Good morning. We will continue with the portion on nozzle. We will find out what is the effect of area ratio and define area ratio. We will also find out how the nozzle operates at different attitudes and look at some typical results. But, before getting into it let us quickly recap where we so that we can connect it with what we are going do today.

We said in the last class that if we need to have a high jet velocity; we need to have a convergent, it should have a throat for which the Mach number is equal to one and then we should have a divergent. We were very clear about the throat, we said it is the place where the velocity is sonic that means the throat velocity is the sonic velocity may be \( V_t \) is equal to \( a_t \).

We also found that any disturbance generated downstream of the throat; suppose I stand on nozzle here and make a loud noise or I make some disturbance this disturbance cannot enter the convergent and therefore, the chamber is isolated. The reason is that the velocity at the throat is equal to the velocity of sound and is greater than the sound...
velocity in the divergent and any disturbance, which generates downstream cannot travel upstream. This is because the disturbances travel at the sound speed. Any disturbance downstream of the throat cannot enter the chamber.

And therefore, the sonic throat essentially decouples the convergent and the chamber from the downstream portion this is because a disturbances travel at the sound speed. Second thing we also told was that the throat is choked. In other words, we told that for the given mass flow rate I can have a maximum velocity, which corresponds to sound speed at the throat and if I want to if I have higher pressure or if the gases are sucked it at lower pressure I cannot exceed this condition of sonic velocity. This means that the throat always will have Mach number equal to 1 or the velocity here should be the sound speed. I think these findings are important.

We also derived an expression for the jet velocity $V_j$ at the exit, which we found $V_j$ as equal to $2 \gamma / (\gamma - 1)$, we will write in terms of the specific gas constant $R$ of the gas into the temperature within this chamber into we had something like the expansion $1 - \frac{P_e}{P_c}$ that is the pressure at the exit over here divided by the chamber pressure here i.e., $P_e / P_c$ to the power $\gamma - 1$ by $\gamma$ and this was under root. We did not consider the convergent divergent shape; we just said that if the chamber pressure is $P_c$ and if the exit pressure is $P_e$ then you have the pressure ratio alone which is important. The temperature in the chamber was $T_c$ and this is how we derived the expression for $V_j$ so many meters per second. Is it alright?

And what we started was with a vent, we derived the velocity and then looked at the shape of the vent; it was necessarily for us to have a convergent followed by the divergent such that if I were to plot it we have a convergent divergent nozzle.
The velocity of the gases as a function of the length of the nozzle mind you increases along the length of the nozzle. Initially I have a converging shape then I have a throat then I have the diverging shape. I get the sonic velocity at the throat and therefore, the velocity will keep increasing along the nozzle in the divergent.

The moment I have at the throat the velocity less than sonic velocity, the velocity builds up to the throat and thereafter the velocity drops. Therefore the necessity to have Mach number one at the throat was essential, right. I think this we must remember. Having said that, in today’s class we will try to see instead of mentioning that the exit pressure is Pe can I put it in terms of the area ratio and area at the exit Ae. What must be the area Ae such that the nozzle will give me the required velocity and that is I want to do today.

Let me repeat again; see when I realize a hardware I do not know the value of Pe; all what I know is I must have a configuration like this. I must have a diameter over here, I must also have the diameter at the exit or rather the exit area ratio. Therefore it becomes essential for me to define something like area ratio of a nozzle to be able to give me the value Pe such that I can get the jet velocity or rather I want to know the configuration of a nozzle which will give me the required velocity.
Let us put it this way: I have convergent, I have the throat, I have the divergent. I want to know what will be my exit area here such that I can get the $V_j$ whatever I want, I want to find out the expression for the jet velocity in terms of a diameter or an area ratio rather than put it in terms of the value of the pressure at the exit, which we called as $P_e$. To be able to do that we are looking at the exit area, I would like to define the area at the throat, because I know that at the throat the Mach number is always equal to 1, therefore I can define it as a critical or an unique particular area for reference and I call the nozzle area ratio as equal to the exit area divided by the area at the throat.

Well. If I have to define something like an area ratio maybe I should think in terms of the area of the chamber over here and relate it to the area at the exit. But whatever be the area here the reference is the throat because that is where the velocity is always equal to the sound speed or Mach number is one. And it is related to the area of the chamber. The gas accelerates from Mach one at the throat in the divergent. Therefore we define the nozzle area ratio as the exit area divided by the throat area and it is denoted by Epsilon and epsilon is equal to $A_e$ by $A_t$. Is it okay? Having said that area ratio of a nozzle is $A_e$ by $A_t$, I want to derive an expression how the area ratio will affect the jet velocity or how should the performance of a nozzle be linked to the exit area ratio.

Let me repeat the problem such that it becomes further clear. Supposing, I have a small rocket, I have the throat Mach number as one. I could also have a small area ratio or a large value of area ratio. I could also have a same rocket in which now I again draw this
nozzle over here, I could have a very large area ratio and how do I find out and compare the performance of an area ratio, which is $Ae_1$ with area ratio $Ae_2$ and if the throat area is the same in the two cases; I have area ratio in one case which is equal to $Ae1$ by at $At$. In the second case, I have area ratio is equal to $Ae2$ by $At$. I want to compare which one gives me higher velocity, I want to compare these two nozzles and therefore, we define area ratio as exit area divided by the throat area.

I could also have had a larger chamber something like this with much larger chamber, but still even for a larger chamber the throat area would describe the same flow condition namely $V_t$ is equal to the sound velocity $a$. The area ratio would be $Ae/At$. Area ratio is always defined with respect to the throat that is exit area divided by the throat area.

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I want to determine $V_J$ and therefore the ratio of $Pe/Pt$ and therefore we are interested in finding its dependence on the value of $Ae$ by $At$ for nozzle. I just look at the continuity of flow; I look at the mass which is coming over here, mass which is passing through the throat, mass which is passing through the exit and I write $m$ dot is equal to the mass which is passing through throat area into rho $t$ into the velocity at the throat is equal to the area at the exit into rho at the exit into $V$ at the exit is it.
Therefore, we find that I need the condition at the throat namely the rho \( t \) at the throat. I need to be able to find out rho \( e \). I do not know Ve; but Ve is the velocity with which the gas is it exiting the nozzle. It will be equal to the \( V_J \), which I have already derived. Therefore, I need to find the conditions at the throat. Therefore, let us first spend a couple of minutes on deriving the expression for the conditions at the throat, which are so critical to a nozzle.

The condition at the throat will specify the mass flow rate, because it is choked here and I will clarify this later on. Let us first find out what are the density, pressure and temperature at the throat.

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We define the density at the throat as rho \( t \), velocity by \( Vt \), pressure at the throat as \( Pt \), and temperature at the throat is \( Tt \). This means that subscript \( t \) denotes the throat condition. And how did we define the chamber conditions? The density is rho \( C \), velocity in the chamber \( Vc \), which we said was equal to 0 pressure in the chamber \( Pc \) and temperature in the chamber \( Tc \). At the exit rho \( e \), exit velocity \( Ve \), which is equal to the jet velocity at the exit, \( Pe \) is the pressure at the exit; well these are all the variables what we have.

I want us to find out the value of rho \( t \) as a function of rho \( c \) may be \( Pt \) as a function of \( Pc \) and \( Tt \) as a function of \( Tc \). Therefore, we again just look at the flow conditions. We are interested in the condition at the throat over here, the conditions are given by subscript \( t \) over here for rho and \( V \). We treat this as a control volume or rather now we
are considering our attention is only in this small region, which I show hatched over here, gas enters at a pressure $P_c$ at velocity 0 at a temperature $T_c$ at it leaves at the throat with a condition of $\rho t$ at a velocity equal to the sound velocity. The pressure is $P_t$ and the temperature is $T_t$.

Let us write the expression for this control volume. Let us again assume adiabatic condition and therefore we can write the enthalpy entering is $hc$ plus kinetic energy 0 is equal to $h$ at the throat plus, I have $V_t$ square divided by 2 which is kinetic energy per unit mass. We must be able to write this is the steady flow energy equation; same mass flow is here, enthalpy in the chamber corresponding to this initial kinetic energy of 0 while at the throat the enthalpy is $h_t$ and $V_t$ square by 2 is the kinetic energy where or $V_t$ is the velocity at the throat giving the kinetic energy per unit mass at the throat as $V_t$ square by 2.

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Now let us simplify this equation. We get from this equation $V_t$ square divided by 2 is equal to $hc$ minus $h_t$. And what is the difference in enthalpy is equal to: $C_p$ into $T_c$ minus $T_t$. Therefore what is $T_c$ minus $T_t$. It is equal to $V_t$ square divided by 2 $C_p$. What is the value of $C_p$ in terms of gamma: 2 gamma divided by gamma minus 1 into $R$. How did this come about? We derived in the earlier class $C_p$ minus $C_v$ is $R$, $C_p$ by $C_v$ is gamma and therefore, $C_p$ is equal to gamma by gamma minus 1 into $R$ and therefore, I can write this expression as equal to gamma minus 1 divided by 2 into $V_t$ square divided by gamma $R$.
And therefore, I can now write the value of $T_c$: I take $T_t$ on the other side to give $T_t$ into what I get now $1 + \gamma - 1$ divided by $2$ into $V_t$ square divided by $\gamma R$ into $T_t$. What did I do? I have taken $T_t$ at the bottom and therefore, I have $\gamma - 1$ into $V_t$ square by $\gamma R$ into $T_c$ into $2$. We know that $\gamma R$ into $T$ is the sound speed or $\gamma R$ into $T_t$ is a sound speed at the throat. $V_t$ is also equal to the sound speed at the throat and therefore this is the Mach number square and I get $1 + \gamma - 1$ by $2$ into Mach number square. Mach number is one and therefore I get the value of $T_t$ as equal to $1 + \gamma - 1$ divided by $2$ or this gives me the value of the temperature at the throat as a function of the chamber temperature $T_c$. $T_c$ by $T_t$ is equal to $\gamma + 1$ by $2$, I take it the other side to get $T_t$ by $T_c$ as equal to $2$ divided by $\gamma + 1$.

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Therefore, for $\gamma$ of 1.4, we find that the temperature at the throat is approximately $1$ over $1.2$ times that in the chamber or rather if the chamber temperature is something like 2000 then it will be something like 1600 degrees; in other words, if $\gamma$ is equal to 1.4 the value of $T_t$ by $T_c$ is equal to $1$ over $1.2$ or depending on the temperature here if the chamber temperature is 2000 the value at throat is equal to 2000 divided by $1.2$, which is equal to may be like 1650. Therefore, the temperature falls at the throat and it is less than the value in the chamber.
Now what will be the value of Pt by Pc? We have derived the expression in the last but one class, what did we tell it is equal to Tt by Tc to the power gamma by gamma minus 1. Please look back in your notes. Let us see how we got this value. We had p by rho to the power gamma is a constant for an isentropic flow and P by rho into T is a constant for an ideal gas. Solving for this we got this particular expressions. In fact, you will remember in the expression for Vt, we had the expression 1 minus Pe by Pc to the power gamma minus 1 by gamma and how did it come, this was essentially Te by Tc and we expressed it in terms of the pressure ratio. We therefore have Pt by Pc as 2 over gamma plus 1 to the power gamma by gamma minus 1.

What is the value of rho t by rho c? I show this here as equal to 2 over gamma plus 1 to the power 1 over gamma minus 1. And what did we have for Pt by Pc: 2 over gamma plus 1 to the power gamma divided gamma minus 1.

We have derived the conditions at the throat namely the value of the temperature at the throat the value of pressure at the throat the value of density at the throat as a function of the conditions in the chamber pressure and which is known to us. The chamber conditions are given to us.

I want us to go back and apply these three relations since we know the density at the throat may be I have to find out the density at the exit. And then find out the value of the exit area ratio as a function of the exit pressure or exit pressure as a function of the area ratio that I am interested in.
Therefore, I hope by now we know how to evaluate the conditions at the throat of a nozzle. If I have the throat conditions from a chamber pressure $P_c$ and temperature $T_c$ density $\rho_c$ I know how to find out the conditions of $P_t \ T_t$ and $\rho_t$.

Now, let us go back and solve the continuity equation namely we said area at the throat velocity at the throat into the density at the throat is equal to area at the exit velocity at the exit rho at the exit or rather from this I get the area ratio $A_e$ by $A_t$ is equal to $V_t$ into $\rho_t$ divided by $V_e$ into $\rho_e$.

I want to substitute the values, I know the value $\rho_t$ in terms of $\rho_c$; therefore, I say this is equal to $2$ over gamma plus $1$ to the power $1$ over gamma minus $1$; I take this particular expression $\rho_t$ by $\rho_c$ is equal to $2$ over gamma plus $1$ to the power $1$ over gamma minus $1$. Now we have the value of $\rho_t$ into $\rho_c$ into $V_t$. We know that $V_e$ is equal to $V_J$. What is the value of $V_J$? $2$ gamma, gamma minus $1$ $R \ T_c$ into $1$ minus $P_e$ by $P_c$ to the power gamma minus $1$ by gamma under root.

Now, we would like to somehow get rid of $\rho_e$ also $V_t$, $V_t$ I can write in terms of the sound speed and this is equal to $a_t$ and therefore here I can write it as equal to under root gamma into specific gas constant $R$ into temperature in the throat.

Please be let us be careful since these are all simple algebraic expressions and we are substituting one into the other and in the process we are also learning how the properties are varying.
Let solve the equation for the area ratio epsilon: It is given by $\frac{2}{\gamma + 1}$ to the power $\frac{1}{\gamma - 1}$ into $\rho c$; $\rho c$ can be written as $Pc$ divided by $RTc$ from the ideal gas equation $P$ is equal to $\rho R T$ giving $\rho$ is equal to $P$ by $R T$.

And now, I have another value $\gamma R$, now let us strike of some of the numbers here, here I have under root $\gamma$ under root $R$ under root $\gamma$ under root $R$; therefore, now I get a value, which in the denominator would be two. We will take $Pc$ outside $\frac{2}{\gamma - 1}$ into $1$ minus $Pe$ by $Pc$ to the power $\gamma - 1$ by $\gamma$ under root. Now, I have under root $Tt$ here I have under root $Tc$ here. It becomes is it $Tt c$, $2$ over $\gamma$ plus $1$, $P$ by $R T c$ into under root $Tt$ on top $\gamma R$ get canceled; therefore, I have $2$ by $\gamma$ minus $1$ into this term $Tc$.

We now have an expression for area ratio as given by the above expression. We would like to simplify the expression by expressing it as ratio of pressures so that we can write it as a function of the $Pc$ and $Pe$ alone. For this purpose, let us take a look at $\rho e$ and I can write the value of $\rho e$ in terms of the pressure at the exit. The pressure at the exit divided by density is equal to specific gas constant into the temperature at the exit. I simplify this expression to give me $\rho e$ as equal to $Pe$ divided by $RTe$. Therefore, now I can substitute the value of $\rho e$ as $Pe$ divided by $RTe$, so that now I get an expression for $Pc$ by $Pe$. I can also make some changes for the value of $Tt$ that is the temperature at the throat.
And as we had seen earlier that the temperature at the chamber divided by the temperature at the throat is equal to 1 plus gamma minus 1 by 2 into Mach number squared at the throat which is 1 square. The value of Tc by Tt is therefore equal to gamma plus 1 divided by 2.

Now, these two equations namely the value of temperature at the throat in terms of the chamber temperature and the exit gas density in terms of Pe by R Te are substituted in this particular expression and now we therefore get the area ratio epsilon as equal to 2 over the same terms: I write again gamma plus 1 divided by 1 over gamma minus 1. Further, Pc divided by R Tc now gives the term of Tt divided by Tc. We can also write the value of Tc as 1 over under root Tc, which is over here under root Tt, I can write as 2 over gamma plus 1 divided by half into under root Tc. We had got this from rho e which was equal to R Te by Pe and this is divided by the same value what I had here namely 2 over gamma plus 1 under root 2 over gamma plus 1 and gamma minus 1 into 1 minus P by Pc to the power gamma minus 1 by gamma close bracket - the bracket should have been closed here. Now, let us simplify this: we have under root Tc under root Tc cancels and now if I were to put it in terms of Te by T c in terms of Pc by Pe.
We now get an expression which gives me epsilon or the area ratio is equal to 2 over gamma plus 1 to the power 1 over gamma minus 1. We also have Pc here, we have Pe here let us write it has Pc by Pe and now R also cancels out. Therefore, now we have Te left over here that is the exit temperature and there is nothing else left. Let us write the value of Te over here and I have taken Pe inside and here I have Tc. And this divided by 2 over gamma plus 1; this is 2 over gamma plus 1, 1 over gamma minus 1 and this I take 2 over gamma plus 1 half into under root of the denominator. This comes out as 2 over gamma minus 1 into 1 minus Pe by Pc to the power gamma minus 1 by gamma and close bracket here.

Now immediately we see that 2 over gamma plus 1, 2 gets cancelled and gamma plus 1 comes on top and therefore now I can write the denominator as equal to gamma plus 1 divided by gamma minus 1 into 1 minus Pe by Pc to the power gamma minus 1 by gamma close bracket over here. Let us now simplify the numerator; I get 2 over gamma plus 1 divided by 1 over gamma minus 1.
And now we can express these terms: express Te by Tc in terms of Pc by Pe. Please observe that we have been doing this by setting Te by Tc using the isentropic expansion process as Pc divided by Pe to the power gamma minus 1 by gamma. You will recall we have done this several times and therefore, if now I say Pc by Pe and I add this together I will get an expression in terms of Te by Tc, we will get Pc by Pe into the value of 1 minus gamma divided by gamma, which is equal to Pc by Pe to the power 1 over gamma because 1 minus 1 plus 1 over gamma gives 1 over gamma.
And therefore, we write the area ratio Epsilon as equal to Pc by Pe to the power one over gamma and therefore, my net expression now becomes 2 over gamma plus 1 to the power 1 over gamma minus 1 Pc by Pe to the power 1 over gamma divided by gamma plus 1 divided by gamma minus 1 into 1 minus Pe by Pc to the power gamma minus 1.

What does this expression tell us, this expression tells us that the area ratio of a nozzle increases as the chamber pressure increases or rather as the ratio of Pc by Pe increases. The increase in Pc/Pe can come about either by increasing the chamber pressure or by decreasing the exit pressure. If I have a very low value of exit pressure my pressure ratio is larger and I require a larger area ratio nozzle. Of course, gamma also plays a role, but only a secondary role. The main aspect of area ratio comes from the change from the variations in the value of the chamber pressure to the exit pressure.

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Let us take a look at some of the values maybe, which I have plotted in the slides which will be now presented. First, we see that area ratio is defined by the value of Ae by At. The expression we had got we had the jet velocity Vj or Ve given by square root of 2 gamma divided by gamma minus 1 into R T c into one minus Pe by Pc to the power gamma minus 1 divided by gamma.
And thereafter we wrote the area ratio in terms of these parameters and got this particular value which worked out to be $2 \over \gamma + 1$, $1 \over \gamma - 1$, $P_c$ by $P_e$ to the power $1 \over \gamma$ and a whole series of $\gamma$ terms.

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We should have had $\gamma$ and we should have got the expression as $\gamma + 1$ divided by $\gamma - 1$ into the $P_e$ by $P_c$ term in the denominator. When we plot this expression, we get:
We get the area ratio as a function of $P_c$ by $P_e$. As $P_c$ by $P_e$ increases, the area ratio increases. Further, as the value of gamma decreases from gamma of 1.4 to 1.1, we find a larger area ratio is required to give me the same value of $P_c$ by $P_e$. 
This slide gives area ratios for larger value of pressure ratio. Again, the value is $P_c$ by $P_e$ is expressed on the X axis while the area ratio is shown on the Y axis. You find that as gamma decreases I need a larger value of the area ratio to give the same pressure ratio. In other words what it tells me is if my gases have a smaller value of gamma then I need a larger area ratio to give me the same value of pressure ratio. This is all about area ratio.

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What is it we have done so far; we found out the value of the area ratio and related it to the value of the chamber pressure divided by the exit pressure. Now in general area ratio of most of the nozzles are between 15 to 400.
If we were to look at the expression for area ratio you find that when $P_e$ becomes zero, I need area ratio, which is something like infinity, I cannot have infinity, because I cannot construct a rocket, which gives me a very large value of area ratio going to infinity, I cannot keep on extending because the mass of my rocket will keep on increasing; therefore, the general practice is to have area ratios between 15 and 400, 15 for those rockets, which operate within the atmosphere or which operate near to the Earth and 400 or values around this for rockets, which operate in the vacuum regions. Therefore, the question which now comes in is, if I have a rocket whose nozzle whose area ratio is either too small or large, how does it perform?

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Suppose, I have a rocket nozzle, which let us say has a small value of $A_e$ by $A_t$: this is the value, I say epsilon 1 which is equal to $A_e$ 1 divided by $A_t$ 1. For the same condition of the throat, I also have another rocket, which has a larger area ratio, let us say $A_e$ 2 for the same value of let us say $A_t$ 1. The latter is epsilon 2 and is equal to $A_e$ 2 divided by $A_t$ 1. Now the thing what I want to describe is suppose the chamber pressure is the same in both the cases. What I am going to get is a smaller value of $P_e$ 2 as compared to $P_e$ 1 since the area ratio is more. Stated in the reverse order, $P_e$ 1 will be greater than $P_e$ 2. The smaller nozzle expands to a higher value of pressure; if area ratio increases as in the case of nozzle with area ratio epsilon 2, the value of $P_e$ 2 is less than $P_e$ 1.

We also know that the ambient pressure decreases as the altitude above the surface of the Earth increases. At sea level, the ambient pressure is one atmosphere i.e., 10 to the power
5 Pascal. As the altitude increases, the pressure decreases till at an altitude corresponding to one at which at the edge of the atmosphere let say around 50 or 60 kilometers altitude, the pressure will go down to a very small value and when I go to geosynchronous altitudes its almost perfect vacuum. The ambient pressure with respect to the exit pressure of the nozzle is expected to play a role. Let us assume that in the specific case Pe 1 for the nozzle with area ratio epsilon 1, the ambient pressure is Pa.

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Therefore, let us put down the values or pressures in the form of a plot or figure. In the figure here, we show the pressure variation along the nozzle. In the first case with epsilon 1 area ratio, the pressure in the nozzle continua. It starts with the value of Pc comes down to a value of Pe 1. In the second case, for the same chamber pressure Pc, it starts from Pc, but continues further till I get a much lower value of Pe 2 at the exit.

Let us consider a situation where in the ambient pressure is equal to Pa. I show Pa to be somewhat less than the value Pe 1 this figure.
The pressure is varying in the nozzle along its length. The ambient value of pressure $P_a$ in the first case of the small rocket nozzle is less than $P_e 1$. In the second case, the rocket nozzle is bigger and therefore the gases expand further with the exit pressure being less than the ambient pressure $P_a$.

Therefore in the first case, what is going to happen since the pressure at the exit is greater than the ambient pressure? In the second case, the pressure at the exit $P_e 2$ is less than the ambient pressure $P_a$. In the first case the expansion is not completed; therefore, we call this nozzle as being an under expanded nozzle. In the second case, I expand it over and above the ambient pressure; therefore, I call this particular nozzle as an over expanded nozzle.

Are there any defects in these two nozzles? What we have done when the nozzle is small – that is, the area ratio is less is that the expansion is lower than what could have been possible. Therefore we are not able to get a high jet velocity because the exit pressure has still not been able to match the ambient value. The expansion is incomplete. We could have got much more jet velocity had we really expand it a little bit more come till the ambient pressure. We are losing some velocity.

What is the problem with over expanded nozzle? Well the pressure here itself within the nozzle is equal to the ambient pressure. At the exit, the pressure is going to be lower. However, at the exit, the pressure is ambient which is higher. Mind you, the flow in the divergent is supersonic and does not know the conditions existing ahead of it. All of a
sudden the supersonic flow finds a higher pressure because it has already been expanded to a lower pressure and this is clearly not possible. Therefore, it is necessary that something like a shock stands over here within the nozzle; that means, I have supersonic flow it is not able to see anything before it, but all of sudden when the flow reaches it sees a higher pressure and therefore something like a shock is required to match the exit pressure.

The situation is like the following: I have a nozzle here and now the pressure has come to the ambient over here itself and therefore, if I were to plot the pressure, the pressure is going to decrease further. I need to have a shockwave and the flow downs stream of the shockwave is subsonic. The divergent nozzle considering the subsonic flow will act as a diffuser instead of a nozzle and the pressure will increase till it reaches the ambient value at the exit. That means there is going to be a shock and the adverse pressure because of the shock would cause flow to separate at the walls of the nozzle. Since we have a higher pressure at the nozzle wall, the performance of this nozzle may be even better than had the flow not separated. But, normally this flow separation does never happen symmetrically, and it’s leads to something like side forces, and therefore over expansion is never preferred at all. I will get back to this point a little later; this point may not be clear at this point in time.

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All what we are discussing is that if the nozzle area ratio epsilon is not properly tailored and I get the pressure at the exit of the nozzle Pe to be greater than the ambient pressure Pa, I have under expanded nozzle. In this case the nozzle performance is poor; I do not get the high value of jet velocity which is possible by further expansion to the ambient pressure. But, in case the nozzle exit pressure Pe is less than the ambient pressure Pa, we will have something like a shock. The increase in pressure at the shock and in the subsonic flow subsequently will lead to flow separation taking place at the walls of the nozzle. And the flow gets separated from the walls due to the adverse pressure gradient. Flow separation doesn’t take place symmetrically along the circumference, with result that in some regions we have higher pressure, where flow separation takes place. In regions where the flow is not separating, the pressure is lower. The low and high pressure distributions along the circumference which give rise to and this is not desirable.

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We will come back to this point after seeing a few pictures on flow separation in nozzles. What do we really mean? Let us go back and ask ourselves, can I make a plot of pressure distribution in flow separation or let us say an under expanded nozzle. How does the flow behave? Let us consider this diagram. A chamber, a nozzle and the center line of the nozzle. We said that for the under expanded nozzle Pe is greater than Pa. Therefore the flow comes here, it meets lower pressure; therefore I have something like flow is going to expand out like this as if it bellows out. And how does the flow expand out, I
have rarefaction fans or expansion waves which are generated at the nozzle exit. And similarly over here that means the plume comes here, this is the low-pressure region; this is higher value of pressure and when the flow expands out as a series of expansion fans. So we have the expansion waves being shed at the nozzle exit while the plume spills out. I have here the back pressure, which is the ambient pressure surrounding the expanded plume. The same expansion fans are shown for a two dimensional geometry.

After the expansion waves, pressure in the plume matches with the low pressure ambient. At this center the flow velocity is still higher. Therefore, what is happening? Here the pressure has been matched, but here I have expansion, therefore here the pressure is going to be less than the value of Pa; that is the nozzle with under-expansion, travel and meets the expanded boundary of the plume. The expansion waves are reflected back from the plume surface as compression waves, the compression waves converge to form shock waves as shown. The pressure behind the shock waves increases more than the ambient pressure and the shock waves intersect as shown. We have compression region over her after the shocks. The compression waves subsequently hit the plume as shown. They are reflected back as expansion waves. And therefore now, the pressure decreases from the expansion waves. In this way a series of zones of pressures more than the ambient pressure and less than the ambient pressure are formed in the plume from the nozzle. The formation of these zones is due to the interaction of the rarefaction fans and shocks with the boundary of the plume.

In regions wherein pressure is high the temperature is also high. If the temperature is higher, the plume becomes luminous and you can see the pattern with alternate bright and dark zones.

We find that because of under expansion, there is further sudden expansion that means there is an expansion fans and this expansion fans impinges on the plume surfaces. And when the expansion fans impinge on the plume surface, the expansion waves are reflected as a compression that means as weak oblique shocks. These oblique shocks further compress the medium. The interaction of the compression waves with the plume surface forms expansion waves and the process of compression and expansion continues.

If we were to have an over-expanded nozzle, we will have different flow pattern in the plume. A shock is formed within the nozzle and this compresses the initially expanded flow. This is because Pe is less than Pa. The flow being supersonic, we need a shock
which will match the higher value of pressure. Therefore, what is going to happen is the plume boundary will come down like this since the ambient pressure is higher. The shock waves interact with the plume boundary and are reflected back as expansion waves. The plumes expand following the expansion or rarefaction fans. The expansion fans are reflected from the plume as compression waves and the pressure in the plume thereafter increases. And so the processes of compression and expansion continue along the plume.

In other words, in the case of overexpansion, we get a higher-pressure region little bit away from the nozzle exist. In the case of the under-expanded nozzle, we get a high-pressure region just at the nozzle exist, that means over here, I get a high-pressure region following the oblique shock waves in the case of under-expanded nozzle.

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To be able to appreciate this point, I show some slides of the nozzle plume and this will become clear to you now.
See this shows a particular second stage rocket of PSLV, and here you see this is the divergence portion of the nozzle. This is the combustion chamber. And if I take the inside configuration of the nozzle, it will have a throat and will come back with a convergent shape like this to the chamber. Therefore, I am looking at the outer portion of the nozzle here.

When the rocket fires for some time, the nozzle runs hot and become red hot mainly we are looking at this part. It becomes at red hot, because it is heated by the hot gases. And then hot gases are converging towards the center after leaving the nozzle like in an over expanded nozzle. You see the plume becoming luminous after the shock wave. The downstream is not clear, because water is sprayed to cool the plume.
Let me go to the next one, I show the same nozzle again. It is red hot. The white part is the luminous zone after the oblique shock waves. The oblique shocks are seen and they interact along the base.

Let us go to some other experimental firing, in this shows the engine test wherein this is the exit of the nozzle, and in this particular case, exit of the nozzle is such that the flow is probably over expanded. And therefore, you have something like shock waves which increase the pressure and temperature in certain regions of the plume. These regions become luminous. And what happens is the high pressure region over here, gives you a higher pressure and higher temperature. If I have oblique shocks like this, which give me
a high temperature region. It looks like a shock diamond you know, I have a high pressure region which is luminous. Afterwards, the oblique shocks comes here, I have rare fractions fans coming another oblique shocks coming; I have another white patch over here. Again the process, I have something like a series of shock diamonds from the shocked high temperature gases..

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In continuation, this slide shows a space plane SR-71 We see the shock diamonds in the plume in this particular case. And we continue with this, this is a test of an engine. And here you find, there is a shock here and something like this. This is because the exit condition is over expanded.
We will continue with nozzles in the next class. We will review overexpansion and under-expansion and then proceed further.