

Industrial Instrumentation
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Lecture - 21
Problems and solutions on Industrial instrumentation

Welcome to the lesson 21 of Industrial Instrumentation. In this particular lesson, we will do some exercises. That means I will give you some problems and also we will provide the solutions. The best way to solve the problem you first do not look at the solution. First try to solve the problem and then see whether the, whatever the answer you got it, that is getting, I mean tally it with the results which we have given, right? So, we will particularly solve in this lesson the problems on the LVDT and then pH probe and McLeod Gauge. As you know, the McLeod Gauge is used for the low pressure measurements, pH is used for the, pH probe is used for the pH measurements and so LVDT, LVDT already we have covered. So we will have, solve different tops of, different types of problems of LVDT, right?

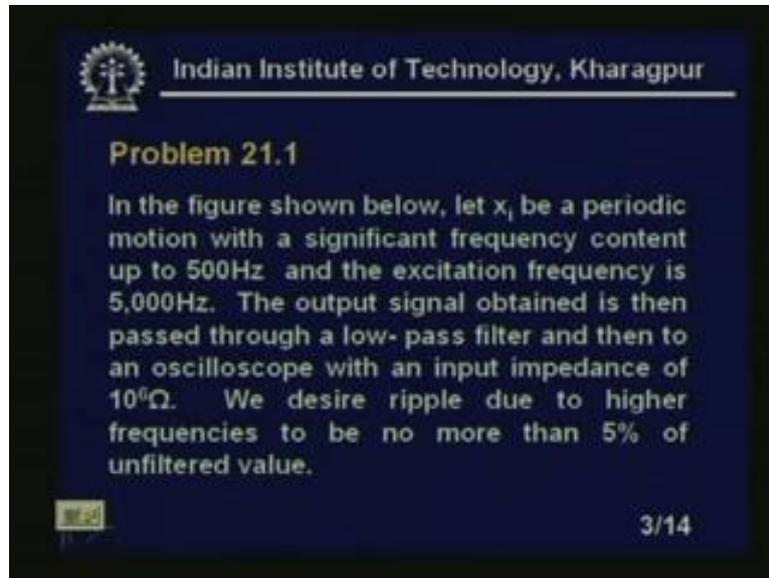
So, let us look at the contents of this lesson. So, these are problems and solutions on industrial instrumentation.

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The contents are the problems and solutions – LVDT, linear variable differential transformer, pH meter as well as McLeod Gauge, right? So, let us look at the problem.

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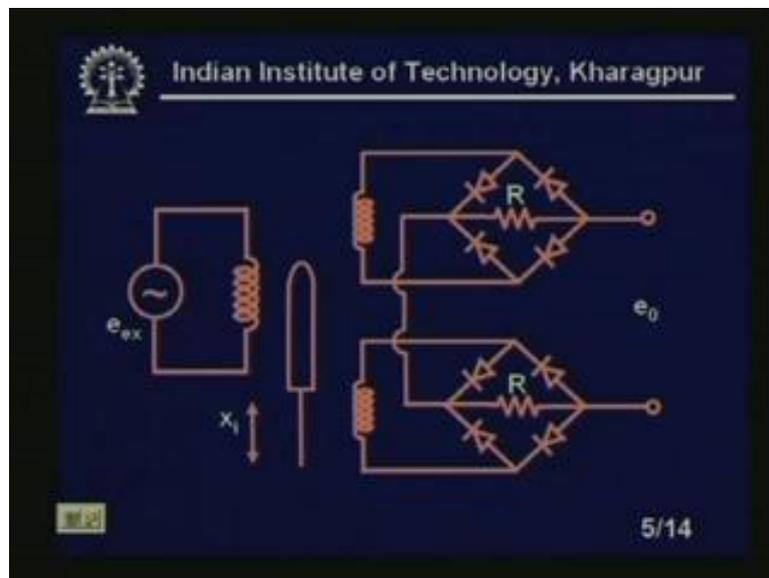
Problem 21.1

In the figure shown below, let x_i be a periodic motion with a significant frequency content up to 500Hz and the excitation frequency is 5,000Hz. The output signal obtained is then passed through a low-pass filter and then to an oscilloscope with an input impedance of $10^6\Omega$. We desire ripple due to higher frequencies to be no more than 5% of unfiltered value.

3/14

The problem 21.1, you see it is telling, in the figure shown below which will come in the slide 4, let x_i be a periodic motion with significant frequency that you can see that is no problem.

(Refer Slide Time: 2:22)



You can see here the, this is our LVDT, right, with a phase sensitivity demodulations. You see the, we have used here four diodes here, four diodes here and we have discussed this in details, right? So, we make the algebraics, I mean summation of the two voltage to get the voltage output and this is the input to the system and this is the excitation. E_{ex} is excitations of our LVDT and x_i is the input motions or the input of the system and e_y is the output of the system of the LVDT, right? Let us go back.

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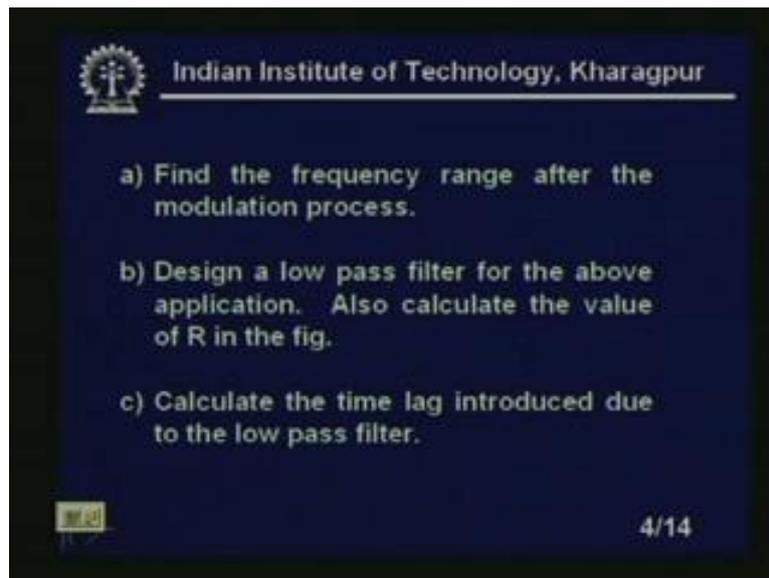
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3/14

In the figure shown below, let x_i be periodic motion. So, this time we are measuring the periodic voltage, I mean periodic signals, right? So, in the figure shown below, let x_i be a periodic motion with a significant frequency content up to 500 Hertz and the excitation frequency is 5000 Hertz. The output signal obtained is then passed through a low-pass filter and then to an oscilloscope with an input impedance of 10 to the power 6 ohm. So, just you can see, just for the sake of problem solving you have to take an, usually the input impedance are slightly more or even 10, 10 mega ohm or more. So it is, in this case we have taken 1 mega ohm. We desire the ripple due to higher frequencies to be no more than 5% of the unfiltered value, right?

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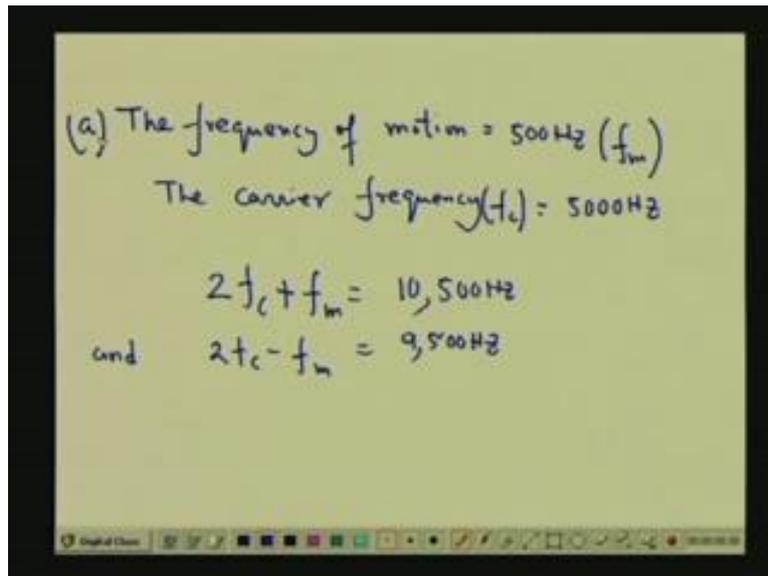
- Find the frequency range after the modulation process.
- Design a low pass filter for the above application. Also calculate the value of R in the fig.
- Calculate the time lag introduced due to the low pass filter.

4/14

Find, number a find the frequency range after the modulation process, excuse me, design a low pass filter for the above application, also calculate the value of R in the figure. The resistance values which we have shown in the diagram that resistance you have to calculate and c, calculate the time lag introduced due to the low pass filter, right? This is our, all the problems. So, these we will solve one by one, right? So, let us see, let us look at how we can solve this problem, right?

Now, let me take a blank page, right? The diode bridge shown in the figure, as you know, act as a phase sensitivity demodulator. Now, frequency of the motion is 500 Hertz, right?

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(a) The frequency of motion = 500 Hz (f_m)
The carrier frequency (f_c) = 5000 Hz

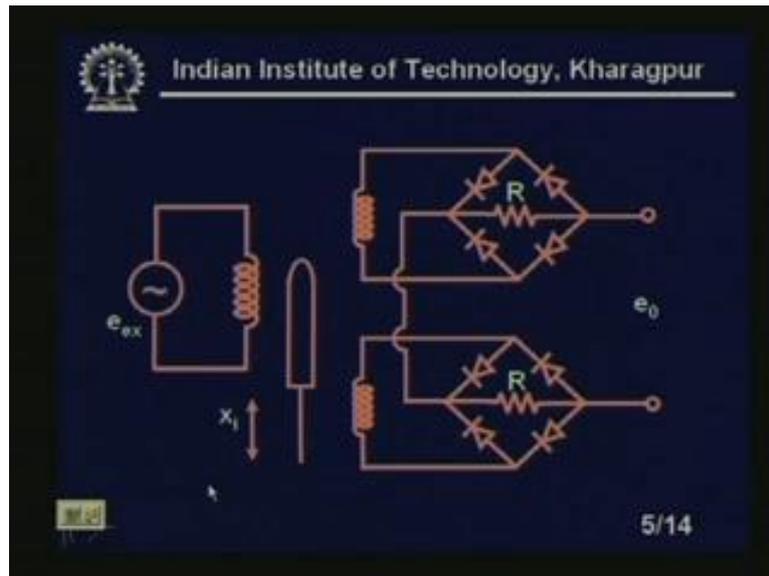
$$2f_c + f_m = 10,500 \text{ Hz}$$

and $2f_c - f_m = 9,500 \text{ Hz}$

The frequency of motion, frequency of motion is 500 Hertz. This is actually is f_m , right? The carrier frequency, the carrier frequency f_c , this is 5000 Hertz, right and after the modulation process, other than the motion frequency we will have frequencies, two frequencies we will get. The two frequencies are two f_c plus f_m plus f_m equal to, this will become 10,500 Hertz and $2f_c$ minus f_m , this is 9,500 Hertz, right, excuse me. Thus, we can say that the frequency range will lie within these particular signals which we have given. Now, this is a problem, I mean a, we have solved. Now let us look at the part b.

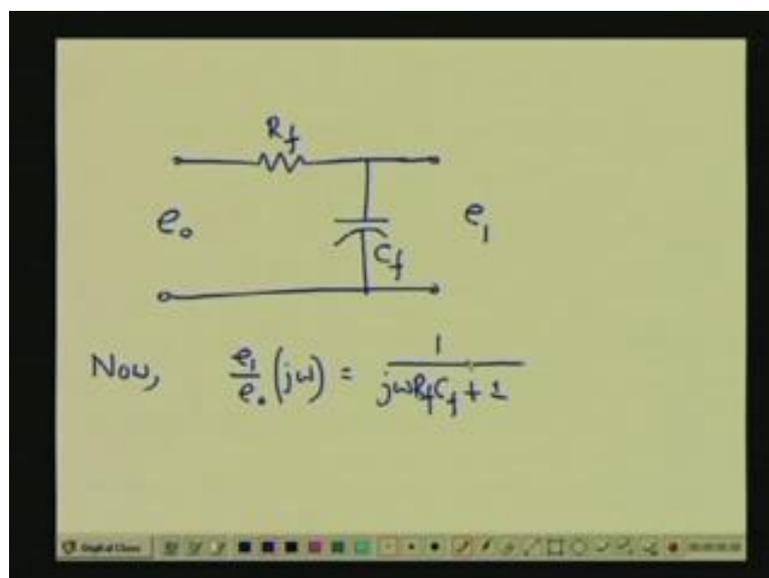
In part b, we are telling that the design a low pass filter for the above application. Also, calculate the value of the resistance R of the phase sensitivity demodulator circuit as shown in the figure, right? You see, let us go to figure, so which will be more clear. So, let us take a new, first go back, right? So, you can see here.

(Refer Slide Time: 7:00)



So this is our figure, so this R we have to calculate and the low pass filter, because we need a low pass filter to get the actual signals that we say movement of this one that correspond to that what is the voltage output that we will get? So, we will find that thing. So, let us take a blank page again, right? Now, in order to design the low pass filter, we first need to choose, excuse me, we first need to choose the order of the filter that will give us our desired result. Now, let us first consider a first order filter, right?

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The first order filter will look like, this input is coming from the phase sensitive demodulator output. This output we are designated as e_1 and this resistance is R_f and this resistance is C_f , right? Let us choose that. Now, as you know we will have e_1 by e_o $j\omega$ will be given by 1 upon $j\omega R_f C_f + 1$. Now, it is given in the problem that the resistance due to higher frequencies to be less than 5% of the unfiltered value. So, for higher frequencies we consider the 9500 Hertz and above. So, if it is 9500 Hertz, then obviously I can write the equation which looks like this.

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The image shows a handwritten derivation on a green background. The equations are as follows:

$$0.05 = \frac{1}{\sqrt{\omega^2 \tau_f^2 + 1}}$$

$$\tau_f = R_f C_f$$

$$\omega = 2\pi \times 9,500$$

$$\therefore \tau_f = 0.0003348 \text{ s}$$

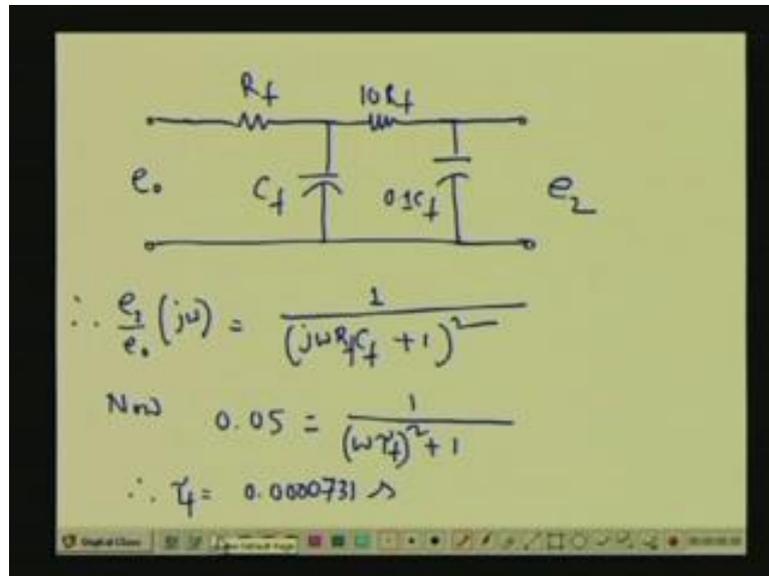
$$\left| \frac{e_1}{e_0} \right| = \frac{1}{\sqrt{(2\pi f \tau_f)^2 + 1}} = 0.69$$

That 0.05 equal to 1 upon root over omega square tau f square plus 1, where tau f equal to R_f into C_f that is the time constant of the filter circuits just we have drawn and omega, obviously is equal to 2 pi into 500, right? Sorry this is, if I take, so omega is equal to 2 pi into 9500, so or tau f, the time constant of the circuit will be given by 0.0003348 second, right? For this T_f we have the amplitude ratio, you know. So, for this T_f , the amplitude ratio we will get that means e_1 by e_o you take mod of that. It will become 1 upon root over 2 pi f m tau f whole square plus 1. So, this will give you 0.69.

Thus we see that the, that the high frequency portions of the x_i will be distorted considerably, right? So, high frequency component of this x_i that means the movement x_i is the movement to the, our LVDT core. So, that will be distorted

considerably. So this, it, inference is that this type of first order section will not be suitable for this type of application. So, let us take a second order section, right? So, if you take a second order section, it will be like this.

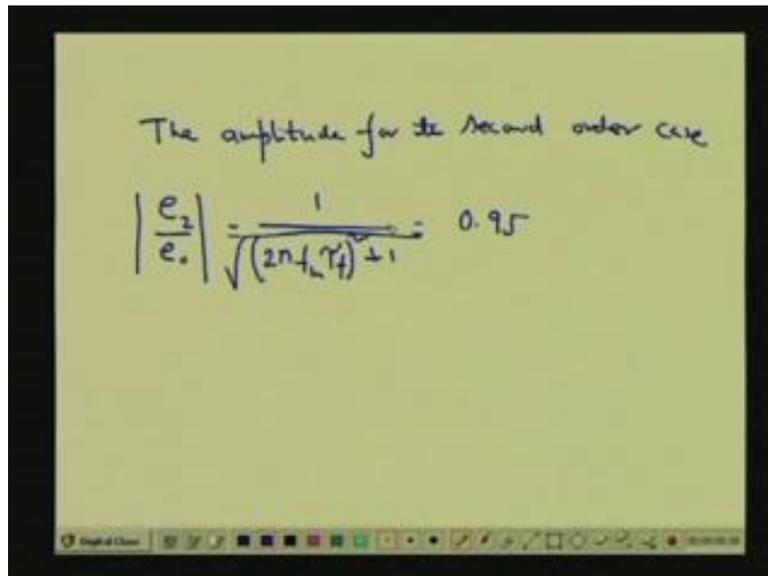
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The filter will look like, these are all passive filters. So, most of the cases it will suffice, because if you use the active filter there are some advantages, as well some there are disadvantages also. This is e_2 , right? This is R_f as before, this is e_1 , this is C_f . So, this is $10R_f$ we have taken and this is we have taken point C_f , right? So, e_2 by, I can write e_2 by e_1 $j\omega$ is equal to 1 upon $j\omega R_f C_f + 1$ whole square, right?

Now again let us check whether this, I mean is good for my work. So, .05 that we have done before also for single order filter, $\omega \tau_f$ square plus 1, right, so τ_f will come up as, τ_f will come up as 0.00731 second, right? So, τ_f will come up as, so I can write like there. So, the τ_f will come up as 0.000731 second.

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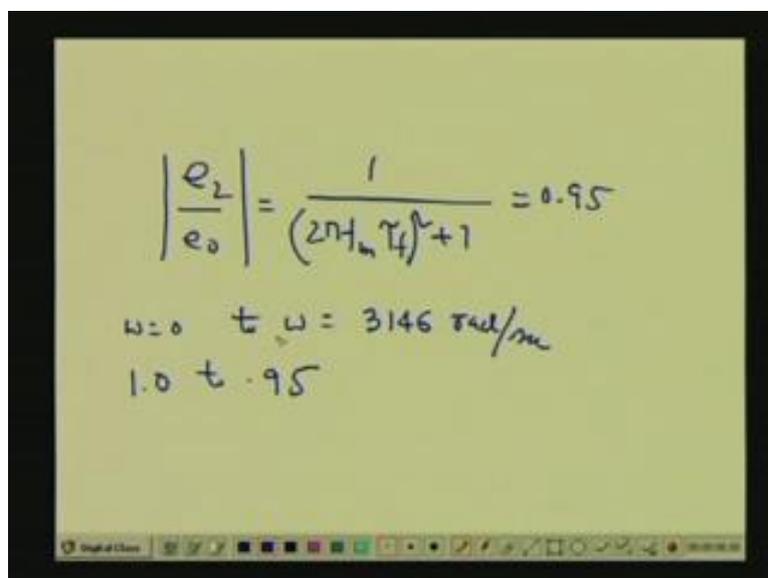


The amplitude for the second order case

$$\left| \frac{e_2}{e_0} \right| = \frac{1}{\sqrt{(2\pi f_m \tau_f)^2 + 1}} = 0.95$$

So, if I look at the amplitude ratio for the, for the second order case, so the amplitude ratio for the second order case will be, amplitude ratio for the second order case is given by e_2 by e_0 equal to 1 by $2\pi f_m$ into τ_f square plus 1 . So, this will give you point, 0.95 , right, clear? So, this will give you this value. I think there will be one square root here, is not it? So, this will give you the, thus you see that the amplitude ratio from ω naught will be, so the amplitude ratio 1 by ω naught, I am sorry, so actually this will be, let me write down again.

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$$\left| \frac{e_2}{e_0} \right| = \frac{1}{\sqrt{(2\pi f_m \tau_f)^2 + 1}} = 0.95$$

$\omega = 0 \quad \tau = 3146 \text{ rad/s}$
 $1.0 \quad \tau = 0.95$

So, the amplitude ratios will be e^{-2} by e^0 , e^{-2} by e^0 equal to $1 / (1 + 2\pi f m \tau)^2$, which will give you 0.95, right, clear? So, if this is the case, so I can say that the amplitude ratio from $\omega = 0$ to $\omega = 3146$ radians per second is nearly flat. That means the amplitude ratio from $\omega = 0$ to $\omega = 3146$ radians per second is nearly flat. That is from 1 it will be 1 and it will fall to 0.95, right. So, it is reasonably good. Now, in order to determine the, the values of R_f and C_f , we use the fact that each stage in the process must have an impedance 10 times more than that of the previous stage. Now, you see the oscilloscope has an input impedance of 1 meg ohm, is not it?

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Handwritten mathematical derivations on a green background:

$$\therefore 10R_f = 10^5 \Omega$$

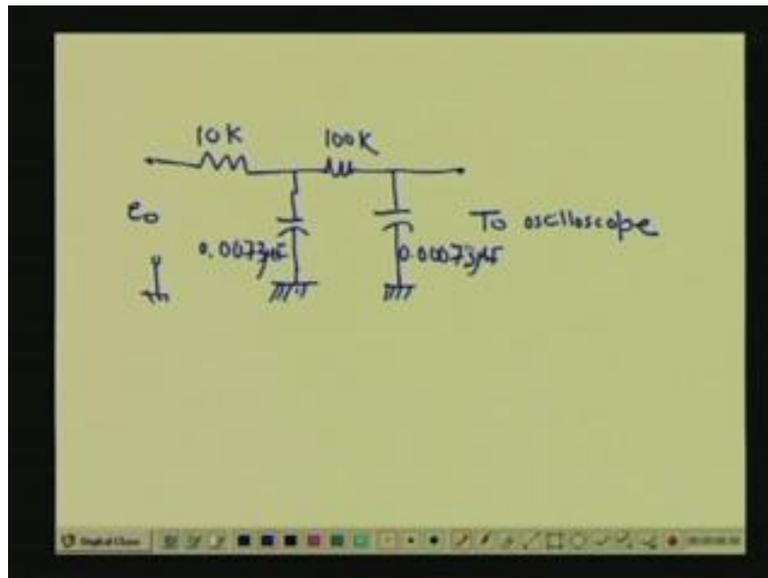
$$R_f = 10^4 \Omega$$

Now, $C_f R_f = 0.0000731$

$$\therefore C_f = 73 \times 10^{-6} / 10^4 = 0.0073 \mu\text{F}$$

If the oscilloscope has an input impedance of 1 meg ohm, I can write $10 R_f$ equal to 10^5 ohm. So, R_f we can take equal to 10^4 ohm, right and now $C_f R_f$ equal to 0.0000731. So, C_f will be equal to 73×10^{-6} by 10^4 . So, it is coming up as 0.0073 micro farad, right? So, ultimately we get the circuit which will look like the filter circuit. Let me take a new page.

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Sorry, this will be again, right, so let us go to pen. So, this will be again resistance. So, let me take the eraser again, so this is gone. So, we take the pen. So, this will be a capacitor, right? So, between this we will take the output, right? Here, I will get the input or I can say or between this and this the output from the phase sensitive demodulator will come and here the output from the, output which will go to the oscilloscope, to oscilloscope, right? The values are 10 kilo ohm, 100 kilo ohm. Then, 0.0073 micro farad, 0.00073 micro farad, right?

Now, the third part, the part c we have to find the phase angle. So, let us do that problem. Let me take a new page.

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(c) To determine the phase angle

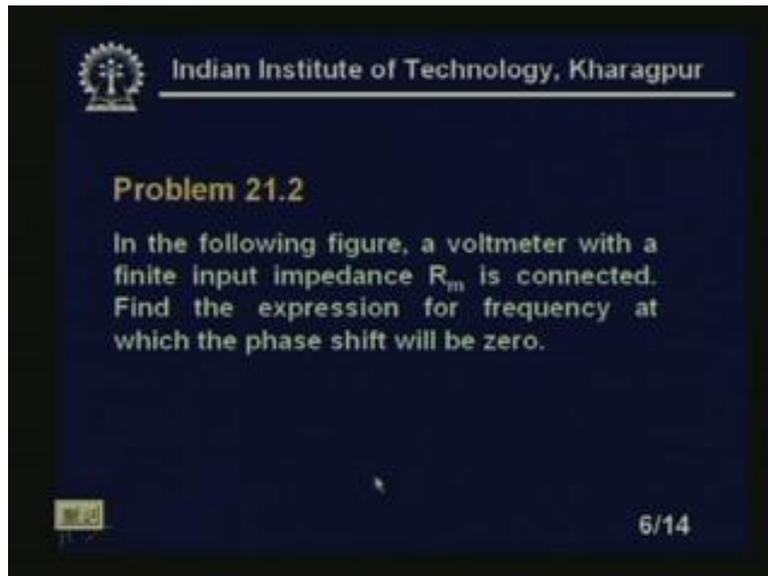
$$\angle \frac{e_2(j\omega)}{e_0} = - \tan^{-1} \frac{2\omega\tau_f}{1 - \omega^2\tau_f^2}$$
$$= -25.8^\circ$$

Delay = $\frac{25.8}{57.3 \times 3140} = \underline{\underline{144 \mu s}}$

So the problem is, solution to the part c of the problem, problem 21.1, to determine the phase angle, a phase angle e_2 by e_0 $j\omega$ minus of tan inverse $2\omega\tau_f$ upon $1 - \omega^2\tau_f^2$, so this will give you minus 25.8 degree, right? So, the phase angle will introduce a delay which can be computed as, so the delay it will produce will be given by 25.8 upon 57.3 into 3140, which is equal to 144 micro second. These are the, all solution to the problem number 21.1.

So, now we will go to problem 21.2 that is also an LVDT. So, let us look at that, right?

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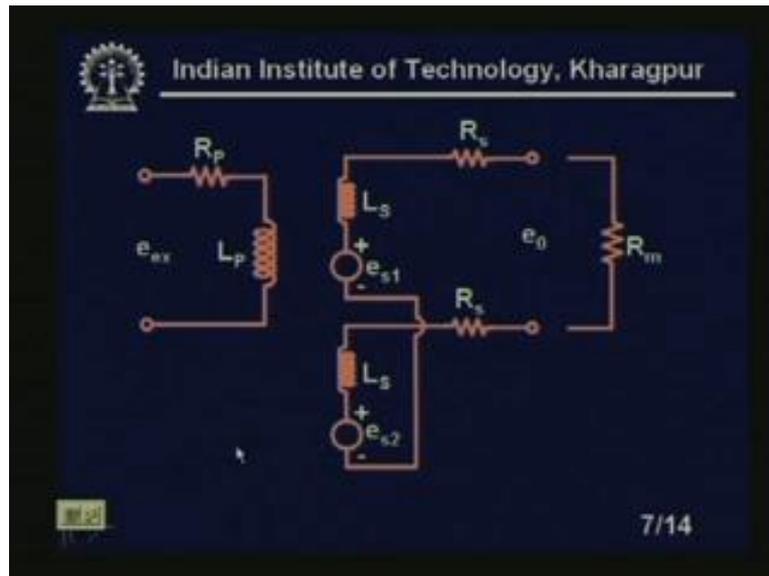


The slide features the IIT Kharagpur logo and name at the top. The main text reads: "Problem 21.2. In the following figure, a voltmeter with a finite input impedance R_m is connected. Find the expression for frequency at which the phase shift will be zero." A small navigation icon is visible in the bottom left, and the slide number "6/14" is in the bottom right.

Problem 21.2, in the following figure a voltmeter with a finite input impedance R_m is connected. Find the expression for the frequency at which the phase shift will be zero. In many situations this is very good, I mean, I mean problem is to know if the manufacturers also will give you some frequency, manufacturer of LVDT will give you the frequency, where the phase shift will be zero, right? So, it will be utilized, that frequency can utilized to make the input output phase shift zero. Because you see the, we have seen when we discussed LVDT to kill this phase shift we have to use lead lag network.

In the case of leading phase angle we have to use a lag network, in the case of lagging phase angle we need a lead network. So, that type of additional circuitry we can dispense off, if I use a particular frequency of signal when there is no input output phase shift, right? So, it is important in that case. So, how will I know, how will I know what is the frequency, where the phase shift will be, input output phase shift will be zero? Once you solve this particular problem 21.2, so you will find that the, we can find the frequency where the phase shift is zero. Typically the manufactures also supply this particular frequency, right? Let us go now.

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This is our circuit, so let us go back.

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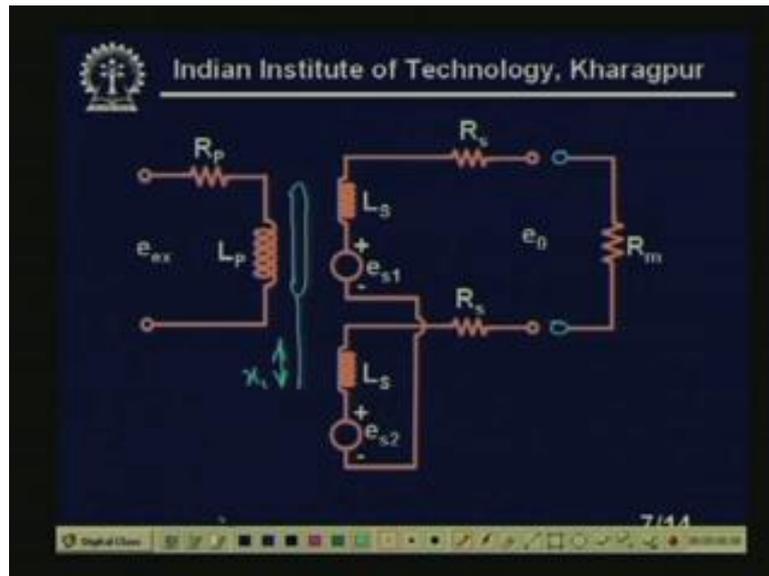
Problem 21.2

In the following figure, a voltmeter with a finite input impedance R_m is connected. Find the expression for frequency at which the phase shift will be zero.

6/14

You see, this is our, in the following figure, a voltmeter with a finite input impedance R_m is connected. Find the expression for the frequency at which the phase shift will be zero.

(Refer Slide Time: 21:43)



This is our circuit. You can see here, so we have connected a voltmeter here **actually**, see, we have connected a voltmeter here. We have connected a voltmeter here, right, to get a phase shift, I mean to measure this voltage, right? So, let us first compute, right, all the values are given. L_p is the inductance of the primary side, in the resistance of the primary side. L_s is the inductance of the one secondary, L_s is the inductance of another secondary. So, we should have one core here which we have not shown, right, which has a moment χ_m , right? So, this is our problem. So, let us solve the frequency, let us find the frequency when the phase shift will be zero, right? Let us take a blank page, right? You see, what will happen here? Anyway, let us first go back to the, let us take a blank page.

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Soln 21.2

$$i_p R_p + L_p D i_p - (M_1 - M_2) D i_s - e_{ex} = 0$$

$$(M_1 - M_2) D i_p + (2R_s + R_m) i_s + 2L_s D i_s = 0$$

$$\frac{e_o(D)}{e_{ex}} = \frac{R_m (M_2 - M_1) D}{[(M_1 - M_2)^2 + 2L_p L_s D^2 + L_p (2R_s + R_m) + 2L_s R_p] D + (2R_s + R_m) R_p}$$

So, this is a problem number, solution to the, sorry, solution to the problem 21.2, right and analyzing the, our circuit we have seen that we can write the circuit equations like this. $i_p R_p$ plus $L_p D i_p$, D is the delta operator, we have seen that thing many times before, M_1 minus $M_2 D$ that means D means as you know d by dt , right? So, in this case you see what is that you see here? This is D means d by dt , right? So, it will be, in this case it will be $d i_s$ by dt . In this case it will be $d i_p$ by dt , right, right? Let us take the pen again, fine. $D i_s$ minus excitation voltage equal to zero, right and M_1 minus M_2 in the secondary side also we can write into $D i_p$ that colour, the pen colour changed, it does not matter, R_s plus R_m into i_s plus $2 L_s D i_s$ is equal to zero. So, this will give you the output which will look like, let us take this pen, e_{naught} by excitations D equal to R_m , meter impedance M_2 minus $M_1 D$ upon M_1 minus M_2 whole square plus $2 L_p L_s D$ square plus $L_p (2 R_s + R_m) + 2 L_s R_p D$ plus $(2 R_s + R_m) R_p$, right? Let us take new page.

(Refer Slide Time: 26:32)

$$\frac{e_o}{e_{cx}}(s) = \frac{R_m (M_2 - M_1) s}{[(M_1 - M_2)^2 + 2L_p L_s] s^2 + [L_p (2R_s + R_m) + 2L_s R_p] s + (2R_s + R_m) R_p}$$

$$\frac{e_o}{e_{cx}} = \frac{j\omega R_m (M_2 - M_1)}{j\omega [L_p (2R_s + R_m) + 2L_s R_p] + [(2R_s + R_m) R_p - \omega^2 \{ (M_1 - M_2)^2 + 2L_p L_s \}]}$$

So, the e_o by e_{cx} I can write in s domain equal to $R_m (M_2 - M_1) s$ upon $(M_1 - M_2)^2 + 2L_p L_s$ multiplied by s^2 , second order system plus $L_p (2R_s + R_m) + 2L_s R_p$ multiplied by s plus $(2R_s + R_m) R_p$, right?

This equations if I write in the $j\omega$ domain, so it will look like e_o equal to $j\omega R_m (M_2 - M_1)$ upon $j\omega [L_p (2R_s + R_m) + 2L_s R_p] + [(2R_s + R_m) R_p - \omega^2 \{ (M_1 - M_2)^2 + 2L_p L_s \}]$, like this one. See, now if I want to make the phase shift, input output phase shift zero, then you see that this term that means if I take a different pen, this term, this term, this term and this term will be zero. Then, $j\omega$, j , j will cancel out. So, it will be only the imaginary parts. So, there is no phase shift between input and output. Otherwise there will be phase shift, right?

So, this entire term that means $(2R_s + R_m) R_p - \omega^2 \{ (M_1 - M_2)^2 + 2L_p L_s \}$ should be zero, right? So that, obviously then I can make that I can say that to make the phase shift zero that to make the input output phase, phase shift zero, the last term of the denominator is to be zero. So, let us do that. Take a new page, fine.

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$$(2R_s + R_m)R_p - \omega^2[(M_1 - M_2)^2 + 2L_p L_s] = 0$$

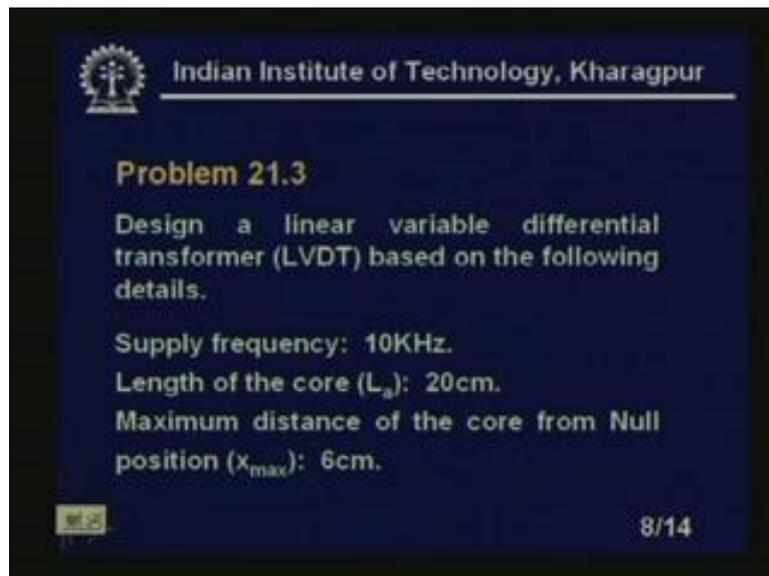
$$\therefore \omega = \sqrt{\frac{(2R_s + R_m)R_p}{(M_1 - M_2)^2 + 2L_p L_s}}$$

$$\frac{e_o}{e_{in}}(j\omega) = \frac{R_m(M_2 - M_1)}{L_p(2R_s + R_m) + 2L_s R_p}$$

So that means to make phase shift zero I will make 2, sorry take another new page, so that to make phase shift zero, $2R_s + R_m R_p - \omega^2 (M_1 - M_2)^2 + 2L_p L_s$ will be equal to zero. Therefore, ω will be equal to root over $2R_s + R_m R_p$ by $(M_1 - M_2)^2 + 2L_p L_s$, right? So, obviously at that frequency when there is no phase shift between input output, our input output relations will be $e_o = e_{in}$ at ω will be equal to $R_m (M_2 - M_1)$ upon $L_p (2R_s + R_m) + 2L_s R_p$, right? So, this is our final expressions, clear?

Now, let us go to the problem number 22.3, right?

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Problem 21.3

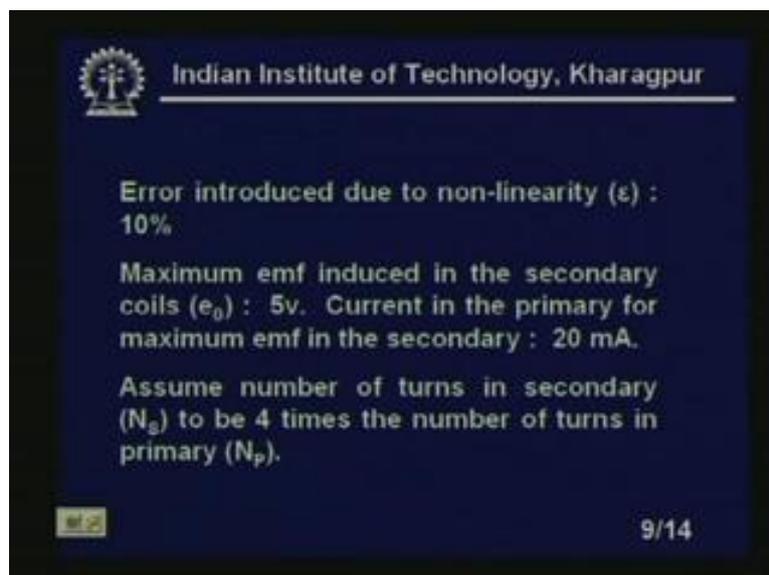
Design a linear variable differential transformer (LVDT) based on the following details.

Supply frequency: 10KHz.
Length of the core (L_a): 20cm.
Maximum distance of the core from Null position (x_{max}): 6cm.

8/14

Now 21.3, I am sorry, design a linear variable differential transformer LVDT based on the following details. What are the details let us look at? Supply frequency is 10 kilo Hertz, length of the core L_a should be 20 centimeter, maximum distance of the core from null position that is that means maximum movement of the core from the null position is 6 centimeter.

(Refer Slide Time: 32:14)



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Error introduced due to non-linearity (ϵ): 10%

Maximum emf induced in the secondary coils (e_o): 5v. Current in the primary for maximum emf in the secondary : 20 mA.

Assume number of turns in secondary (N_s) to be 4 times the number of turns in primary (N_p).

9/14

Error introduced due to non-linearity is 10%, maximum emf induced in the secondary coils e_0 that is .5 volt, I am sorry. Current in the primary for maximum emf in the secondary is fixed also, 20 milliamperes. We assume that number of turns in the secondary to be the, which is the N_s , 4 times the number of turns in the primary.

As you know, the number of turns in the secondary is always higher, right? So, you have to design this LVDT, right? So, let us proceed in this particular design, Let us take a blank page.

(Refer Slide Time: 33:05)

Soln Prob 20.3

The net induced emf $e_0(j\omega)$

$$= j\omega I_p \left[\frac{4\pi N_p N_s \mu_0 \mu_r x}{3s \ln(r_o/r_i)} \left(1 - \frac{x^2}{2l^2}\right) \right]$$

$$|e_0| = \omega |I_p| \left[\frac{4\pi N_p N_s \mu_0 \mu_r x}{3s \ln(r_o/r_i)} \left(1 - \frac{x^2}{2l^2}\right) \right]$$

$$\frac{r_o}{L_s} = 0.05$$

So, the net induced emf $e_j \omega$ of the secondary coil is given by $e_j \omega$. So, this is the problem, solution to problem 20.3, right? So, the net induced emf, so the net induced emf, emf $e_j \omega$ of the secondary coil is given by $j \omega I_p \frac{4\pi N_p N_s \mu_0 \mu_r x}{3s \ln(r_o/r_i)} \left(1 - \frac{x^2}{2l^2}\right)$, right? Now, we have taken the standard notations for this and we can write obviously that that e naught if I take the mod of this, so this will be, we have used all the standard notations. As you know that to solve this problem we must refer to the, our, our lessons on the LVDT, right?

So, because I do not like to repeat all these notations which I have used, so if you, once the person attended to that particular lesson, he can also solve this problem. So, same notations we have used in solving the problem also, right? Now, e naught, I

have, obviously I can write ωI_p equal to $4 \pi N_p N_s \mu_0 \times 3 s$ natural log r_o by r_i $1 - x^2$ by $2 p^2$, like this. We know from the text that the ratio r_o by L_a is about right is a , the ratio r_o by L_a , that means r_o by L_a typically is 0.05, right? Both are in meters or in centimeters.

(Refer Slide Time: 35:43)

Inner radius of LVDT (r_i) = 0.05×20
 $= \underline{\underline{1 \text{ cm}}}$

$\frac{r_o}{r_i}$ varies between 2 to 8

$\frac{r_o}{r_i} = 4$, $r_o = 4 \times 1$
 $= \underline{\underline{4 \text{ cm}}}$

Then it is, therefore the inner radius of the LVDT assembly, so the inner radius of LVDT assembly will be that means r_i will be equal to .05 into 20, because 20 is the length, already given length of the, I think primary, right? Length of the primary is 20 centimeter. So, it is coming as 1 centimeter, right? The ratio r_o by r_i varies between 2 to 8. So, let us take that means ratio r_o by r_i it is also given, varies between 2 to 8. Let us take r_o by r_i 4, then the outer radius of the LVDT assembly is equal to 4 centimeter, right? Now, length of the primary winding, so the outer radius let me write r_o . So, since this is 1 centimeter, so 4 into 1, so it is 4 centimeter, right? Now, length of the primary winding will be given by, take a new page.

(Refer Slide Time: 37:05)

Length of the primary winding

$$p = \frac{x_{\max}}{\sqrt{2\epsilon}}$$
$$= \frac{6}{\sqrt{2 \times 0.1}}$$
$$= 13.4 \text{ cm} \approx \underline{14 \text{ cm (Say)}}$$

Length of the secondary winding

$$S = p + x_{\max} = \underline{20 \text{ cm}}$$

Length of the primary winding, primary winding will be equal to p equal to x_{\max} upon 2ϵ . So, this will come up as 6 upon 2 into 0.1, right, right 2 into 0.1, where ϵ is a non-linearity, so which is we are saying 10%, right, 10% non-linearity. So, .1; so, it will give you the value of p and since it is, 6 is the maximum displacement of your LVDT core, so it is coming up as 13.4 centimeter. So, let us take it 14 centimeter, right, say right?

Now, length of the secondary windings will be, so the length of the secondary winding, secondary winding S will be equal to p plus x_{\max} is equal to 20 centimeter, right, 14 plus 6, it is 20 centimeter, right?

(Refer Slide Time: 38:58)

Soln Prob 20.3

The net induced emf $e_o(j\omega)$

$$= j\omega I_p \left[\frac{4\pi N_p N_s \mu_0 p x}{35 \ln(r_o/r_i)} \left(1 - \frac{x^2}{2l^2}\right) \right]$$

$$|e_o| = \omega |I_p| \left[\frac{4\pi N_p N_s \mu_0 p x}{35 \ln(r_o/r_i)} \left(1 - \frac{x^2}{2l^2}\right) \right] \dots (1)$$

$$\frac{r_o}{L_a} = 0.05$$

Now, in equation 1, if I go back, the equation 1 means that the, which is a mod 1, right, see if I go back whether you can see, yes so this is the equation 1, right? So, so the equation 1, let me take a new page.

(Refer Slide Time: 39:16)

$$|e_o| = 5V, |I_p| = 20mA$$

$p, s, r_o/r_i, \omega$ are given

$$\mu_0 = 4\pi \times 10^{-7} H/m$$

$$5 = \frac{6.28 \times 12070 \times 10^{-8}}{4.2} (N_p N_s)$$

$$N_p N_s = 27427, \therefore N_s = 4N_p$$

$$\therefore N_p^2 = 6856$$

Now from the equation 1, when e naught, mod of e naught equal to 5 volt, I p equal to 20 milliamper, putting the values of p, s, r o by r i and omega that means putting the value of p, s, r o by r i, omega in the equation 1, equation 1 and noting that the

maximum emf is induced at the maximum distance from the null positions, we find that taking permeability of the free space as $\mu_0 = 4\pi \times 10^{-7}$ Henry per meter, so we will get the value of, we can write down that means in the left hand side if e_{naught} is 5 volts, so 5 volt it will come up as six point, after plugging in all these values p, s, r or by r, i, ω in equation 1 and putting μ_0 equal to $4\pi \times 10^{-7}$ Henry per meter, we will get, on the left hand side we will get 5, because e_{naught} is 5 volt.

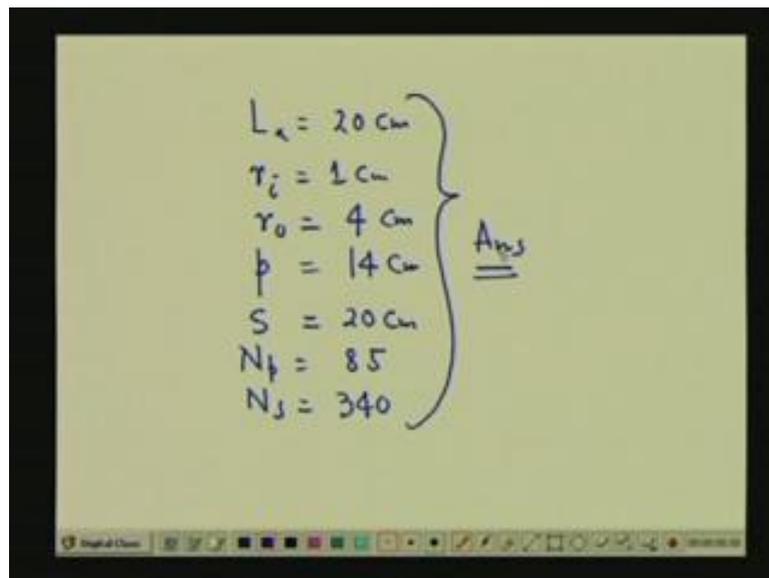
That will be equal to $6.28 \times 12070 \times 10^{-8} \times 4.2 \times N_p \times N_s$, right? Now, $N_p \times N_s$ we will get 27427, right or because N_s we have given that N_s is equal to 4 times of N_p it is given already, right? So, I can right that N_p^2 I will get equal to 6856, right?

(Refer Slide Time: 41:24)

The image shows handwritten calculations on a greenboard. At the top, it says $N_p \approx 83$. Below this, there is a horizontal line. Under the line, it says $N_p = 85$ (Say) and $N_s = 340$. The board also has a toolbar at the bottom with various drawing tools.

So, N_p will be equal to, so N_p almost equal to 83, right? We take N_p , now we take N_p equal to 85, say and N_s we have taken equal to 4 times, 340, right? So, if I write down all the design parameters of the LVDT which we have designed, so it will look like this.

(Refer Slide Time: 41:58)



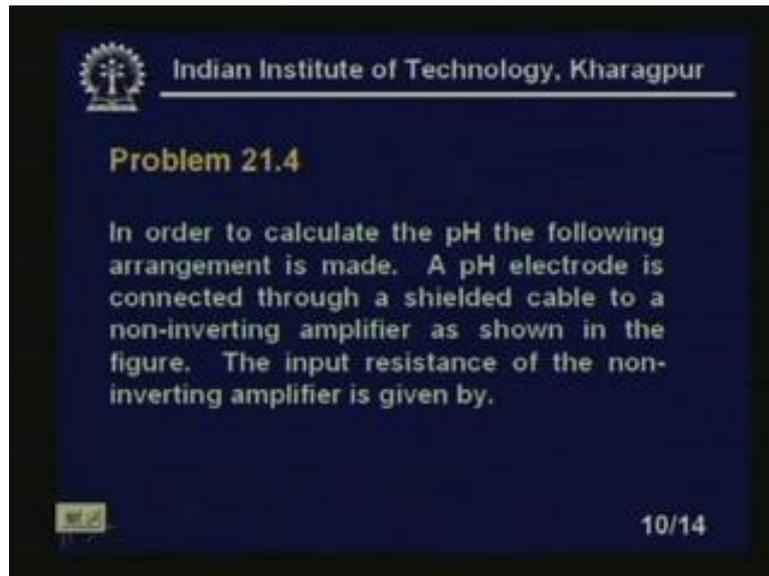
Handwritten notes on a green background, enclosed in a black border. The notes list the following parameters for an LVDT design:

$$\left. \begin{array}{l} L_a = 20 \text{ cm} \\ r_i = 1 \text{ cm} \\ r_o = 4 \text{ cm} \\ p = 14 \text{ cm} \\ S = 20 \text{ cm} \\ N_p = 85 \\ N_s = 340 \end{array} \right\} \underline{\underline{Ans}}$$

So, complete design of LVDT L_a equal to 20 centimeter, right? r_i equal to 1 centimeter, then r_o equal to 4 centimeter. Then, p primary length 14 centimeter, S is 20 centimeter, number of turns in the primary is 85, number of turns in the secondary is 340, right? So, this is a complete solution to the problem number 12.3, right?

Now, let us do one problem on the pH meters. We have solved the problem on the pH meter, we have already done pH sensors in the lesson, I think 20. So, we will solve some problems on the pH also. Let us go back.

(Refer Slide Time: 43:01)



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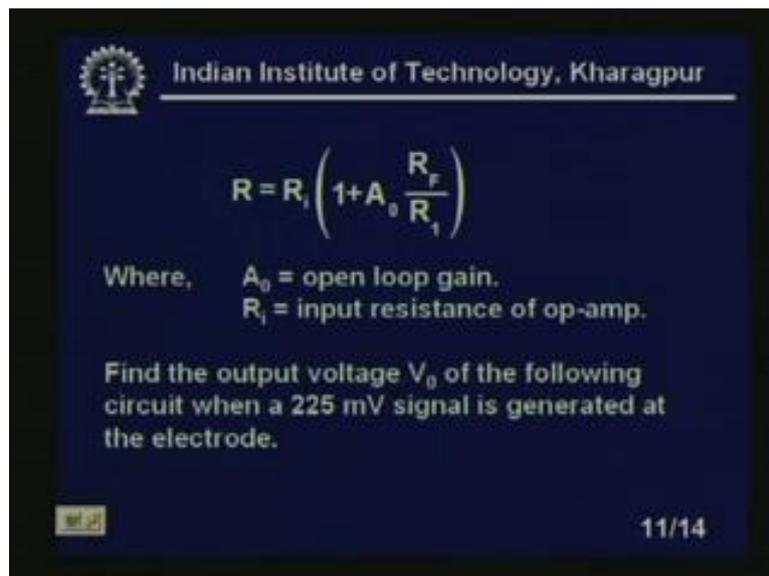
Problem 21.4

In order to calculate the pH the following arrangement is made. A pH electrode is connected through a shielded cable to a non-inverting amplifier as shown in the figure. The input resistance of the non-inverting amplifier is given by.

10/14

Problem number 21.4: the problem is in order to calculate the pH, the following arrangement is made. A pH electrode is connected through a shielded cable to a non-inverting amplifier as shown in the figure, we will show the figure.

(Refer Slide Time: 43:22)



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$$R = R_i \left(1 + A_o \frac{R_f}{R_1} \right)$$

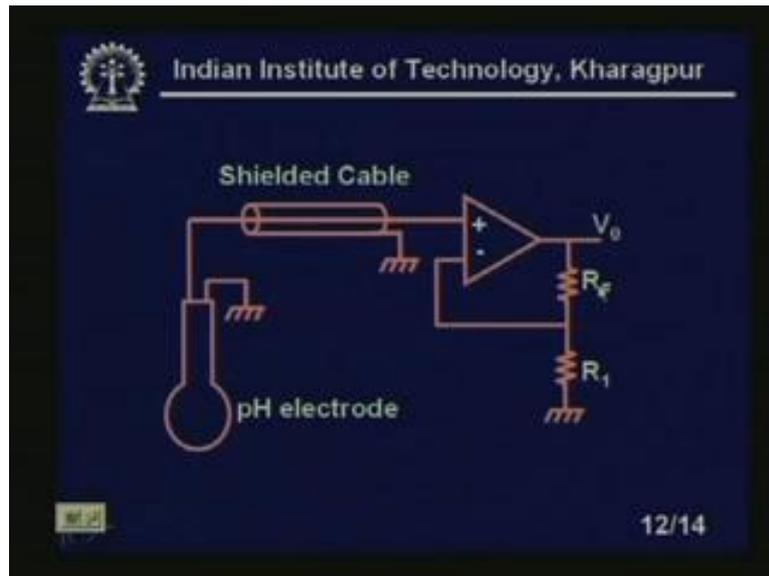
Where, A_o = open loop gain.
 R_i = input resistance of op-amp.

Find the output voltage V_o of the following circuit when a 225 mV signal is generated at the electrode.

11/14

The input resistance or impedance of the non-inverting amplifier is given by R equal to $R_i \left(1 + A_o \frac{R_f}{R_1} \right)$. Everything will be clear in the circuit, so let us go back.

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Let us look at the circuit once again, R_1 , R_f .

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The slide contains the following text and equation:

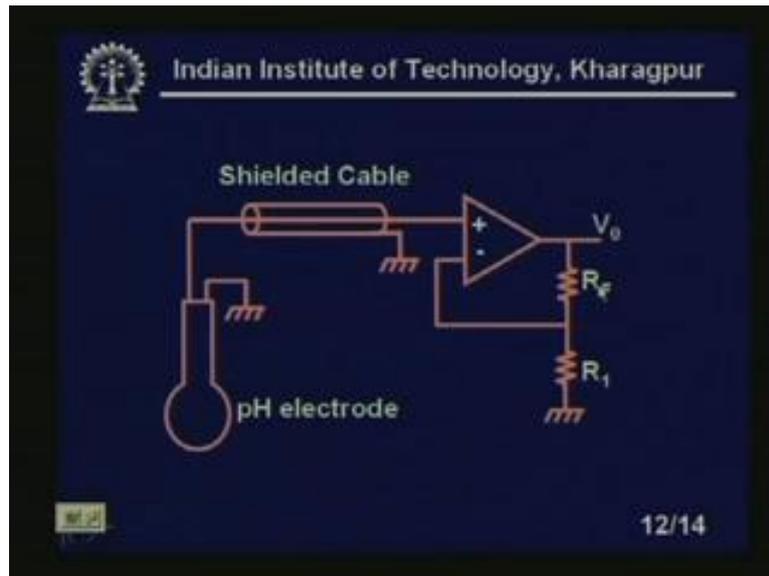
$$R = R_i \left(1 + A_o \frac{R_f}{R_1} \right)$$

Where, A_o = open loop gain.
 R_i = input resistance of op-amp.

Find the output voltage V_0 of the following circuit when a 225 mV signal is generated at the electrode.

Now R_f , R equal to R_i 1 plus A_{naught} R_f by R_1 . A_{naught} is the open loop gain and R_i is the input resistance of the op-amp. Find the output voltage V_{naught} of the following circuit when 225 millivolt signal is generated at the electrode.

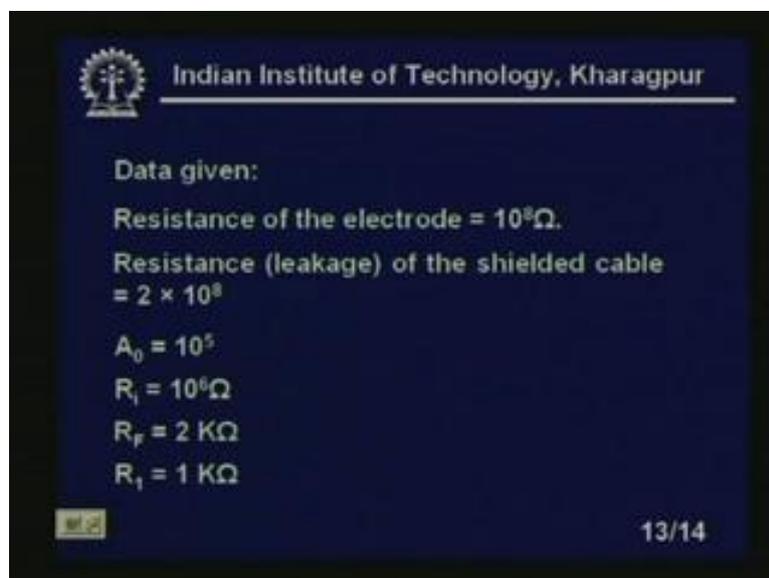
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Shielded cable is, looks like this pH electrode and this looks like this way. So you see, this is buffer amplifiers we have used, supposed to be very high input impedance, so we have used shielded cable.

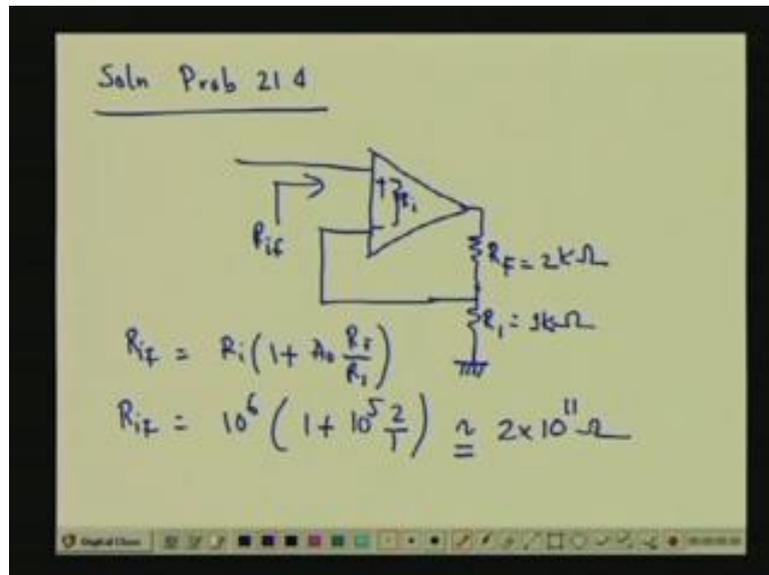
Now, data given, let me see whether we have, yes; find the output voltage V naught of the following circuit when 22.5 millivolt signal generated at the electrode, right, very small signal were generated.

(Refer Slide Time: 44:26)



The data given are: the resistance of the electrode is 10 to the power 8 ohm, right? The resistance leakage of the shielded cable is 2 into 10 to the power 8 ohm, right? A naught, open loop gain of the amplifier is 10 to the power 5, R i is, the input impedance is 10 to the power 6 ohm and R f is 2 kilo ohm and R 1 is 1kilo ohm. So, this is our... Now, let us look at the solution.

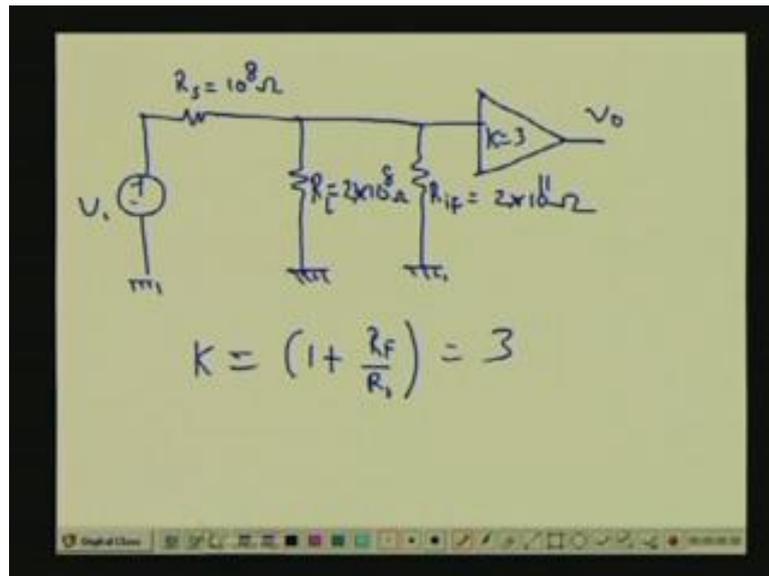
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The solution looks like this, solution to problem 21.4, right? The op-amp given in the problem can be represented like below. You see that is we can, we can draw it like this. So, plus minus, so it has a very high input resistance. This is R i, right, R i and this signal is coming here and this signal is coming here, connected here, connected here this is R f, this is R 1 1 kilo ohm and this is 2 kilo ohm, right? Now, obviously I can write you see that R if, R if is looking from this terminal, R if. R if equal to R i 1 plus A naught R f by R 1 and so, R if equal to 10 to the power 6 1 plus 10 to the power 5 by 1. So, this will give you almost 2 into it is 10 to the power 11 ohm, quite high input impedance, right?

So, now if draw the equivalent circuit for, for measurement systems it will look like this.

(Refer Slide Time: 46:45)



Let me draw the first equivalent circuit of the entire system. This is the pH voltage which we are getting, source impedance, then resistance, this is the impedance of the meter, then we have an amplifier, amplifier has a gain of 3, because if you look at the value of R_f and R_i , you will find the gain as, is of 3. So, this is the output voltage, so this looks like this, right? So, this R_s is 10^8 ohm, source impedance, this is our pH voltage and this is a length R_L actually the, the cable resistance, right? So, this cable resistance or leakage resistance of the cable, leakage resistance of the cable is 2×10^8 ohm; cable resistance to the 8 ohm and R_{if} , input impedance becomes 2×10^{11} ohm that we have just computed, right?

So, that leakage resistance 2×10^8 ohm, I mean input impedance 2×10^{11} ohm, please note it is 11 if I take, so it is 11 ohm. So, this is 11 ohm. Then, the source impedance is 10^8 ohm and gain of the amplifier is 3 that is we can find because the closed loop gain is 3 . So, the R_s is the resistance of the probe or the resistance of the sensor and R_L , R_L is the leakage resistance. Now, the closed loop gain, obviously what is the closed loop gain of the circuit? That is I already put it, it looks like this - $1 + \frac{R_f}{R_i}$.

(Refer Slide Time: 48:41)

$$\begin{aligned}
 V_o &= K V_i \left(\frac{R_L || R_{if}}{R_s + (R_L || R_{if})} \right) \\
 &\approx 3 \times 225 \times \frac{R_L}{R_s + R_L} \quad \left(\because R_{if} = 2 \times 10^{11} \gg R_L = 2 \times 10^8 \right) \\
 &= 3 \times 225 \times \frac{2 \times 10^8}{10^8 + 2 \times 10^8} \\
 &= \frac{3 \times 225 \times 2}{3} = \underline{\underline{450 \text{ mV}}}
 \end{aligned}$$

So, this will give you 3 and analyzing the above circuit we can write, analyzing the above circuit we can write that V_o will be equal to $K V_i R_L \text{ parallel } R_{if}$ upon R_s plus leakage resistance $R_L \text{ parallel } R_{if}$, right, so which becomes, almost equal to 3 into 225, because this is voltage generated by the pH probe into R_L upon R_s plus R_L . Please note this is almost equal to ..., right, like this. Why? Because you see that R_{if} is equal to 2 into 10 to the power 11 ohm, right, which is much, much greater than 2 into that means R_L which is equal to 2 into 10 to the power 8 ohm, right, quite obviously. So, this we can write ultimately, this can we can write 3 into 225 into 2 into 10 to the power 8 upon 10 to the power 8 plus 2 into 10 to the power 8, right? So, this will give you 3 into 225 into 2 upon, let me take eraser, upon 3. So, it is coming up as, since this is in millivolt, 225, so this also will be in millivolt, 450 millivolt. This is the answer, clear?

Now, let us one solve, let us solve one problem on the McLeod gauge. In low pressure measurements we have seen that there is a, McLeod gauge has a, it is typically used for measurements of the low pressure. Now, McLeod gauge has non-linearity. So, if you discard that nonlinearities we can sense as a, the linear sensor, but nonlinearity introduce some amount of error. So, let us look at that. The problem will be on that, let us look at that, right?

(Refer Slide Time: 51:20)

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McLeod Gauge

Problem 21.5

The McLeod Gauge has a bulb of volume 110 cm^3 . The capillary diameter is 1.2 mm . Initially the reading was found to be 3 cm . Later it was found that the observed reading was wrong. Find the error in the measured pressure if the true reading is 2.5 cm .

14/14

This is problem on the McLeod gauge. This is 21.5 I think, yes, 21.5. The McLeod gauge has a bulb of volume, you must refer to the lectures or to the video lectures of the low pressure measurements of the McLeod gauge, right? The McLeod gauge has a bulb volume of 110 centimeter cube, right? The capillary diameter is 1.2 millimeter. Initially the reading was found to be 3 centimeter. Later it was found that the observed reading was wrong and find the error in the, find the error in the measured pressure if the true reading is 2.5 centimeter. So, there some error has been introduced, let us look at that, right?

(Refer Slide Time: 52:21)

Soln Prob 21.5

$$V_B = 11 \times 10^4 \text{ mm}^3$$

The volume of the capillary for the initial reading

$$V_{c1} = \frac{2 \times (1.2)^2}{4} \times 30$$
$$= 33.93 \text{ mm}^3$$

So, solution to problem 21.5, right? The volume of the bulb is given by, volume of the bulb will be given by 11 into 10 to the power 4 millimeter cube, right? The volume of the capillary, of the capillary for the initial reading is given by V_{c1} , let us take that volume, 2 into 1.2 to the power square upon 4 into 30, so which is giving you 33.93 millimeter cube. So, the pressure p_1 in that case, I will take a new page.

(Refer Slide Time: 53:42)

The pressure $p_1 = \frac{3393 \times 30}{11 \times 10^4 - 3393}$

$$= 0.009256 \text{ torr}$$
$$= \underline{1.2332 \text{ Pa}}$$

The pressure p_1 is equal to 33.93 into 30 upon 11×10 to the power 4 minus 33.93 , which is coming up as 0.009256 torr, right? So, it is coming up as 1.2332 Pascal, clear? However the exact capillary volume is, what is the exact capillary volume?

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The exact capillary volume is

$$V_{c2} = \frac{2 \times (1.2)^2}{4} \times 25$$

$$= 28.27$$

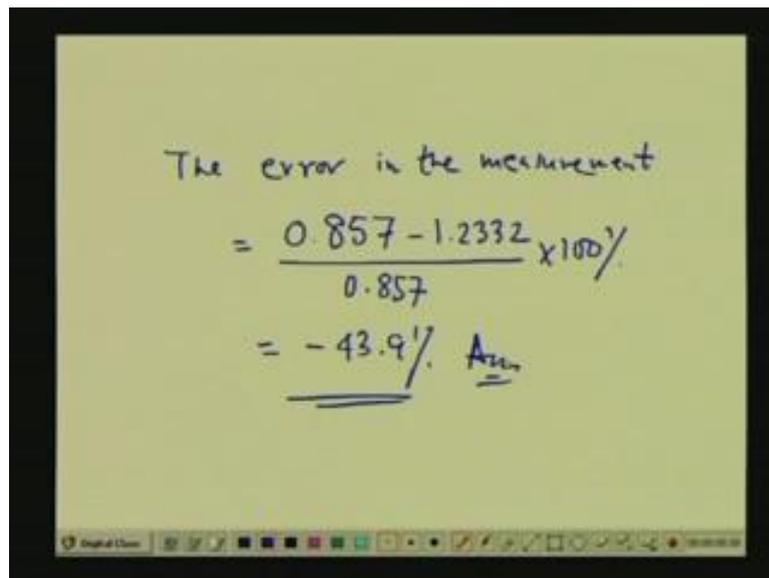
The corresponding pressure

$$p_2 = \frac{28.27 \times 25}{11 \times 10^4 - 28.27} = 0.00642 \text{ torr}$$

$$= 0.857 \text{ Pa}$$

The exact capillary volume is V_{c2} equal to 2 into 1.2 whole square by 4 into 25 . So, this will give you 28.27 millimeter cube, right? So, the corresponding pressure, corresponding pressure will be p_2 if I take that is 28.27 into 25 upon 11×10^4 to the power 4 minus 28.27 is equal to 0.00642 torr, right? So, it is coming up as 0.857 Pascal. So, this is the true pressure.

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The error in the measurement

$$= \frac{0.857 - 1.2332}{0.857} \times 100\%$$
$$= \underline{\underline{-43.9\%}} \quad \text{Ans}$$

So, therefore the error is, so error in the measurement is 857. Let me take, it is 857 minus 1.2332 upon .857 into 100%, which is giving a error of 43.9%, right? So, this is the answer to the system, right? So, with this I come to the end of lesson 21, where we have solved all the problems.

Preview of next lecture

Welcome to the lesson 22 of Industrial Instrumentation. Actually in this lesson and subsequent lesson, we will find that, we will discuss some of the basic signal conditioning circuits. As you know, the signal conditioning circuits are very much necessary in various phases of the sensors, because we need the, whenever the signals are electrical, we need, we need to process, we have to process that about signals and we need some signal conditioning circuits. So, in this lesson and the next lesson we will discuss some of the signal conditioning circuits commonly used in instrumentations. This is lesson 22. Now, this is the signal conditioning circuits I.

(Refer Slide Time: 58:04)



Contents of this lesson - positive and negative feedback topology we will discuss, we will discuss the active filters, we will single amplifier structure.

So, it does not matter if the sensitivity parameters are high, because I have exactly designed the resistance value and the capacitance value. So, in that case even though sensitivities are high, I will get the desired value of the, desired value of the filter parameters, ω_p , Q_p , ω_z , Q_z and capital K, right? So, it is very cheap. It is very small, noises also, because if you increase the number of amplifiers your noise problem will also, I mean will be predominant. So, these are the typical problem in the higher amplifier structures. However, we will see that, we will in a subsequent, I mean lessons that the, we will go for three amplifier structures where we can achieve this orthogonal tuning. Also, the sensitivity figures will also, will be less.

With this I come to the end of the lesson 22.Thank you!