In the previous lecture, we looked at some safety stock models and we also studied the impact of lead time on the safety stock. We assume that the demand during the lead time is going to be probabilistic and we have looked at several cases, where the demand follows a discrete distribution or the demand follows a continuous distribution. We also looked at lead time and we looked at two cases, where the lead time is deterministic and the lead time is probabilistic and followed a distribution. And we also saw that, this would lead us into more safety stock than the case of deterministic inventory.

And one of the things that we said at the end is, it also has an impact on supplier selection. Ideally, we would like to choose a supplier, who has a deterministic lead time rather than choosing a supplier, whose lead time performance follows a distribution. The moment the lead time is not deterministic and shows variation, we are fully aware that the safety stock is very high. So, the first impact or the first application or use of these concepts lies in supplier selection, where we have to be careful in choosing a supplier, the suppliers should not show variation in lead times. We will now see a couple of more
areas in supply chain management, where this theory of safety stock versus probabilistic demand has some application. So, we look at the second one, which is called aggregation.

(Refer Slide Time: 02:33)

Now, let us look at a case, where there are five retail outlets and the demand for a particular item, let us say is 50 and sigma of demand is equal to 10. We could also use s to represent the standard deviation of the demand, but let us use sigma to represent the standard deviation of the demand, which is 10 units. Now, if we have this situation and if the lead time is say 2 weeks and these are weekly demands. And if the lead time is 2 weeks then demand during the lead time is equal to 50 into 200 and sigma during lead time is equal to root 2 into 10, which is 14.14.

Now, each of these retail store wishes to maintain 95 percent service level and the demand distribution is normal. Then, we need to go into the standard normal curve to find out the z value, for which the area is 0.95 and that value is 1.645, is the value of z, for which it is 0.95. So, the safety stock that each retail store will be having is z sigma, which is 1.645 into 14.14, which would give us 23.26 and the five retail store will have a total safety stock of 23.26 into 5, which is about 116.25.

Now, if we aggregate these five retails into a single aggregated unit then when you aggregated that the, when you look at the case of aggregation and aggregate all of them together, so mu will be 5 times 50, which is 250, sigma will be root 5 into 10, which is
22.36. And once again, if lead time is 2 weeks then demand during lead time will be 500, which is the same as the demand of 100 over five retailers, but the standard deviation of lead time demand would now be root 2 into 22.36, which will become 31.62.

And once again, if we want to maintain 95 percent service level, z will become 1.645 and safety stock will become 31.62 into 1.645, which is 52.01. So now, you see the impact of aggregation, so when we are able to aggregate all of them and bring it together in the same place then we realize that the safety stock is actually 52.01, instead of 116.25. The place we actually gained was in the fact that, the standard deviation of lead time demand is 31.62 here for the aggregated, while it is 14.14 and its going to effectively get multiplied by 5.

So, it is at root over, which gives us the saving in the safety stock, because moment we aggregated the positives and negatives, somewhere cancel out each other at steady state and then it is enough to have a slightly lesser or lower safety stock to take care of this. But then there are several ways of achieving this, from a very practical point of view if we look at five retail stores aggregated into a single one then there are some advantages of having five retail stores in five different locations.

And having all aggregating them into a single location will have different effects, for example, the demand can change when it is aggregated. But then one of the ways of handling today to enable this is, through information sharing, where if these five retailers even though they are located in five different places, they are able to share information. And in situations, where the actual demand exceeds the reorder level, if the other one that has positive stock can come and help the retail, which is going to run into shortage, one can achieve the benefits of aggregation.

Of course, the time taken to transport between one place to another would also have an impact, but assuming that it is possible to transport things quickly, one can gain the advantage of a lesser safety stock when you aggregate. So, it becomes a tradeoff at sometimes between the inventory holding of the excess inventory versus the cost of transportation between one place to another. So, in several instances in supply chain management, the tradeoff is between holding inventory and additional transportation cost.
The same idea can be applied to warehouses, now instead of five retailers, one could think of five small warehouses versus a single large warehouse and essentially the concepts are the same. So, through information sharing, one entity can help the other in situations, where the actual demand exceeds the reorder level and once again, tradeoff between transportation cost and excess inventory. The second aspect that happens is called component commonality.

(Refer Slide Time: 10:32)

Common components, now suppose we look at manufacture of an electronic item, say it could be a computer, an assembly, largely assembly dominated, but manufacture of as electronic item. And let us assume that, about 10 components that we are considering, nowadays electronic items come with a lot of variety and since there are lots and lots of variety, each variety essentially constitutes a particular product. So, let us assume that, there are 10 different types of components, 10 different components, 10 different components go into the final assembly.

Let us also assume that, the lead time demand shows a distribution with mean equal to 4000 and standard deviation is equal to 2000 and lead time is equal to, let us say 1 week. Now, in order to maintain 95 percent service level then your safety stock is $z$ sigma, which is 1.645 into 2000, which becomes 3290 and for 10 different components, the total will become 32900 items. Now, if we combine an aggregate or have n common
components as it is called then we would get μ equal to 40000, σ will be equal to \sqrt{10} into 2000, so this will become 6324.55.

And once again, for lead time equal to 1 week and 95 percent service level, z equal to 1.645 and z σ is equal to 6324.5 into 1.645 is 10403.8. So, once again we observe that, the safety inventory that we have in the case of common components is much lesser than the safety inventory that we have in the case of different components. So, organizations now try to have a common component in several of their assemblies.

And the uniqueness of the variety comes towards the end, where the unique components which are few, will differentiate products from one another and bring variety into the system. Now, this idea which is being increasingly used is called delayed differentiation, where up to a very large time in the assembly, the components are very similar. And after which the differentiation actually happens and the longer it takes to differentiate the variety, the better it is from a safety stock point of view.

As long as we are able to aggregate and as long as we are able to have common components that keep happening, it is always possibly to do that. Another way, by which organizations handle this, is also to try and have what are called substitutable products. The same product for example, can come in two different specifications and they may meet the same purpose. So, when we have a situation, where the actual demand is for, let us say the two different things are A and B.

(Refer Slide Time: 15:30)
A good example is A and B could be hard disks of different capacities, so when we have to make A and have to make B, when we make the product that has component A and then the product that has component B. Now, if A and B are treated separately as two different components then we get into this part but then there can be situations, where A and B need not be of the same cost. In fact, one of the assumptions that we made here when we compare these two is that, the actual cost of the item is comparable.

Because, here there are 10 different components and total of 32900, 3290 of each component is in safety, whereas here there is only one common component with 10403. Now, if the cost of this common component is the same as these ten or is the is the same as average of these ten then these two figures are directly comparable, otherwise the costs are comparable, now when we look at A and B, which are the same component, but two different specifications.

Now, by the same idea we understand very quickly that, if A and B are substituted for each other or if A is a component with the higher specification and B is a component with the lower specification, but A can substitute B. While we may not want B to substitute A, but A can substitute B, we know that, when if we are able to do this then we are able to achieve what has been told here. The safety stock will come down when we are able to substitute which means, we are aggregating the demand for A and B.

But then we are not sure that, A and B will have the same cost, B being a component with lesser specification, can be a little less costlier than A. But then organizations may still like to do it, not always but sometimes, because in some sense, the additional costs that is incurred or lost by substituting A for B will be compensated by the fact that, when you combine both of them together, the safety stock comes down and the overall cost of maintaining the safety stock will come down.

So, today in supply chain management, we have all these things, which are very common. This aggregation, common components and substitution of components, delayed differentiation and postponement which means, to try and delay the differentiation of different varieties of the same product as much as possible so that, up to the point where we delay, we can do all the three of them. Essentially, the principles are the same, the principle is that, you are able to aggregate a demand and once you
aggregate a demand, the standard deviation comes down, so we are able to use the same principle and get it.

This superimposed by the additional principle that lead time is not probabilistic, the moment the lead time becomes probabilistic, all these safety stocks jump up and they become order of magnitudes higher than what they actually are. So, if one wants to exploit this idea and try to gain by the fact that, my safety stock is coming down if I aggregate something or if I substitute something or if I delay the product differentiation then I have to ensure that, I do not get into this territory.

So, the first thing that is required is excellent supplier relationship, where the supplier is able to give items in time which means, we are looking at deterministic lead time. The moment supplier starts showing variation and lead time is non determinstic then this safety stock shoots up. So, if we are in this territory then even though this can give us a slight advantage, the total cost would be very high, because in any case, safety stock will be high.

So, the first thing that organizations have to do in order to benefit from things like aggregation, component commonality, delayed differentiation is to ensure that, lead times are deterministic and work with the suppliers to get deterministic lead time. So, this is the impact of safety stock in the context of supply chain management, and in the performance of organization, Now let us look at another topic.

(Refer Slide Time: 20:49)
Now, let us go back to basic inventory problems, now in the basic inventory model, we have looked at two types of demands. We first looked at a case, where annual demand is given, annual demand is continuous or demand is continuous and we looked at case, where demand is time varying. We could use the economic order quantity formula to get the order quantities here, we could use a lot sizing heuristic like Silver-Meal to try and get the order quantities here.

But, both of them require, the first thing both of them require is the D which is the annual demand in this case and the same D given as periods in this case. So, let us get into the question of, how do we calculate this annual demand and once this annual demand is given or demand is given in any form, for a particular item or a set of items, how do we do it. And in inventory we looked at, most of the times we looked at a single item and on some occasions, we looked at multiple items.

Whereas, in forecasting or aggregate planning, we were not looking at an item level, we were looking more at a product level or at an aggregate level. So, how do we find out the item wise demand from the forecasted value, which is product wise. So, there has to be a relationship between the product and the individual items and that relationship is called the bill of materials, also called BOM. We use the simple schematic diagram to represent some ideas of bill of material and how we are able to move from the product wise forecasted demand to item wise demand.

So, use a simple schematic diagram in reality; these diagrams are much bigger and far more complex. So, let us assume that, we make a product P and for the sake of illustration, let us say that, P is made up of two components, two assemblies A and B, both are assemblies and then these assemblies go to make the final product P. So, let us also assume that, A is made up of, A and B are sub assemblies, which finally make P.

Let us say A is made of C and D, where D is a bought out item and C is a machined - something that comes from the supplier, undergoes manufacturing and then it becomes a component C, which goes to the assembly. So, D is bought out, so it comes from the supplier, let say C is actually made out of E, which also comes from supplier, which we call some S 2 as another supplier. And only for the sake of illustration, B let us say is also made out of D, but it is machined or let us say, for the sake of completion, B comes out of E.
E comes from the supplier S 2 and then E is machined and then it becomes B, which goes to the assembly. So, it is a very simple diagram, in reality if you take any manufactured finished product, this bill of material would be a much bigger network, which could sometimes involve thousands of components and assemblies and sub assemblies. Just for the sake of understanding the concept and for the sake illustration, I have used a very simple figure here.

Now, let us assume that, this month we want to make 400 of P, this 400 could either be a forecast or it could be a demand, which we have to make, a firm demand that we have to make. Now, let us assume again for the sake of discussion, that it requires 1 of A and it requires 2 of B to make 1 of P, so automatically, 400 of P would involve 400 of A. So, it would involve 400 of A and it would involve 800 of B. Now, once again let us assume that, this would require 2 of C and 1 of D which means, to make 400, this would mean 800 of C and 400 of D.

Now, once again we would say that, in order to make 1 of C, I need 2 of E, so I need 1600 of E and here for example, I say to make 1 of B, I need 1 of E, so I need 800 of E. So, at the end, I would go back and say that, if I want 400 of P, I need to buy 1600 of E from the supplier, I need 400 of D from the supplier and I need 800 of E from the supplier. Now, let us also try and bring some lead time into the picture, now there are several lead times or several times taken.

So, let us assume that, it takes about 1 week to - for example, if I want P today, the assembly takes 1 week to do this and if I want to make A, let us say the assembly takes 2 weeks and to make B, the assembly takes 1 week. We will later convert these weeks into days, but right now let us look at weeks and then D comes directly from the supplier and within 2 weeks, it will become A. Now, let us say E comes from the supplier and let us say, it takes 1 week to become C. So now, let us go back and represent the same thing pictorially in a different manner.
So, let us assume that, we have something like this and let us say we are talking of weeks. So, here I want to make 400 of P, so if I want to make 400 of P then I need 400 of A here, here I need 400 of A, because it is going to take 1 week, here I need 800 of B. Now, if I need 400 of A here then 2 weeks before that, I need 800 of C and 400 of D and if I need 800 of C then 1 week before, I need 1600 of E, so here I need 1600 of E. Now, if I need 800 of B here, 1 week before, I need 800 of E, so I need 800 of E.

If I need 800 of B then 1 week before I need 800 of B, 1 week before, so I need 800 of E somewhere here, so here I need 800 of E. So now, if I need 400 of P at the end of this month then I need, now what are the things I have to go to the vendor and buy. So, things that I go to the vendor are E and D at the end of it. So, I need to have 800 of E, I need to have 400 of D and I need to have 1600 of E, these are the three things that I need here.

So now, this has been translated to this and then I know that, if need 400 of P here, this is what I need. Now, let us go back and say that, we are now changing this 400 of P at the end of the month, let us say and we change that to, we say that, this month is going to have 4 weeks. And then we say that, we are going to have 200 100 50 and 50 as the demand for 4 weeks. So now, let us draw this diagram again considering that, let us say I need 200 P here, if I need 400 P here, I need 800 E.

So, if I need 200 P here, I need 400 E here, I need 200 D here and I need 800 E here, now if need 100 P here, this is the 4th week of the month, this is the 3rd week of the
month. So, if I need 100 P here, then I need, for this 100 P, I need 200 E here, I need 400 D here and I need 100 D here and 400 E here. Now, if go back and say that, I need another 50 P here, now this 50 P would mean, 100 E here, 50 D here and 200 E here.

Now, another 50 P here would mean that, we need 100 E here, 50 D here and 200 E here, so now, the moment we shift this demand into weeks, we are going to have this as the requirement for the bought out items. So now, I can summarize it and say that, the same thing I am representing here. So, I am saying, if this is the plan for P in 1 month then I summarize here and say, at this point I need 400 E, at this point I need 200 E, at this point I need 900 E, at this point I need 500 E, at this point I need 200 E, at this point I need 200 E.

I go back and say, somewhere here I need 200 D, I need 100 D, I need 50 D and I need 50 D, so I can sum it up and say, this is my requirement for this. So now, all I need to do is, the moment I get this structure right, I need to keep inputting values here, which are my expected production quantities of P, let us say in every week. Now, the week or the time period can be changed, the algorithm will remain the same, now weeks can become days.

So, if this for next 4 days then this whole thing will become day and the whole thing will shift here and say, one day before I need so much, another day before I need so much, another day before I need so much and so on. So, depending on how we define the time period, which is either a week or a day, the algorithm will remain the same and the way we compute the quantities will also remain the same. So then all we need is, if we have something like this and on a time scale we are able to say, so much of P required it in all these times.

We can back calculate and say, this much of D and E we require here but then in reality, P is not dependent only on D and E, but P would be dependent on some 100 other items or 500 other items, but it does not matter, but the algorithm and the way to compute is the same and needless to say that, this can be done extremely on a computer. So, when for the first time, manufacturing organizations started computerizing this whole thing, bill of material and then translating the, this is the bill of material in a pictorial form, you could write the bill of material also on a file.
Now, it is easy to write a program that will translate this into this, now the program that started this thinking process, which is to translate this to this is called the materials requirement planning or it is called the MRP. So, the essential function of the MRP is to incorporate the bill of material and it is ability to look at quantities of individual demands when the aggregate demands are fed into the MRP system. Now, the additional thing that can be done is, this can be extended for the next, assuming that the period is a week.

This can be extended for the next month or next 2 months, it can either work on a rolling calendar or rolling time frame or one can do right at the beginning of the year, what is the demand for the 50 weeks or 52 weeks and then bought it out. Now, if we are working only with P, continuing on this numerical illustration, if we are working only with P, which depends on D and E then we have one calendar for E and another calendar for D.

So, once we have these two calendars, we can separately use a lot sizing model here like a Silver-Meal heuristic or part period balancing, to try and find out what quantities we have to order here at each point. Not necessary that we have to order 200 200 500 900 200 and 400 because then the supplier lead time comes into the picture that is, 1. So, we need to order before the supplier lead time, giving enough time for the supplier to deliver plus bunching would also save, because every one order would make us incur an order cost.

So, if it is cheaper for example, to order 400 here, instead of a 200 and 200, because we will save one order cost incur an extra inventory carrying, which is the general principle of lot sizing heuristic such as Silver-Meal. So, one could apply heuristics like Silver-Meal to try and find out, exactly at what point, how much we order, so that can be done for each of these items. Now, the next thing that we look at is, it is not that the organization produces only P, so it could producing another product say, Q, which is another product.
And then this Q would also depend on for example, let us say this would depend on two sub assemblies R and S. And just to make the illustration simpler, let us say R would depend on E and S would depend on D, so one would say that, if you want make 200 of Q or if you want to make... Before we do that, if you want to make 1 of Q, let us say we need 1 of R and say 2 of S and then we would say 1 of R would want 1 of E and each would involve 2 of D.

So, if we want make 200 of Q, now 200 of Q will become 200 of R and 400 of S and this would mean, 200 of E and 800 of D. So, once again if we go back and write these lead times as we did earlier then we can draw another diagram like this for Q, as we did for P. And once we are able to draw another diagram for Q, now from that diagram, we can draw separate time calendars for D and E, which would depend on the amount of Q that we want to produce, the total as well as the split up of that week wise.

So, if the organization is making 10 different products then we can get 10 different calendars for E and D, not only for E and D, every item that is bought out. Now, we can merge these 10 calendars of D and E to make only one calendar for each of these items and then apply the lot sizing heuristic so that, you are able to order these effective. The other alternative in the pre MRP days was, since this was time consuming activity even when done manually, it was always easy to go back and do a single multiplication.
And say, totally in the year this much P I am producing, totally in the year this much Q I expect to produce. And I know that, for every P I need this much of E and this much of D, for every Q I need this much of E and this much of D. Simply multiply by the requirement and aggregate it, we would get that single D, which is the annual demand for that item and one could use an economic order quantity there. The only catch or difficulty there was, if this annual P is distributed unequally over the months and unequally over the weeks then the economic order quantity can sometimes result in shortages.

But, the moment they are distributed unequally over the 50 weeks and then if we have a system like this in place, which is computerized and MRP system that is computerized, most MRP systems are and then we have a MRP system in place then the system would provide us the kind of calendars that we could require for each of the item. And once we are able to get this aggregated calendar for E, this was for P, similarly there could be for Q, some other item and so on, aggregate all of them then we could place individual orders for E.

Then, gets through the next complication, we have seen basic inventory models and lot sizing models, but in this lecture series, we have not seen lot sizing models and discount. Whereas, we saw discount in the regular inventory model, we did not see discount in the lot sizing models then we could build a lot sizing model that takes care of discount. We could build whatever analysis or whatever additional models that we saw in the normal regular inventory with annual demand, we could bring all those dimensions into the lot sizing.

So, things like discount can be brought in, things like multiple vendors can be brought in, things like aggregating items to a single vendor and that can be brought in. So, when items are aggregated to a single vendor, we could apply multiple item inventory models, where by ordering items together, we can save on the truck cost. So, all these models can be brought in to make effectively the entire cost of the materials management system as less as possible.

At the same time, keeping in mind service levels, keeping in mind that we do not tie up too much of inventory in the system, because today’s manufacturing does not require organizations to tie up so much of inventory in the system. So, minimum amount of
inventory is certainly require to do and to find out that minimum and to exploit ideas from safety inventory, that we saw ideas like substitution, postponement, delayed differentiation, aggregation.

Use all of these to try and benefit from minimizing the total cost of regular inventory as well as the total cost of maintaining the safety inventory. Now, before we proceed, we take a quick recap of, what are all the things we have covered till now in this lecture series and try and relate all of them very quickly and then move on to the next topic in this lecture series.

(Refer Slide Time: 45:05)

So, the first topic that we saw was forecasting, which essentially helped us in getting an estimate of the demand for the product. The kind of forecast that we saw were aggregate forecasts for example, we were not trying to look at forecasting models that would forecast the demand of the product tomorrow. The time series models that we saw, which we concentrated, were looking at periods like 1 month and so on. So, forecasting was done at a reasonably aggregate level and from forecasting, we saw models for aggregate planning.

Now, aggregate planning would give two outputs, one is the production capacity that we are going to have in terms of regular time and overtime. If there is some outsourced capacity then that is going to come in, what is the production capacity that we are going to have in a certain period and let us say, the period that we were looking at, is like a
month. If the other output from the aggregate plan is the work force but then let us not dwell too much on work force changes.

Let us assume that, work force is a slightly different thing and let us look at the production capacity that is available in each period in terms of man hours aggregate to meet the aggregate demand of the product. So, till this aggregate planning, we did not come to individual products. Now, at the end of aggregate planning, we looked at two things, we looked at economic lot scheduling problem and we also looked at disaggregation models.

In all these disaggregation in particular, we said if a certain amount of man hour is or time is available, how do we divide this time to make various products. So, the first time we actually came at a product level or an item level was in disaggregation. So, if a certain amount of capacity is available in a month, how much of that is going to be used to make product A, how much is used to make product B, C, D and E, is what we saw in this disaggregation.

Then, after we cover disaggregation, somewhere in between we started inventory, though it does not come sequentially in the system, inventory is somewhere here, but inventory provides with our management of materials. So, if we want to produce item A at this point in time, when do we order, how much do we order so that, we have that much of the item available with us. So, here in inventory we saw couple of things, we started by assuming that each item has a certain annual demand and then how the material is ordered and how they are brought in so that, they can be produced.

At the same time, we also saw models when this annual demand is broken to months, broken to weeks, broken to periods, where the demand was be different in each period. And then later we saw the bill of material and how we are able to relate inventory to the final product. Item to product is what we saw right now, where certain production of a certain amount of product P is now broken down item wise for D and E, item wise as well as time wise.

So, this actually helped us to try and relate the item to the product, so this is exactly what we are right now and we have seen all these topics. And then we need to move to another topic, where we know that, at the end of disaggregation, we are going to make or produce a certain product or an item in the next say, 1 week or so or the next 2 days or so
whatever is the cycle time that we are talking out, we are going to make this product. We also assume that, through a good materials management system or inventory planning system, the material to make, these are also available.

Now, we need to carry this material and time into individual machines or resources and then try and say, how much of time each machine is going to spend on each of these products or in general, each of these jobs. Now, that leads us to a topic called sequencing and scheduling, where given a set of machines and machine resources and the set of jobs and activities to be performed and these products have to be made. Now, how are we going to utilize these machines to make these jobs or how are we going to ensure that, these jobs and all the individual requirement of these jobs are going to be met within a certain time on a given set of resources, which are the machines. So, we would go into detail on sequencing and scheduling in the next lecture.