Good morning everybody. We were looking at the solving the partial differential equation for the acoustic field. And now, as supposed to a classical equation, we have source term, which is a heat release rate which is really driven by some kind of fluctuating heat source and we actually took the partial differential equation and did the modal expansion on it or so called Galerkin technique. And then we derived the ordinary differential equation for this.

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So, what we have this two equations d eta j over d t equal to eta j dot and the second equation which is coming from the energy equation is written for d eta j dot and..... just a moment.... yeah this is fine. So, there was a question as to what is the damping term that is this term that I used. So.... which represents acoustic damping. Now, speak a little bit about acoustic damping, but modeling damping is very complicated thing and...... but I just want to raise the issues which some of you may....at least from the industry may appreciate. So, in reality damping is a very complicated thing. See, acoustic waves
actually, have to get converted to vorticity waves and they are the one which get dissipated. It is in the boundary layer.

So, it is just like... acoustic, by itself, doesn't have dissipation but it is the vorticity wave that can be dissipate. So it sets up a boundary layer and in the boundary layer these things dissipate. And it is somewhat involved to calculate this and also we have losses. So, we have radiation losses from the end. If you have open pipe, you can have radiation losses, sound is going out otherwise you would not hear any sound and if you have.....I mean, in reality, no matter how thick walls you get, some amount of sound get into the wall, the wall should vibrate and take away some energy. So, there is volume losses and depending on what gas is there. If it is lot of water vapor, you will have lot of damping because water vapor damps much more than dry air. If you have humid air, it damps much more and so on. Now, there is also one more complication if you are having a combustion experiment.

So, you have a steady flame with certain shape and all that and you measure the damping, but the flame shape, temperature profile, concentration profile.....everything would be different when instability happens. Because flame shape itself would have changed, temperature profile would have changed and so on. So, the acoustic damping value itself changes during the combustion instability. There is no easy way how to find it, there is probably no way at the moment. Radiation losses are modeled fairly, reasonably well. So, at the moment I am not going to get into this although in reality it is very important problem because whether the instability comes or not, depends on how much you drive versus how much you damp.

It is like whether you have enough money to spend and whether you fill rich depends on... not just on how much money you get but, how much money you spend; Like I read that, when Amitabh Bachhan declared bankrupt, he has only ninety three crores with him. I am having not even one lakh with me and I think I am very rich.

So, it depends on what is coming in versus what is going out. So, the amount of damping in this system is very critical because you may have instability even with a small driving, if the dumping is very small. But, you have same amount of driving, if the dumping is very huge, you won't have instability. So, we are.... this is a big topic and I guess a lot of
research need to done but, I am not going to get into this but, I have made a simple model.

This model come from Culick and Matveev. I will write the reference also. So, what their model is? So, omega j is wave number of the... frequency of the jth mode. Now, with non-dimensional sense, it is all the same. So, j correspond to the same j-th mode, oh, yesterday we have written n but we can replace n with j. That is the notation which have here and c 1 and c 2 are just constant which you get from experiment. You perform experiment and estimate c 1 and c 2 based on...lot of people have done experiment and then we put this value in here.

So, this is kind of a adhoc way of treating damping and but, rest of the derivation is fairly type. This term we are, to some extent, putting in the hand although there is some basis for it. I hope Rajesh you are okay with. This is the first mode.. yeah.. fundamental. There are n eigen values right! For the natural mode, you just take the first one. Again, there is a question has to the actual eigen value may be different from the natural mode and so on. But, this is like a model damping, so we know the modes and based on that we put in a damping. So, damping depends critically on the boundary layer, damping depends on viscosity, heat conduction and so on. But, here, every is empirically obtained and put into the c 1 and c 2 and this.... you can see that this term this is you have high frequencies, this term's contribution will be high.

So, which is..... that is the way they modeled it, which is correct because higher modes are damped easily because higher modes have higher gradient in terms of heat conduction and viscosity; because the omega is high, wave number is also high. I will just leave it there as I don't want to go into greater detail but, this is the topic where you can do Phd thesis on. But, within this level only I want to speak about not lot of.... there are.... people have measured damping for different kinds of pipe and so on. And again the losses can be nonlinear for example, if you have radiation from the duct end.....
.....when you look at a pipe and the flow is coming this way, what will happen? How will be the flow out side? It will be like a jet with shed vortices and when you go in...... We are sucking this way so, you have a inherent asymmetry, so, this will introduce non linearity in the boundary condition and so on. So, you can have a losses... can have non linearity. People who have studied musical instrument, have model the losses due to this kind of vortex shedding and so on so forth. So, I mean, a lot a complication are associated this term (Refer Slide Time: 00:41) and it need not be linear also but, we will just leave it. Here any other question?

So, this is like our breathing. We are alive because on this asymmetry if dout stops flow separation we would all die within the 3, 4 minutes. Because we will be..... if this was symmetric we will breath out, what we breathe in and breath in what you breathe out, which is eventually CO2 level in our blood will rise. So, thankfully we have asymmetry and we are alive. It is good for us, the flow separation makes dealing with equation difficult. It is ok rather we alive and deal with complex equation than have a simple equation and be died. It depends, what solver are you talking about? If you are having a DNS solver which can deal with.... the compressible DNS....deals with everything. Then your p prime will include everything : acoustic and hydrodynamic.

Even if you have LES source, compressible LES, compressible RANS, the moment you say compressible you have a acoustic you have as well....if it is a Navier-Stokes
equation, you have hydrodynamics in built in to it. What is acoustic, what is hydrodynamics....that is the issue and it is hard to separate them out unless you do some kind of analysis and also, there are ways to separate. I will not go into that. But the other thing is to do a full compressible analysis of real combustor, turbulent combustor is not so easy. People don’t do that they split it into hydrodynamic zone, combustion and try to couple them in some way to the other. And there are issues associated with this because what are the right equation to use and so on. So, it is a open problem. Yes, it is included that makes things even more difficult because you have to very careful. If you can solve everything numerically it will have everything. But, if you....I think, may be, 50 years later or 20 years later, that will be the way things will be the done.

At the moment you can do that for hydrodynamics or even combusting flows but thermoacoustic instability because the acoustic length scales are of the order of meter and the hydrodynamics length scale where the flame is and so on is of the order of centimeter and reaction zone may happen over millimeter. So, the several length scales and therefore, it makes calculation way difficult. Despite of length scale, despite of time scale, it is not so trivial to real just take a solver and run in and do get the answer. In principal if you can do? Yes everything is there, you do not have to..... So, why are we having a wave equation? Because we don’t want to deal with anything else and we have simple wave to deal with wave. So that’s all.

So, we are trying to make things simple, so we throw away everything else. Exactly, yeah. So, we are saying that hydrodynamic, in this particular example, at least, hydrodynamic zone is very thin and its affect can be distilled into heat release rate. And then you go into acoustics. But, I am not even solving hydrodynamics here, strictly you have to solve for the hydrodynamics and obtain the equation for heat release rate and then you have to take these in to solver and merge in a coupled manner. Hope it is somewhat clear.

But, you are asking difficult, very deep question actually. And, even if you can.... Again, in principle what to do is to do in practice. At this moment it is very difficult. I do not know how many years more it will take, that is it. Any other questions? Very nice question, thank you. Unsteady RANS, you can use unsteady RANS. We have to have unsteady base flow which will give unsteady q prime. Without that, I think, it would be difficult. People are using for solving hydrodynamic flow field and I think, eventually
industry will move in this direction and LES is the still for the academic. Or may be
linearize LES solver or something may be still feasible but full nonlinear solution….. My
feeling. These things all go wrong, predictions. Very good point, anything else? Because
it is talking about 10 million grid points, 20 million grid points and so on, so it is……It
is just getting one solution, you have to get time evolving solution. It is the first
difficulty, and you have to do this for wide range of parameters. Because somewhere you
may have instability, somewhere you may not have instability. So you have to check
everything. So it is very expensive calculation. So I will proceed. So, I will mail you this
references which give all these things. So, next thing is to linearize the time delay. So,
what u do is, u f of …….So, u is, of course, based on our Galerkin modes and we have to
write a expression for this. So, it is little tricky but not that tricky, it is just straight
forward once you figure out this kind of algebra. Now, you do not need to do this, you
can actually solve it as a delay differential equation and software packages are available
to solve delay differential equations but I will not deal with in this class.

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times.....times cos 2 phi x f times eta 2 plus zero times eta 2 dot plus ....plus cos n phi x f times eta n plus zero times theta n dot. So, I have written it out in long hand.

So, this I can write in a matrix form as cos phi x f........so, although it is quite involved to write this thing on the board. When you do matlab programming or fortran programming, it will be quite simple, because all this can be done by matrix multiplication and simple do loops and so on. Matlab is very suited for multiplying vector and matrices. It is optimized for that. You don’t even have to write these loops. You just define the matrices and vectors and tell to multiply and it will multiply very fast. They are optimized for that. I will write everything in the long hand so that things are clear. I hope, this is clear. I will pass for a minute. So, this could be written as u transpose kai where U transpose cos phi x f zero.... So, if kai transpose is this ....eta 1, eta 1 dot, eta 2, eta 2 dot and.....

The kai would be this column vector and u transpose would be this......if u have transpose and u can write everything in a same row. I think that is the reason why they invented this notation. They should multiply U transpose kai and call this is as U and call this as kai. You can see this will give this and so, this is a slick notation that is all. So, we need one more thing for the second term.

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So, they have to expand this. So, what I will do is to follow the same step. So, this big term now became just a small symbol. We can write big matrices but, they symbolically
be very small and even constructing big matrices is involved in writing a paper, but, on computer it is very peaceful. When I differentiate this u ...(Refer Slide Time: 15:27) if I differentiate this with respect to time, the first term will be eta 1 dot time cos pi x f. The next time will be eta 2 dot time cos pi xf. So. There is no eta 1 term, there is no eta 2 term. Because when we differentiate this term is stay but dot coming here and this term will go away. So, there is eta 1, eta 2 term, there is only dot terms.

So, if you say that..... we can call this in a same manner where p would be equal to 0 cos pi x f 0..... In any dynamical system we do in time domain, I mean, you will take this approach. It doesn’t have to be in thermoacoustics, even if you study some complicated vibration problems or structural problems or some problems in magneto-hydrodynamic instability and in astro-physics. I thing you will follow the same approach, if you want to convert it into the final solution of the form d k by dt equal to f of kai and…. So, it just looks like several pages and in the end when the dust settles down you have something really neat and trimmed.

You may be wonder why you have written so many pages. That is good thing. Now, our next step is to deal with this big thing. We have delt with u f transpose. Now, we have to assemble this thing as a matrix. Again I have must keep on emphasizing n number of times that it is just the assembling procedure looks difficult but, actual assembling in the computer is trivial.

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So, let me write that because you do not have to write in long hand in the computer. You can assemble them in this way or you can put all the eta first and all the eta dot next. That is all up to you …… So, if you go back and look at this equations (Refer Slide Time: 00:41) d eta dot by d t equal to eta j dot or minus it a dot is equal 0. So, that equation will assemble so, what we need is some kind of matrix here times eta 1 eta 1 dot eta 2 eta 2 dot …..eta n eta n dot.

Now, it is clear what is the ….that is the equation we are going to do first. D eta 1 d t equal to eta 1 dot. So, what happened to this coefficient? What would be the first one? Let me write that equation here. We write d eta 1 over d t minus eta 1 dot equal to 0. So, what are the coefficients? So this is the first equation. Yeah, that will be 0 minus 1 and everything else will be 0. Ok. Excellent. Can you look at the next equation and say what would be the coefficients? So, you will have this j pi which I call mega j.

So, that will be multiplying eta 1 and this term will be multiplying eta 1 dot and so, you will have the omega one squared mega one… 0 0 and then comes 0 0 0 minus one and here would be 0 0 mega 2 square 2 do it a mega 2 0 and here last one will have lot of 0 0 minus 1 and this one would have again call 0 and omega square and 2 square n omega n.

Yeah, this is….so it was peaceful to assemble the left hand side. So, if you look at the right hand side, 1st equation will have 0 on the right hand side. Then d eta j by d t minus eta 1 dot equal to 0 so, we can put 0 the 2nd one will have is 2 k j pi over gamma m …. over times….pi x f . The 3rd equation will have again 0 the 4-th equation will have again is…….. times sin 2 pi x f its clear right. So, I will call this whole thing as some kind of alpha or something. So, this would be like…. let me write it out. Some more symbol can you tell me which we have not used. I want to use alpha later ….some symbol, I will say g ….I don’t think we have used it before……times 0 pi sin pi x f…… So, this g would be 2 k over gamma m times this square root of 1 by 3 square of p….. t minus tau minus 1 by 3.

So, we have put it in some matix form. In last semester I talked whole thing in one day. So, I guess nobody understood anything. So, they were not asking anything. So, we said that we want to do a linearized analysis. So, we want to do linearization and ….how would you….. we will have a non-linear term…… yes square roots. So what would we
do? Binomial expansion, we will do a binomial expansion but, we have to make sure that u f prime is small. So that we do not run into any problem. Is it clear?

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Use Binomial expansion for right hand side .....This is valid for ......u f prime less than one third. So, we expand this term, we get 1 plus 3 u f prime times t minus tau divided by 2. And next term would be ........what is the expansion of 1 plus x power half? So the next term will be minus. So 9 over 8, we can cancel this 1, this 1 and now, we can simplify by taking it out. For the non-linear analysis, we do not have to do this. But, I will do the non-linear also with this, when you can see some nice form. Is this ok so far? I will pass for a minute.
We know how to expand this. So, we will use that..... root 3 over 2.... This is ok. (Refer Slide Time: 29:47) The reason for this square term ......I am writing u f prime times u f prime and one of them I replace by this expansion the other one I am keeping it in that way and...... I have worked out the whole thing and I see which way it comes nicely. So, that is the reason for this and first wrote the other one and became a big mess. OK! Look at this. Is this ok?

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Now, we can assemble. So, let me call this matrices A 1 (Refer Slide Time: 22:57) I wanted to come to this form, this is my hidden agenda or whatever..... So, that is the reason I am doing all this. 1 minute, let me check. There is a minus. We are half way there but, we want to make the whole thing compact and in this form. We can use the machinery to....what the things in maths, you get it to some problem that is solved, then you use that machinery to solve your problem.

So, we always reduce to some standard thing. And the mathematician worked …..they have been working for 100 of years, getting these standard things so that we can use it. I must take a little time to explain how we did these things? So, I did not know anything dynamical system theory. Neither I did know anything about matrices and I did linear algebra in B.tech but…..I did problems but why we are doing all these things. When we did this we did first in the standard way….use of second differential equation and solve it and so on. We were seeing all these interesting thing like transient growth and sub-critical bifurcation and so on. Of course, we could have continued that way …..bulldozing our way but, suddenly it occurred to me that all of these things have been done. So, then we took a time out, put everything in to the frame work of things, that already been done and this procedure and then that really help. Otherwise we were reinventing the wheel.

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There is a wheel, take it, put it in your vehicle and go….that is the idea. Should I write out what is a 1, I will do that. So, a 1 equal to 0 minus 1 0 0 dot dot dot 0 0 then next one will be 0 0 0 minus 1 0. Then here 0 0 0 minus omega 2 square minus 2 si 2 omega 2 0 and come back to 0 minus 1 and omega n square…… sorry this should be here. Thank you. Over-confident and make a mess. These dots mean 0. So, given that this is a 1, my objectives here is to…. I am doing it step to step. Because I could do whole thing step but, then it may be very confusing.

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So, once you do this, next time onwards you can do this much faster. I am purposely going slow. This is the plan. So, to get this…. so, what we do is….. we need something to call this (Refer Slide Time: 32:34) term. So, we define a new constant, not constant, variable, beta j, or parameters, is route 3 over gamma m k j pi times sin j pi x f. As a subscript beta j is not a constant but, its bita 1 bita 2, bita 3 like that. So, we will look this term, we know that this has to….we have to take this multiply the 1st term. So, a 2 can be constructed as follows. 0 beta 1 0 by the beta 2 and 0 beta n times u transpose. This is would be 0 minus beta 1 0, beta 2 0 minus beta n times cos pi x f 2 0 cos x 5. I am trying to expand. This is the definition of u transpose.
We just have to multiply……what kind of dimension of matrix here? This would be ….. 0 0 0 0 and what would be the next term, minus b multiplied this by this….. minus beta 1 co pi x f. Next term will be 0. What will be the next term? right? 0 minus beta 1 cos and n pi x f 0 minus beta 1 cos x y 0, next what will be the 0 0 0 0 0 next I would be minus beta 2 cos f x minus beta 2 cos x f 0 minus beta n sorry beta 2 n 5 0 come dot 0 dot. Then here minus beta m this multiplied (Refer Slide Time: 39:45) this is multiplied this to multiplied. pi x m 0 minus beta n cos by x f 0 to…….. This is our a 2. This is clear. Any problem in this? So, try to do A 3 also. We are looking at this term (Refer Slide Time: 39:45) here this comes from here.
So, we have to write A 3 equal to …. this is same for all term. This is nothing but, (Refer Slide Time: 32:34) this column written with those constant added…..times p transpose which was 0 cos pi xf plus 0 cos 2 pi xf plus 0 cos n pi xf. Is this clear? So, we assemble this. Where shall I do it? I will try to do here itself with another color. So, this qual to….. so, first term will be 0 0 right and next term be the first will be the 0 (Refer Slide Time: 42:12) and next term have to will be the 0 that is different from here. The first term 0 have and next term 0 that is because u transpose and p transpose have alternate set of terms. So, the next terms will be 0 and after that comes 0 minus beta 2 cos pi f x 0 minus beta 2 cos 2 pi x minus beta 2 cos n pi x and this right last 0 minus beta n cos y x n 0 minus beta n cos 2 pi x n 0 beta 2 cos pi x n.
So, we have it here now. we can redo into this following form. All are set to assemble it d k over d t plus. This is a linear matrix and this is non-linear function here. Linear and non-linear and you can check this at home. So, we have a1 kai (Refer Slide Time: 39:45) minus a 2 kai and then this term and rest will go into to b f sorry b l and b n l non-linear. So, all though this terms are kind of messy, I have to admit this is as prety equation I can get out this. I think it is really beautiful. So, if we linearize, this term will drop out. And, we have d k dt plus some linear operator times kai0. For full thing you have linear part and nonlinear part. I stop here.

Keywords: Galerkin technique, Evolution equation