But now I say this equation may not be really correct, why? Because, what comes out can happen only tc later after the injection, or rather I must say the quantity which comes out through the nozzle at time t is equal the injected flow rate at t minus tc. This is what goes out at time t. The equation let us say dm by dt, mass at time t: this leaves at time t, but what should really burn is what is injected tc time earlier. This is my dynamical equation or this is my actual equation. To solve this equation is difficult. But, we must have a procedure and that is what we will be doing in this class. Therefore, we find that there is a time delay before something happens and let us get back to this example. Mass is injected. Therefore, initially let us say chamber pressure is 5 MPa, which is the steady value of the chamber pressure.

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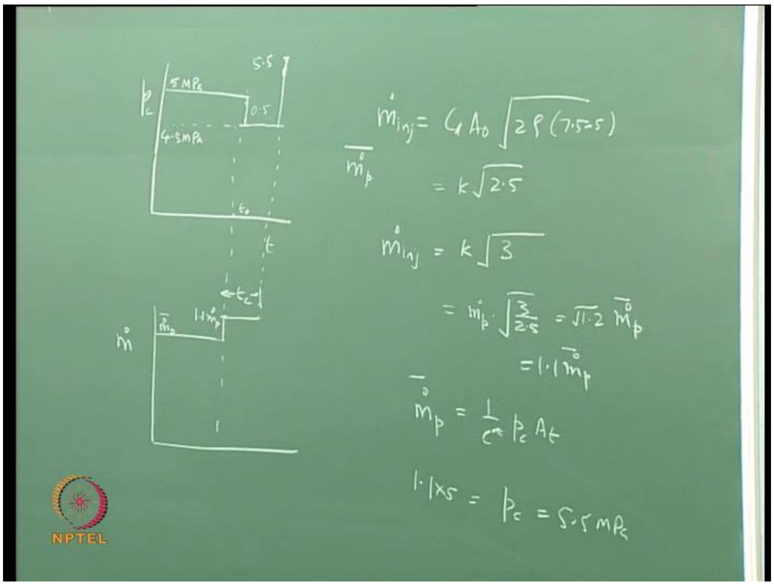

I can now write the mass which is injected is equal to Cd A_0 under root 2 rho into the injection pressure minus the chamber pressure. We do not bother about units because we are not concerned with actual values. The chamber pressure I say is 5 MPa, injection pressure is 7.5 MPa, that means, 7.5 minus 5 is the value of pressure drop across the injector, or rather I say it is equal to a constant k into root of 2.5. This is the mass flow rate from the injector. I take all the fixed values other than pressure in the constant k: maybe I have to multiply by 10 to the power 6, the values of Cd, A_0 and density and all these are consolidated in the value of k.

I say this is my nominal value and I call it as m dot of propellant which is getting injected. Nominal value is because it corresponds to the steady conditions in the chamber.

Let at this particular instant of time t_0, let the chamber pressure drops by let us say 0.5 MPa; that means, this magnitude is 4.5 MPa; something happens in the chamber, maybe there is some problem with the injector or something happens within this chamber and the pressure to fall from 5 MPa to 4.5 MPa. Let the pressure fall instantaneously to this lower value. Therefore, now my chamber pressure which is pc is 4.5 MPa.

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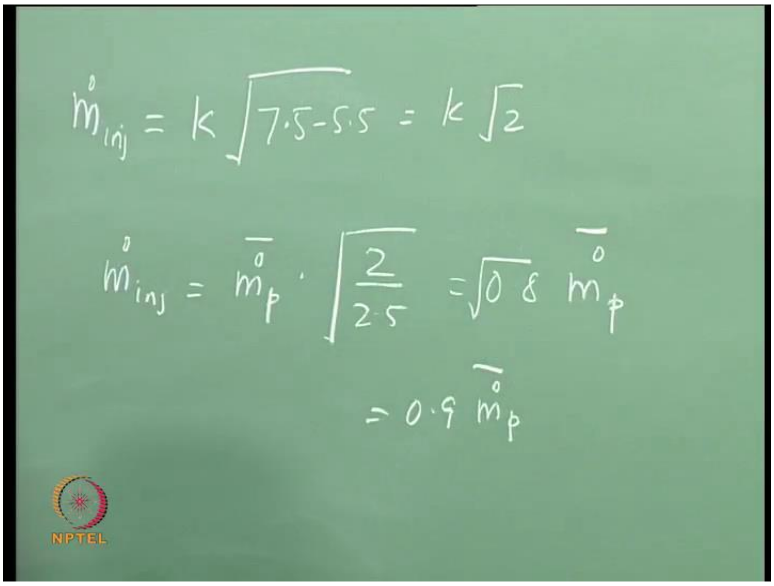

If it is 4.5 MPa, the mass which is injected is going to be k times, now 7.5 minus 4.5 which is 3 under root 3. Therefore, if I were to put it in terms of the steady nominal value of mp dot which was the steady value, it is going to be mp dot into under root 3 divided by 2.5, which is equal to something like 1.2 times under root mp dot, which is equal to 1.1 times the value of m p dot which is the nominal value at steady state conditions.

That means, all of a sudden I am injecting little more which is now 1.1 times m dot p over here. See, the pressure has fallen, because of that the quantity which is injected has now gone up by 1.1 times. It takes a certain time delay and let me expand on it. It takes tc time to evaporate mix and burn together, and when it mixes and burns what is the chamber pressure I get? Originally, I had when the nominal value was m dot p, I have the chamber pressure which is given by 1 by C star into pc into A_t, and now I get a value which is 1.1 times and therefore, my chamber pressure will be equal to 1.1 times the value which was 5 MPa earlier. After this delay time of tc seconds, when the new parcel of gas burns and exhaust through the nozzle the pressure increases to 1.1 into 5 which is 5.5 MPa.

At a time tc after the chamber pressure dropped to 4.5 MPa, the increased mass flow rate through the injector which burns causes the pressure to increase to 5.5 MPa. What is now the implication? The implication is maybe at this point in time when I said the chamber pressure is 5.5, the mass which is injected is now going to be the value of k into under root, the supply pressure is still 7.5 minus 5.5 that is equal to k root 2. Therefore, if the mass injected has is now k root 2, the value of mass injected in terms of the nominal value, which was mp dot which was based on the pressure drop across the injector of 2.5 MPa is now equal to mp dot into under root 2 by 2.5 which is equal to 0.8 under root of mp dot, or rather this is equal to 0.9 of mp dot nominal. 9 into 9 equals to 81 of the old value m p dot which is the nominal value.

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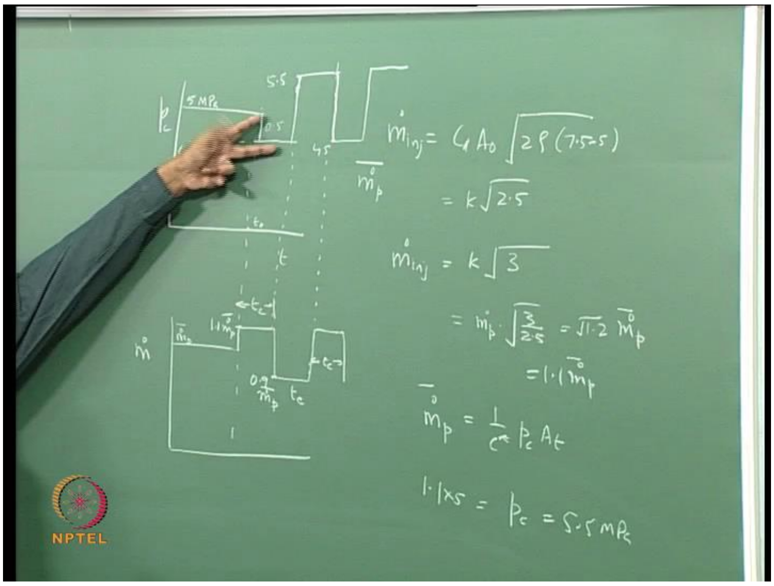

And therefore, what happens when the pressure increases? The mass flow rate now drops to a value 0.9 of the nominal value corresponding to mp dot over here. The chamber pressure remains at the higher value of 5.5 MPa till this new reduced parcel of liquid evaporates burns over a time tc. During this period of tc nothing really happens, and the chamber pressure remains at 5.5 MPa. When this reduced quantity burns over here the chamber pressure now corresponds to a mass of 0. 9 times the nominal and this would be 0.9 into 5 which is 4.5 MPa. This is from equilibrium of 0.9 mass flow rate which passes through the nozzle. The chamber pressure now drops to 4.5 again. And this sequence would continue again after a delay time it increases to 5.5 MPa, falls again to 4.5 MPa and so on. This sequence of event sin mp dot and chamber pressure is seen to be primarily due to a delay in the burning of prepellant after injection by a time tc seconds. And therefore, you find that a momentary drop of 0.5 MPa from a steady value of 5 MPa makes the chamber pressure oscillate between values between 4.5 to 5.5 MPa.

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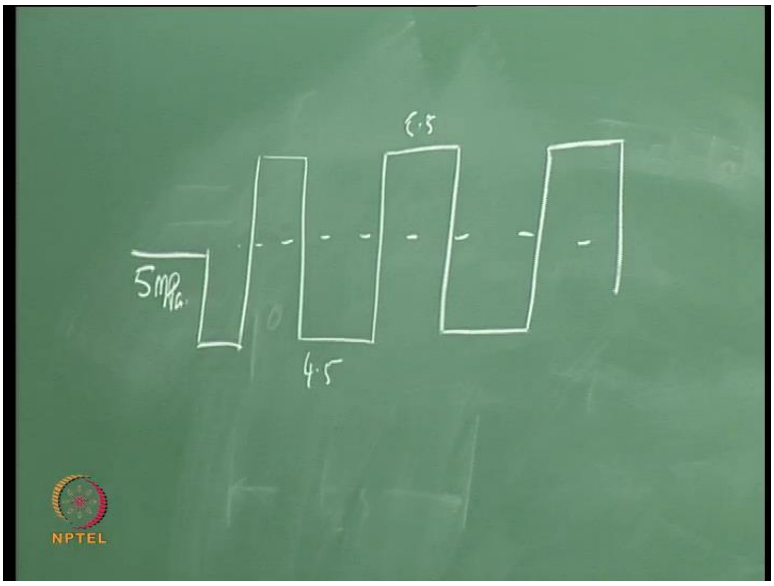

And out of a steady situation, when you have this delay term, what happened? You started getting oscillations. Well, the oscillations are neutral in that, maybe it keeps fluctuating between a value of let us say 4.5 to 5.5 whereas, the nominal value before it dropped down to 4.5 MPa was 5 MPa. Is the process clear? I think if this is clear, we would have understood some part of combustion instability. Namely, a drop or equivalently an increase in chamber pressure whenever there is a delay causes this problem of fluctuations. I will come back to it. Let me give a physical example before I proceed.

We would have seen many toys in the market; I just brought one such toy here. It is very illustrative of the phenomenon. You have these toys which are available in the market, and what you do is you just take this toy, you sway it once. It will keep on oscillating up and down. I put it on the table. I push it here it goes, it keeps on rollicking up and down, it is in a state of neutral oscillations. Why does it have to do this?

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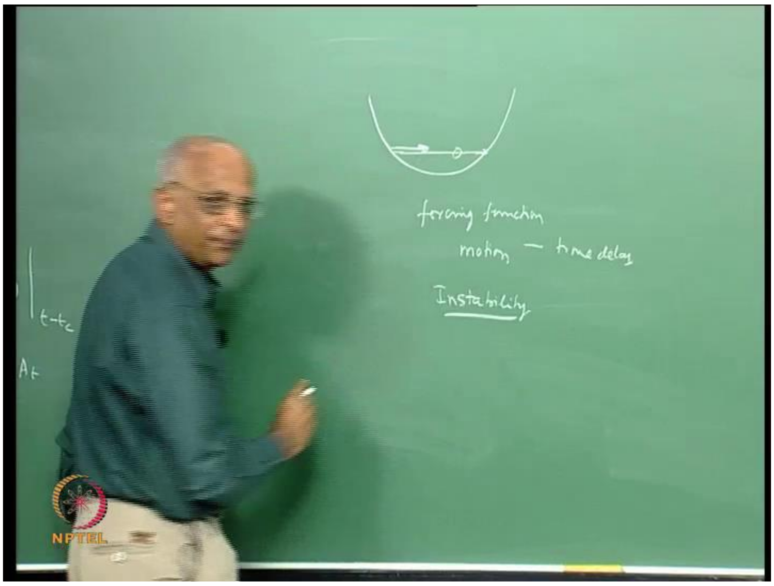

Let us take a look at the construction of this particular toy. There are various such toys. There is a small marble here inside it that is what is making this particular noise. When I shift it, when I shift it there is a sling and a marble over here, when I shift it here the marble comes and hits over here and rebounds. It goes to the opposite side causing the toy to deflect after a short time. There is a delay between the forcing function and the motion and it is this time delay which keeps this toy oscillating. Once the oscillations are started it keeps on going.

This particular toy is different from a toy at the bottom of which you have a mass placed and the momentum about the center of gravity continues the motion. But in this case there is a marble inside and it is this delayed response which keeps it going up and down, until the friction finally stops the motion. Something similar is happening when you inject propellants into the combustion chamber; there is a time delay, the time delay precedes whatever be the change that happens. This is what happens in the case of instability. Can I say this is understood? If this is understood let me go back to the next example. I just do once more example and then we can generalize it.

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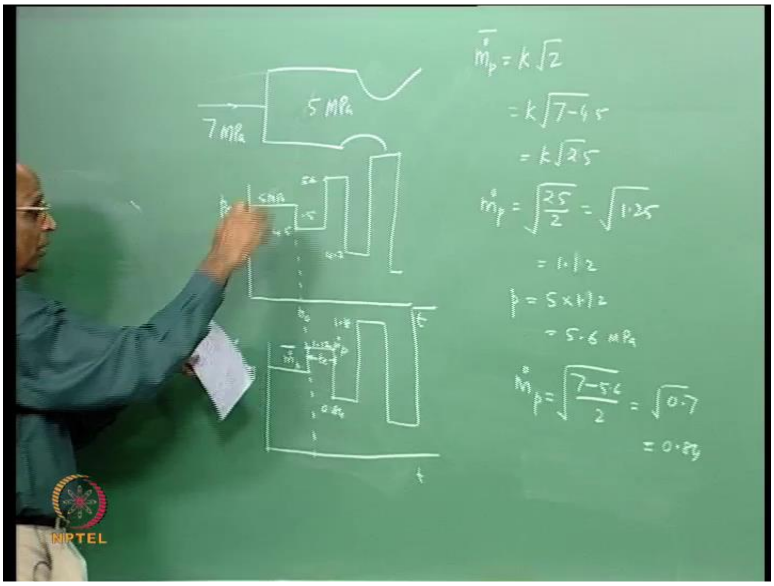

Let us now assume that now I have another chamber; another rocket chamber in which my injection pressure is 7 MPa and the steady value of chamber pressure is still the same viz., 5 MPa. The injection pressure is now reduced to 7 MPa, and we ask what is going to happen in this case? Now, I do not have to repeat much. All what I want to do is maybe make these 2 plots, the chamber pressure as a function of time, the mass which is getting injected as a function of time. The initial steady state pressure is 5 MPa, and maybe at time t_0 we reduce the chamber pressure like in the previous example to 4.5 MPa; that means, I decrease it by 0.5 to 4.5 MPa.

Now, what is going to happen in this case? Let us say corresponding to 5 MPa, I have a steady value of mass injected corresponding to mp dot. This value is k under root of 7 minus 5 viz., k root 2.

Now, I have reduced it the chamber pressure from 5 to 4.5 MPa. Now mp dot would be different from the value under steady conditions that equals k root 2. When the chamber pressure has got reduced to 4.5 MPa, the mass injected becomes k into 7 minus 4.5, that is equal to k into under root 1.25, no 2.5. Or rather the new value of mp dot is going to be under root of 2.5 by 2 which is original, which is equal to under root of 1.25 and this under root of 1.25, will be something like 1.12. Therefore, now I find that the at this point t_0 it increases to 1.12 of the value of mp dot. And what is the repercussion?

Well, it is going to burn after some tc time, and once it burns the pressure increases from the steady value of 5 MPa by the corresponding mass ratio. This value will be 5 into 1.12 which is 5.6 MPa. Therefore, the pressure now goes up to 5.6 MPa. When the chamber pressure is 5.6 MPa well the mass flow rate injected now decreases and it becomes equal to with respect to the steady difference of 2 MPa now it is going to be 7 minus 5.6 equal to 1.4 times the value of a constant k. Expressed as a fraction of the steady injection flow rate it is equal to under root of 1.4 divided by 2 viz., root of 0.7. Root 0.7 is 0.84. The flow continues at this rate until after a time tc when this parcel burns and the pressure in the chamber reaches a value 5 into 0.84, which is 4.2 MPa. The injection pressure drop now increases to 7 minus 4.2 viz., 2.8 MPa. The increase in pressure drop causes more flow compared to the nominal chamber pressure of 5 bar and this after the combustion time of tc seconds results in a further increase of chamber pressure.

The magnitude of the step from 0.5 bar keeps increasing in this case. The oscillations diverge and kee on increasing. The oscillations need not be neutral such as it was for an injection pressure of 7.5 MPa. With a reduction of injection pressure to 7 MPa the oscillations diverge.

I now do a third problem in which I choose the injection pressure as 9 MPa, and the chamber pressure is kept the same at 5 MPa.

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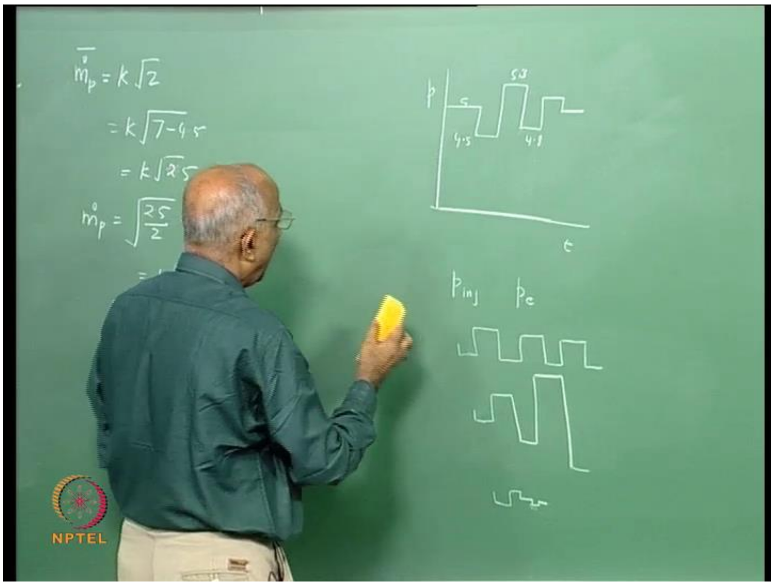

And what do I get for this pressure? I again start with 5 MPa chamber pressure; the chamber pressure drops from 5 to 4.5 MPa, then as time progresses the steps in the amount of 0.5 MPa decrease as time progresses. The pressure with respect to time follows a decreasing trend.

And let me put the numbers because it is important to understand. From the value of 4.5 MPa, the chamber pressure rose 5.3 MPa, the value comes back to 4.8 MPa and so on. It will be something like 5.1MPa next. Therefore, what we have just done is that we have taken a simple case wherein, we varied the injection pressure keeping the chamber pressure constant, and we found after a step drop in pressure the oscillations are so evolving such that it keeps on oscillating in a limit cycle mode with the same amplitude or else the amplitude would diverge and ultimately explode or would the amplitude would decay as a stable system. Either, it could be neutral, it could diverge or increase in amplitude or be a decaying oscillations as a stable system. It is possible to get all these oscillations in the rocket under some condition or the other by variation of the injection pressure and chamber pressure.

And the example of the toy, which I said is very similar to the chamber, kept on oscillating. Now, let us try to understand a little bit more, because I think it should be possible for us to write an equation, by varying the chamber pressure and the injection pressure; and what is the equation?

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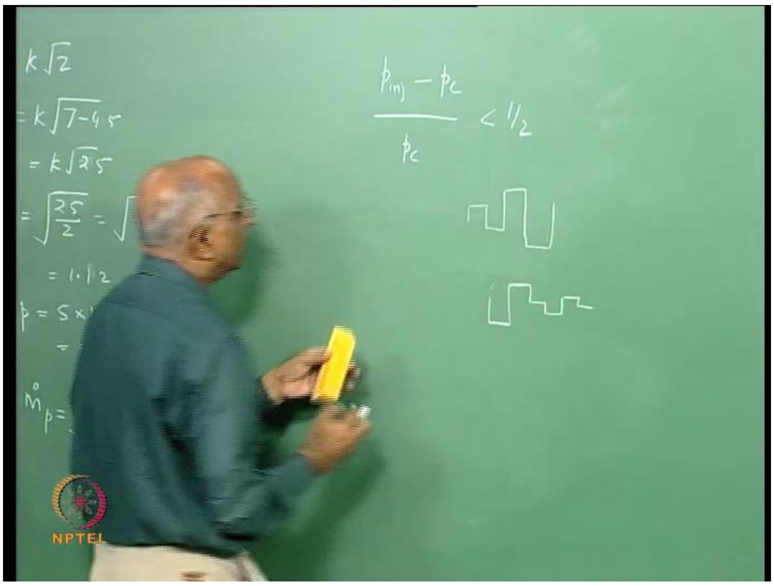

When I have p injected minus the value of chamber pressure divided by pc, if it is equal to half I get neutral oscillations provided I do a lot of trials with different values of pc and injected pressure like we did in the three trials. If it is going to be less than half, I get diverging oscillations, and if it is greater or if it is than half then I get a stable system, wherein the amplitude of oscillations decay.

It is possible to do it numerically, let us try to solve the basic conservation equation. And get the same result. After all, we know how to solve a differential equation; let us try to solve this equation. But, as I told you solving is not going to be very straight forward. I have mass that is injected tc earlier that leaves the nozzle. How do I do this? Now, we must recognize one or two small things as a precedent before I solve this equation.

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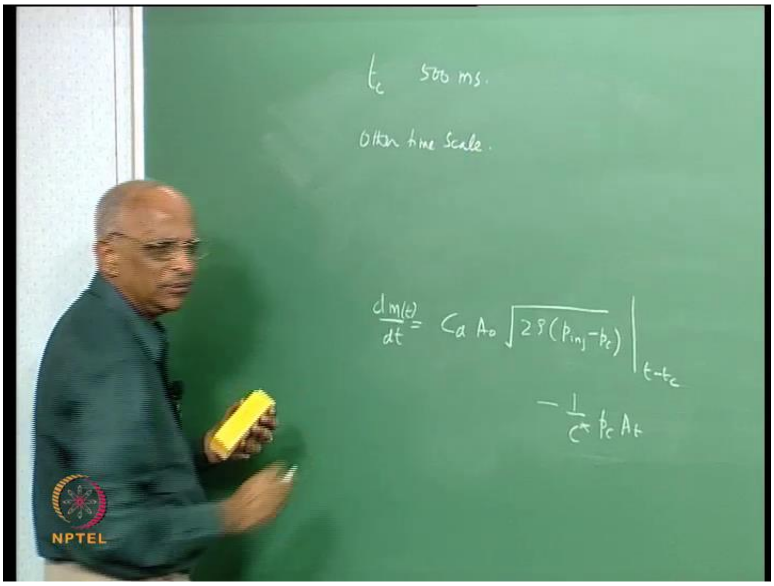

See, when I say combustion time delay tc, let us say it is equal to 500 milliseconds, I just choose a number a large number. See, in a large rocket 500 millisecond may have a different impact than in a small rocket; therefore, I must say I must compare the delay time or the time taken for reactions to occur with respect to something else. What is that other time scale which I should have in mind? Like for instance, let us consider what are the other times which could probably enter into our system or into our computations.

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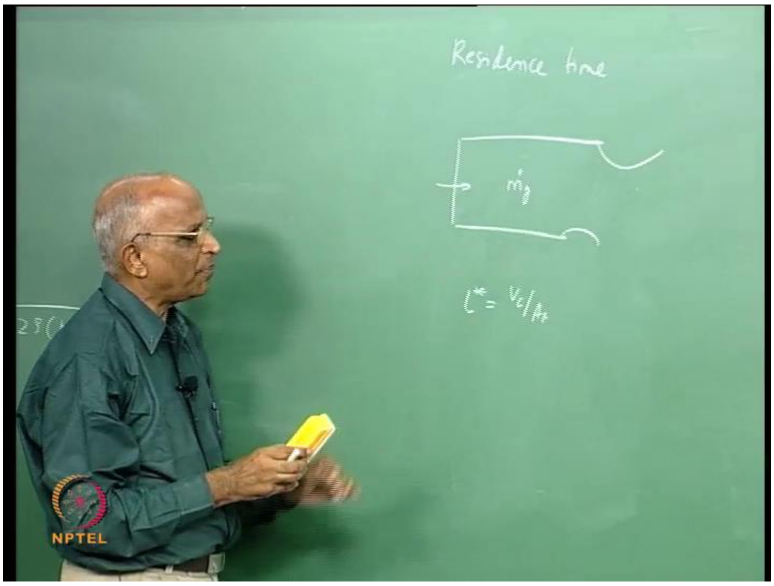

It could be the residence time, that is the time over which a propellant stays in the chamber; propellant vapor stays in the chamber. What is the time which is available for the propellant to stay in the chamber? Maybe, I inject something here, I form gases here m dot g. The m dot g stays in the chamber for sometime before it is evicted. We said maybe L star which is equal to Vc by At gives us an idea of the time over which a propellant stays in the chamber. If the length is large, the gases are going to stay for a longer time, maybe the chamber will require a large value of tc before it becomes unstable. Why are we considering these durations? Maybe, I should go back to that problem again and ask you one more question. If tc was 0, would there be oscillations?

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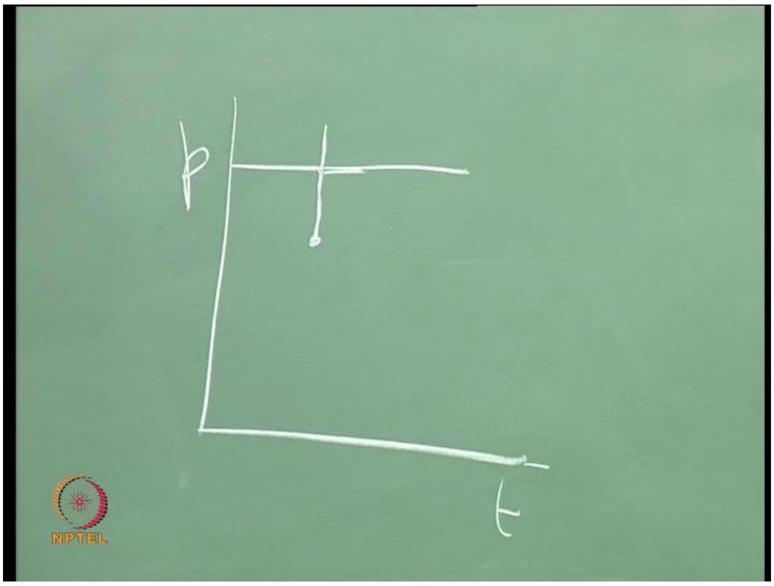

If tc is equal to 0 what would happen. The chamber pressure with respect to time, originally the pressure was 5 MPa it drops to 4.5 MPa. But there is no time delay for burning the excess fuel admitted. Therefore, immediately it burns and immediately it goes back here and adjusts to the chamber pressure of 5 MPa. Therefore, without a time delay I cannot think in terms of these oscillations, and what is the role of this time delay? Well, I compare the time delay with respect to some other time here? Because, if I have a very large chamber, maybe my time delay which is small compared to the residence time, does not have much impact on the burning and subsequent events.

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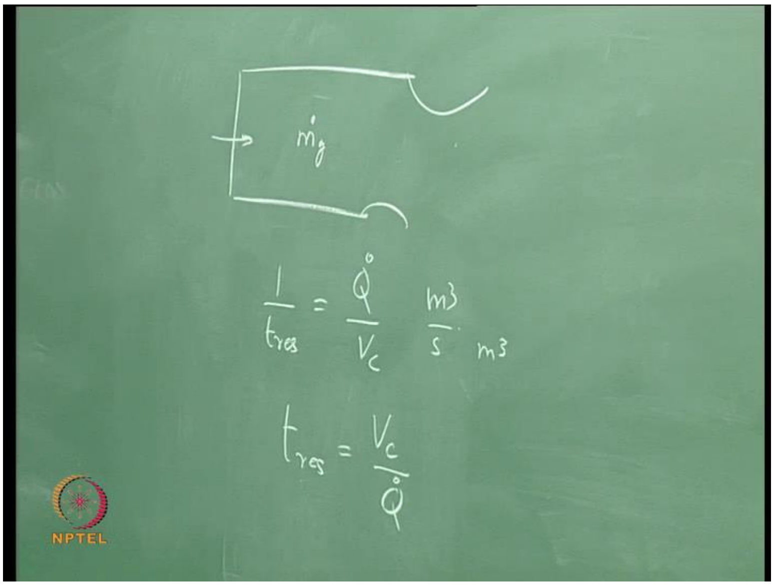

Therefore, I define something known as residence time, which is the time over which the propellant stays in the chamber, and how do I define residence time? Maybe, the rate at which gases are generated in the chamber divided by volume of chamber, if I now put it down it is meter cube by second divided by meter cube no, it must be 1 over t residence right? Or rather, I now write t residence is equal to Vc by Q dot, volume of the chamber divided by the rate at which volume is getting generated in the chamber.

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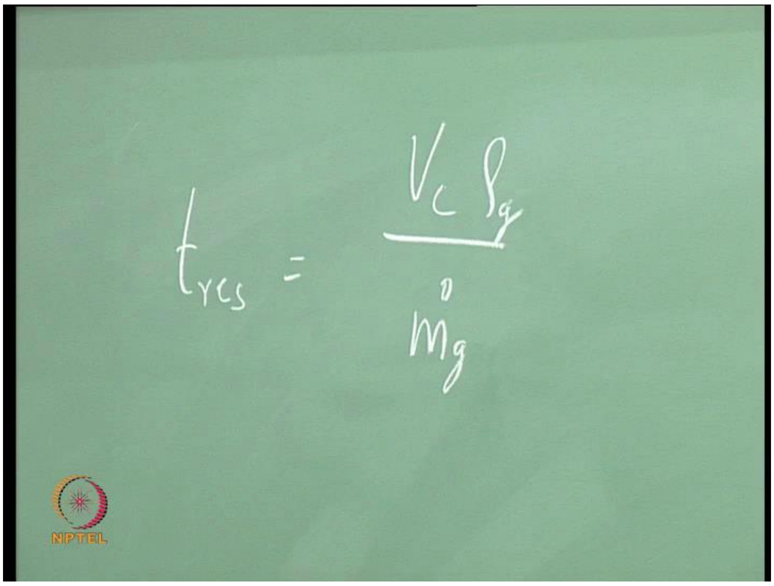

Now, let us put it as t residence equals volume of the chamber, the volume of the chamber I mean this is the volume over here divided by the rate of mass being generated in the chamber divided by the density of these gases.

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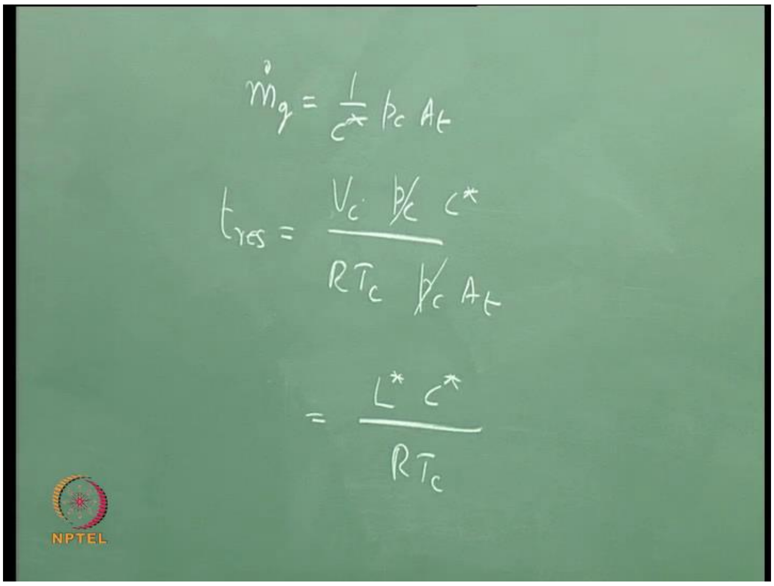

Now, can I write an expression for m dot g what is generated? Whatever is generated goes out to the nozzle under steady state conditions and equals 1 over C star into pc into At. Therefore, I can write t residence as equal to Vc, what is rho g? PV is equal to m R T, m by V is density therefore, rho is equal to pc by RTc. C star comes upstairs so here, pc At in the denominator. And therefore, now I can cancel the pc out, it is a not a function of chamber pressure anymore and I get Vc by At we have defined as L star. We therefore have residence time as L star into C star divided by R Tc. And do you remember, what is R Tc equal to in terms of C star? We have done it earlier.

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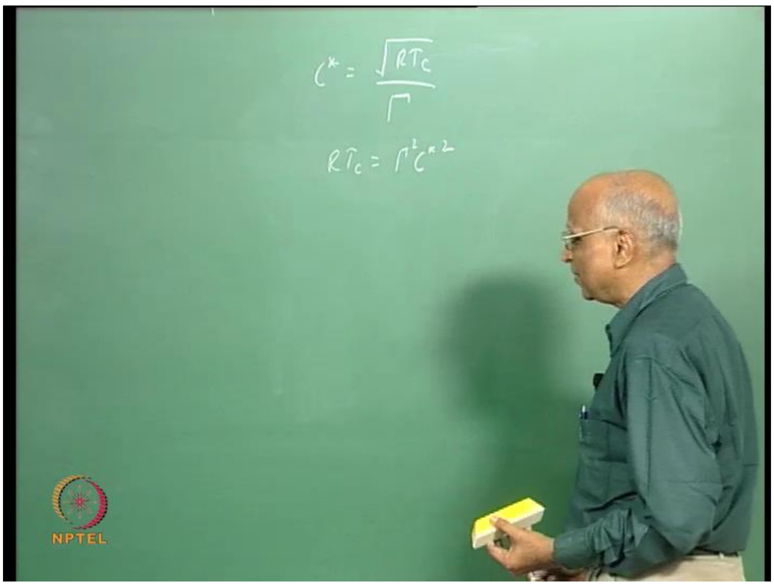

We said C star is equal to under root R Tc by capital gamma therefore, we get R Tc is equal to capital gamma squared into C star square, substitute the value of R Tc here.

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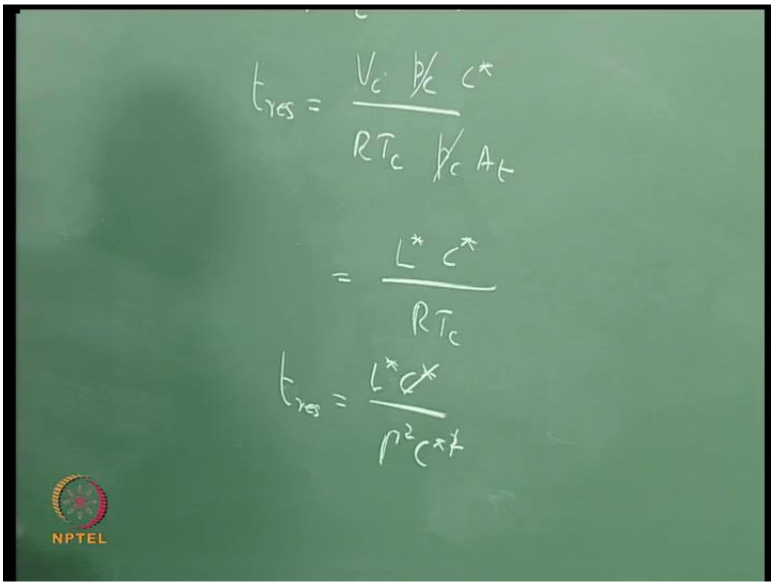

We get t residence is therefore, equal to L star C star by capital gamma squared C star squared. The square in C star goes off and we get:

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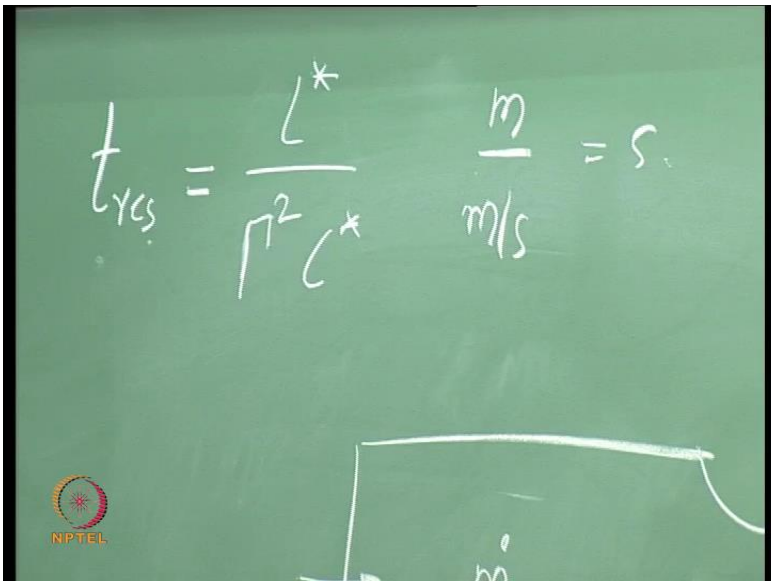

And the value of t residence is therefore equal to L star by capital gamma square, you know capital gamma is a function of small gamma into C star. Well, it makes sense, L star has units of meter, capital gamma has no units, C star has units of meter per second, this is second and this is your value of residence time. You are able to get the residence time. And now I will say my tc that is the combustion delay, should have something to do this residence time. I would like to bring in the residence time and combustion time into this equation. Combustion time is already there, only thing I do not know how to solve the equation. We will first try to bring in the value of residence time into the equation.

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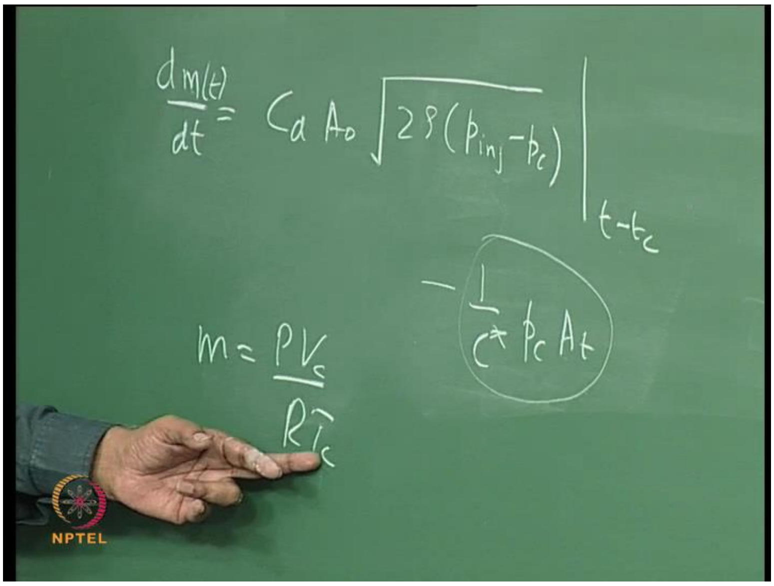

And therefore, let me first take a look at this particular term which is the mass which is leaving through the nozzle. Before that let us try to find out what is the variation in chamber pressure; we say m is equal to P V by R T, we keep doing the same thing again. P Vc by R Tc is a mass in the chamber, we assume Tc hardly varies and take it as a constant because small changes in pressure or oscillation is not going to change this temperature. Let us simplify this equation in terms of dpc by dt.

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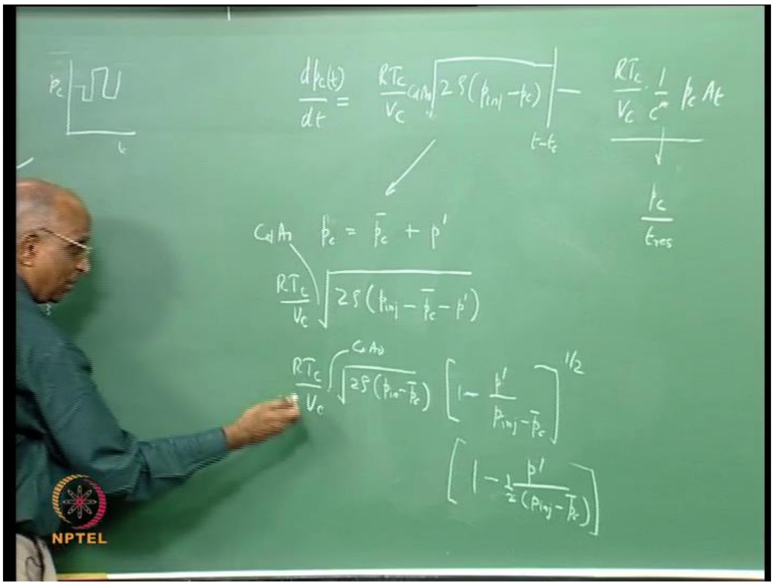

We get dp in the chamber divided by dt is equal to R Tc by Vc into under root 2 into density into p injected minus pc minus, I write the value again R Tc by Vc into one over C star into pc At. This should be multiplied by Cd into the area of orifices A_0: Cd A_0. Essentially, what is it we are looking out? We are looking at the variation of pc with time, we are trying to see whatever we studied just now with the numerical examples, can we predict the changes using this equation? We found under some conditions it goes as a limit cycle oscillation, sometimes it diverges and sometimes decays with respect to time. But we note something here; we must not forget that the mass which is injected t minus tc earlier leaves the nozzle at time t.

Therefore, this is as a function of time therefore, pc is also at time t. Let us simplify this part of the equation. Let us express in terms of residence time. Precisely, what did I write? R Tc was equal to gamma squared C star squared; Vc by At is L star into I still get a value of Cstar into pc over here and therefore, the second expression over here I can write it as equal to pc into L star divided by gamma square C star and this is equal to residence time and therefore, the last part of this gives me a value pc by t residence. I still need to solve this but it does not look that straightforward. Let us see how I can manipulate this equation in some way, and this is quite interesting and simple.

Well, let me say the chamber pressure at any instant of time is denoted by pc which is equal to the steady value. In our example the steady value was 5 MPa plus I have p prime some oscillation, some disturbance in it. Therefore, I can write this expression now as R Tc by Vc into under root of 2 rho into p injected minus the value of pc, that is the steady value minus the value of p prime, or rather I take the steady value outside I take it as R Tc by Vc into the steady value that is 2 rho into p injected minus the value of pc bar over here, and then I write the balance as 1 minus p prime divided by p injected minus pc bar, I have to divide it over here p injected minus pc bar to the power half. All what I have done is I have taken this outside and therefore, I have p bar minus this term over here.

I also know that the magnitude of p prime may not be very large and therefore, p prime divided by the value of p injected minus p bar maybe a small number and therefore, I can write this part of the expression as 1 minus p prime by p injected minus pc bar by 2 of p injected; that is the injection pressure minus the value of pc bar over here. And what is the value I get over here?

Let us see R Tc by Vc, we have just now seen what it is. And what is the value of this? 2 rho into pc somewhere we dropped the value of Cd, where was Cd? You should have got mass which is Cd A_0 here; that means, I should have had here Cd A_0, why did we drop this? We should have carried it forward, and if I take the value of Cd A_0 into 2 rho is the regular mass which is leaving through the nozzle because and these are under steady state conditions.

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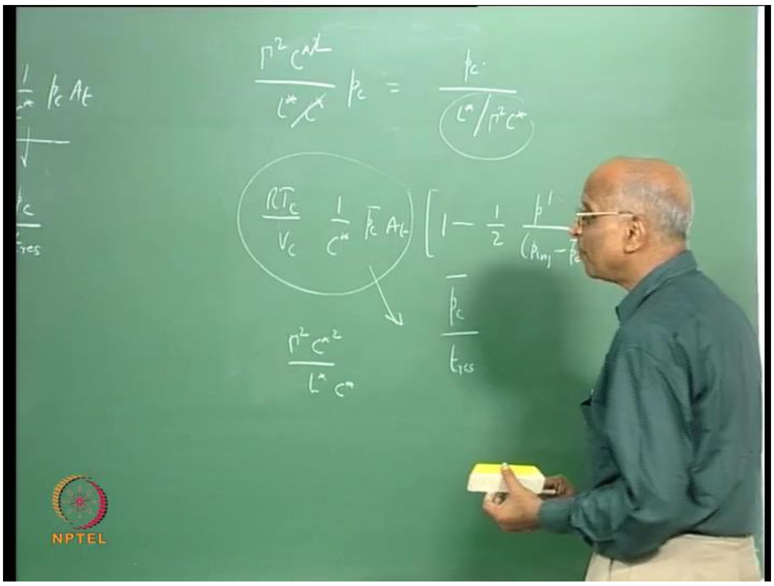

What is it that we now g?et I get R Tc by Vc into Cd A_0 into this is the value which is steadily going, that is one over C star into pc bar into At steady value of pressure. Please be clear about it. I write the same equation here one minus the value of half of p prime divided by the value of p injected minus the value p c bar.

You have a steady value of what is getting injected, and under steady conditions whatever is getting injected is leaving through the nozzle; that means 1 over C star into p into At. We take R Tc by Vc which was a coefficient of 1 over C star into pc At, and if I were to look at this value and again substitute R Tc as equal to gamma square into C star square, and the value of Vc by At which is L star over here, I get the value of C star over here, and what is it that we get? C star L star by gamma square C that is one over residence time and therefore, this expression now becomes pc bar by the value of t residence and therefore, what becomes of my equation?

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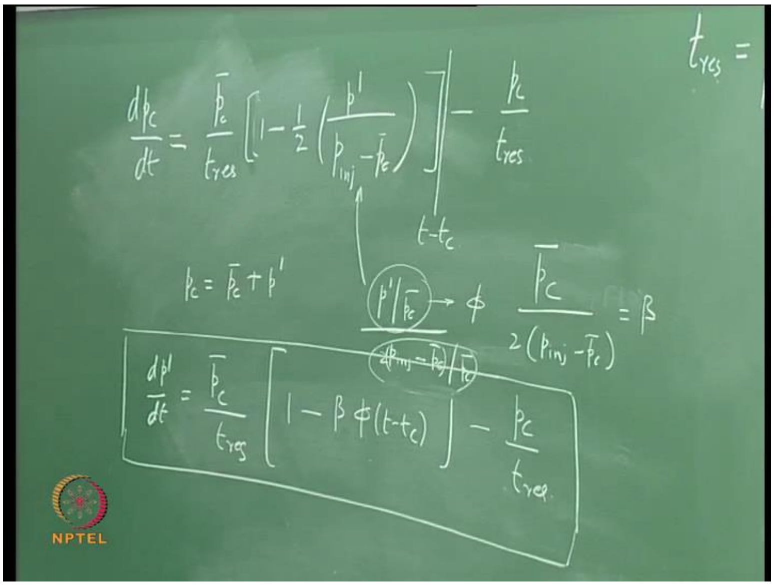

The equation gets terribly simplified, and now I am able to write this equation as dp c by dt as equal to, let us put that down I get the value of p bar by t residence into 1 minus half of I get the value over here as the pressure perturbation divided by this 1 minus half into pressure perturbation divided by the pressure of injection minus the steady value over here minus the value corresponding to the last term, which we call it as pc by t residence.

Therefore, you see I am able to simplify the equation quite a bit now. I have to do to solve this equation. To solve this equation I need to get some way of relating the value of tc, but mind you something which we forgot. And what we forgot was this is at time t minus tc. I need to make some changes in order to understand this and therefore, now I say we have already told well, pc at any time is equal to pc bar plus the value of p prime. And therefore, if I were to change; pc bar does not change, I can write this equation as dp prime by dt as equal to the value pc that is chamber pressure steady divided by t residence into I get the value 1 minus. If I were to make a manipulation, and divide by numerator by pc bar and denominator by pc bar, I can write this thing within brackets as maybe p prime by pc bar divided by p injected minus pc bar divided by p c bar, and this is the magnitude of the perturbation above the mean. I denote it by something like p prime by pc bar.

Let us denote it by phi, and this value which again has a value 2 at the bottom, I say the value of pc divided by 2 of p injected nominal value, this is what I divided p injected minus p c bar I denote by the value beta. I can write this equation as 1 minus the value of beta which is just the steady value, phi denotes the perturbation and I say it is phi of t minus tc is what I get here minus the value of pc divided by t residence, and this is my final form of this equation. I drop the half because I brought half here. I just need to solve this equation to be able to get the value of how the pressure changes with time.

I will continue with solving the equation in the next class. What did we do in this class?

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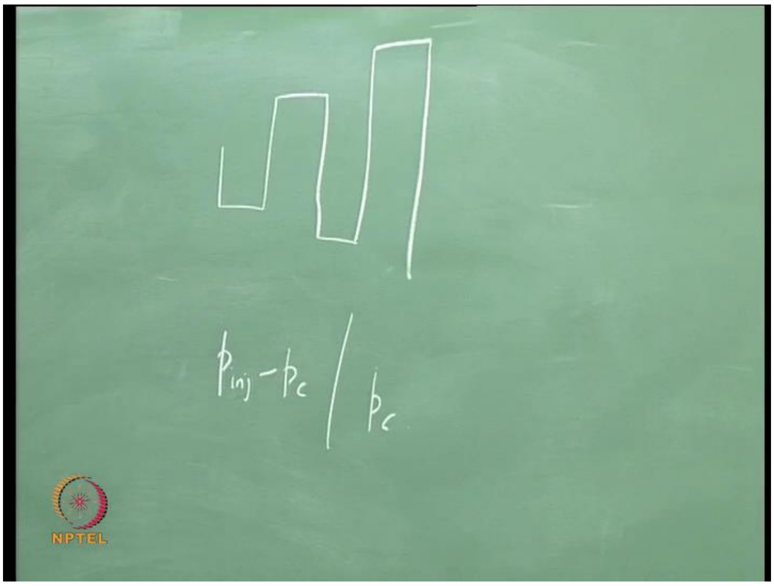

We found that after the liquid propellant gets injected, and it takes a certain time delay to burn and generate the hot gases, which are subsequently exhausted through the nozzle. Because of the time delay, any change in pressure could get magnified depending on the values of p injected minus the value of pc with respect to pc, and that is what we are trying to do through this particular equation. We will illustrate the solution of this equation, and then move forward in the next class.