

Foundation Engineering
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Module - 02

Lecture - 02

Lateral Earth Pressure Theories and Retaining Walls – 2

We were discussing about Lateral Earth Pressure Theories and Retaining Walls, in the last class I told you that what exactly do you mean by lateral earth pressure, how it is generated, what is the need for retaining walls. And then, I told you that the soil, which exerts pressure on retaining structures, due to that there can be three kind of situation, because of that lateral pressure the wall may be at rest, it may move towards move away from the soil or it may move towards the soil.

So, we discussed in detail that how we can analyze the case when the wall does not move at all. And then, we started with the case when the wall moves away from the soil, in that while analyzing the situation, we came across two kind of theories.

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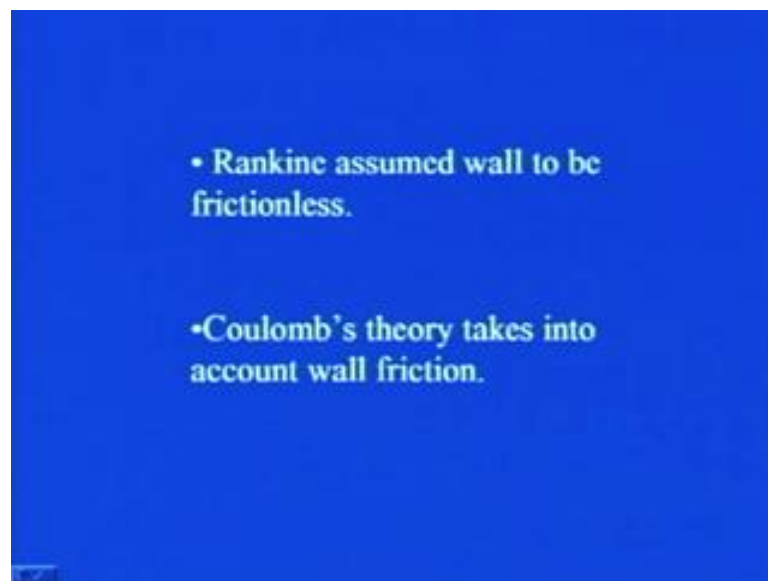


They were Rankine active earth pressure theory and Coulomb's active earth pressure theory. In the last class we saw in detail, that how we can analyze the situation using Rankine's earth pressure theory, in that we saw that two cases in one case the backfill was horizontal and in another case the backfill was inclined at an angle alpha. Then we

saw what is the difference between these two cases and then how your active force per unit length of the wall can be estimated along with its line of action.

So, now let us start with the Coulomb's active earth pressure theory, let us try to have a look that how it is different from Rankine's active earth pressure theory, what are the benefits or advantages or disadvantages of this theory. How it is different from Rankine earth pressure theory, what are the additional parameters that it takes into account, which Rankine theory was not able to take into account.

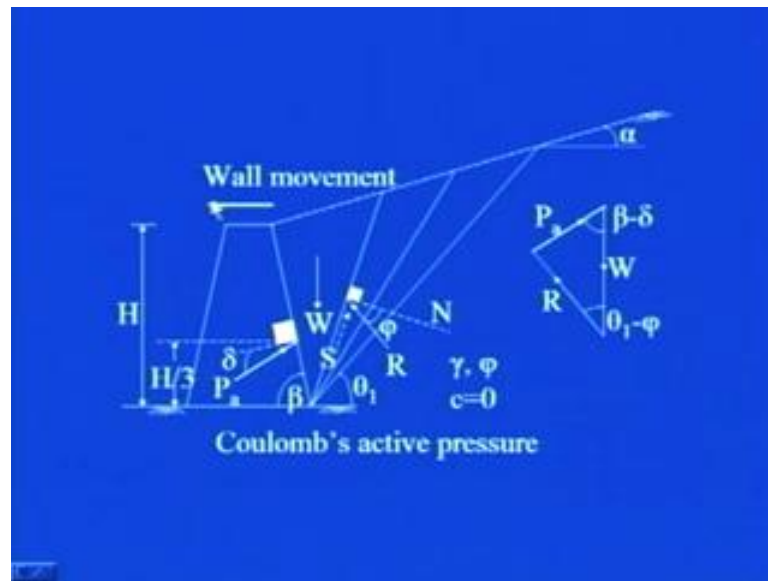
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So, let us start with the Coulomb's active earth pressure theory, see in the previous class we saw that Rankine assumed wall to be frictionless, that is the wall was very much smooth, but Coulomb's theory can take into account the wall friction, which is more realistic in nature. As you see, if there is any retaining wall or any retaining structure, it is just not possible that it will be perfectly smooth, some little friction will be present into that, and that takes into account I mean in the your Coulomb's theory takes into account that wall friction.

Now, let us try to see as we saw in the case of Rankine pressure theory, that what all are the forces, how they are taken into account into the analysis in Coulomb's theory. I will first show you a figure in that all the forces, trial wedges and everything has been shown, we will try to make or develop an understanding of about this theory.

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So, you see here this is a wall its movement is in this direction, this is backfill, this is existing ground surface, this soil is backfill. So, you see the wall movement is away from the backfill, because of that the soil will form a triangular wedge right, it can fail along this wedge, it can fail along this wedge, it can fail along this wedge. This is not a fixed thing, but we talk in terms of any particular trial wedge and then we will try to generalize the process of analysis.

So, this wall is moving away from the soil, which is indicating the active condition, now the backfill is inclined at an angle of α with horizontal. Let us, say if I consider this wedge to be trial wedge and then this edge of the wall is inclined at an angle of β from the horizontal as has been shown in this figure. So, you see the δ is the angle of wall friction, that is this one, then what all are the forces they which will come into picture.

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- β = inclination of back face of wall with horizontal
- α = slope of backfill
- δ = angle of wall friction
- Assuming a failure wedge, one has to obtain the forces acting on it and subsequently active force.

Let us try to see one by one the notations that we have used in the figure beta is the inclination of back face of the wall with horizontal, alpha is slope of backfill, delta is angle of wall friction. Now, if we assume a failure wedge that is this one you see I have assumed this failure wedge, then we have to obtain all the forces which are acting on this particular wedge. And then accordingly by taking the resultant of all the forces, we can find out the active force per unit length of the wall.

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- Weight of wedge, W .
- Resultant, R , of normal and resisting shear forces along the surface. R will be inclined at an angle ϕ to the normal drawn to surface.
- Active force per unit length of wall, P_a . P_a will be inclined at an angle δ to the normal drawn to back face of wall.

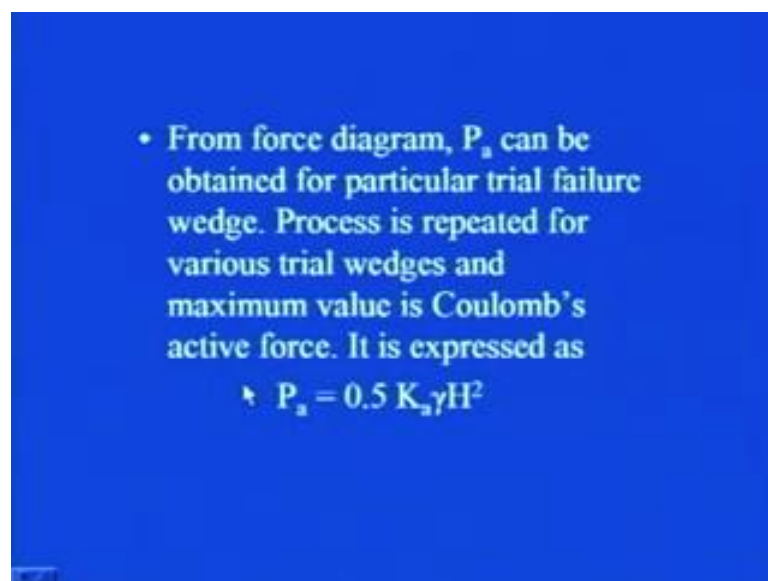
So, let us see that what are the different forces, which is acting on any particular trial wedge. First of all weight of the wedge, which I am denoting as W , second the resultant

R of normal and resisting shear force along the surface, that is this ((Refer Time: 06:14)), you see if I consider this to be surface, this is the resultant R which what resultant it is, it is the resultant of this normal force N and the shear force S and this resultant force will be acting at an angle of phi from the normal to this surface.

See, R will be inclined at an angle phi to the normal drawn to the surface, as in this figure ((Refer Time: 06:44)) you can see, that if this is the figure this is the normal and then from there you make an angle phi and that will be the direction of R. And then along with this weight of the wedge and resultant R as I explained you there will be active force per unit length of the wall, which is I am denoting as P a, this P a will be inclined at an angle delta, which is from the normal to the back face of the wall ((Refer Time: 07:20)).

As you can see here, that if this is the face of the wall, this is the normal to the face of the wall. And from there if I draw a line, which is making an angle delta in this manner, so this will be the direction of active force and then this will be acting at H by 3 from the bottom of the wall. So, what are the main forces, there are three forces which will be acting on any trial wedge, that is first is the weight of the wedge that is W and then the resultant R, which is acting at an angle of phi from the normal to this surface, then active force P a, which is acting at the angle delta from the normal to the back face of the wall.

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Now, from these three forces we can draw the force diagram and then P a can be obtained directly for any particular wedge ((Refer Time: 08:24)). You see, I draw how to

draw this force diagram because, once you know the force diagram simply by measuring any particular length you can know the magnitude of that particular force. I draw now let us try to see that how we can draw this force diagram, I draw a line which is parallel to the line of action of this W , that is this.

And since, we know this particular trial wedge, so we know that what is the weight of this wedge. So, let us say you choose any particular scale, you draw a line which is parallel to the line of action of this W and to that appropriate scale, now you we really do not know that what is the magnitude of this R and the magnitude of P_a , magnitude of P_a we have to find out. So, but we known the angles, if you see from this figure the angle which is subtended between W and P_a is $\beta - \delta$, which will be known to us.

The angle between W and R will be this $\theta_1 - \phi$, where what is θ_1 , θ_1 is the inclination of trial wedge consider trial wedge from the horizontal, as you can see here I am showing this angle by θ . So, if I consider this trial wedge what will be the value of θ , θ will be the angle which this phase will be subtending from the horizontal. So, we know the magnitude of W we know $\beta - \delta$ angle, we know $\theta_1 - \phi$ angle, so first of all we draw a line parallel to the line of action of W and of that particular magnitude.

Now, we draw a line which is parallel to this line that is the line of action of the resultant, what is that, that makes an angle ϕ from the normal to this wedge surface. So, simply by taking simply by with the help of set square what you can do, you can draw a parallel line from this point. Since, we do not know the magnitude of this resultant force R , what we do we really do not know what will be the length of this line, so you simply just draw that line.

Now, what you can do, you can draw a parallel line from this point that is making an angle $\beta - \delta$ from the line of action of W . So, we draw a line parallel to this again we do not know the magnitude of P_a , so we do not know what will be the length of this line. So, you have drawn this line of arbitrary length, you draw this line of arbitrary length, wherever they will intersect that will make this force diagram complete, you see W , R and then P_a .

Now, measuring the length of this particular line and multiplied by the scale that you have chosen to plot this force diagram, you can get the magnitude of active force per unit length of the volume per unit length of the wall. And so you can get the magnitude of

this resultant R also, I hope that this is clear to you let us put it in words in the subsequent slides. You see, that from force diagram this P a can be obtained for any particular trial wedge, as I told you that we really do not know that along which wedge the failure will be taking place. So, we have to try for many wedges, so any particular trial wedge we pick we find out the value of P a, so the same process is repeated for various trial wedges and the maximum value of the active force is known as your Coulomb's active force.

So, you see what will happen you do the analysis for one particular trial wedge, you go for the next one, then you go for the next one what will happen that the magnitude of this force will first go on increasing and then it will go on reducing. So, whatever is the maximum value of the active force that you obtain by this procedure, that will be known as your Coulomb's active force. And it is expressed as P a is equal to 0.5 K a gamma H square, where you know K a we have already discussed as it was in the case of Rankine earth pressure, it is known as active earth pressure coefficient, gamma is unit weight of the backfill of the soil, H is the vertical height of the wall.

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Coulomb's active earth pressure coefficient

$$K_a = \frac{\sin^2(\beta + \phi)}{\sin^2 \beta \sin(\beta - \delta) \left[1 + \frac{\sin(\phi + \delta) \sin(\phi - \alpha)}{\sin(\beta - \delta) \sin(\alpha + \beta)} \right]}$$

- In actual design of retaining walls, angle of wall friction, δ is assumed to be between $\phi/2$ and $2\phi/3$.

Now, you see the although the K a is coefficient of active earth pressure, but here it is how you can determine is little bit different from the Rankines analysis. Because, what happens is that Rankines takes care of the inclination of the back face of the wall plus angle of wall friction and all other this phi and beta aspect. So, you see this K a expression is given by this expression, it is not necessary to remember this long

expression as, the standard values for different corresponding values of beta, phi, delta and alpha this K a value has been given in standard books.

So, you can use those values directly in spite of using this particular expression, now what about this angle of wall friction. See, we really cannot estimate it properly that this will be the exact value of the angle of wall friction delta, so usually from experience point of view from engineering judgment point of view, it has been seen that the value of this delta that is angle of wall friction varies between phi by 2 and 2 phi by 3. So, accordingly you know you have to judge at that particular point, that what should be the value of delta for any particular retaining structures.

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Values of K_a for $\beta = 90^\circ$, $\alpha = 0^\circ$

Φ (deg)	δ (deg) \longrightarrow		
	15	20	25
28	0.3251	0.3203	0.3186
30	0.3014	0.2973	0.2956
32	0.2791	0.2755	0.2745
34	0.2579	0.2549	0.2542
36	0.2379	0.2354	0.2350
38	0.2190	0.2169	0.2167
40	0.2011	0.1994	0.1995
42	0.1841	0.1828	0.1831

As I told you, that the K a value for different beta, alpha and delta angle the these are available. So, I am showing you the typical values of K a for beta is equal to 90 and alpha is equal to 0 degree, what does this mean that is when I say that alpha is equal to 0 degree; that means, that the backfill is not inclined. The backfill is horizontal and when I say that beta is equal to 90 degree, that shows that the back face of the wall is making an angle of 90 degree from the horizontal, that is the back face of the wall is vertical.

So, if these two angles are fixed that is the backfill is horizontal and the back face of the wall is vertical. Then, for different values of angle of internal friction phi and different values of delta corresponding values of K a have been tabulated in this particular table.

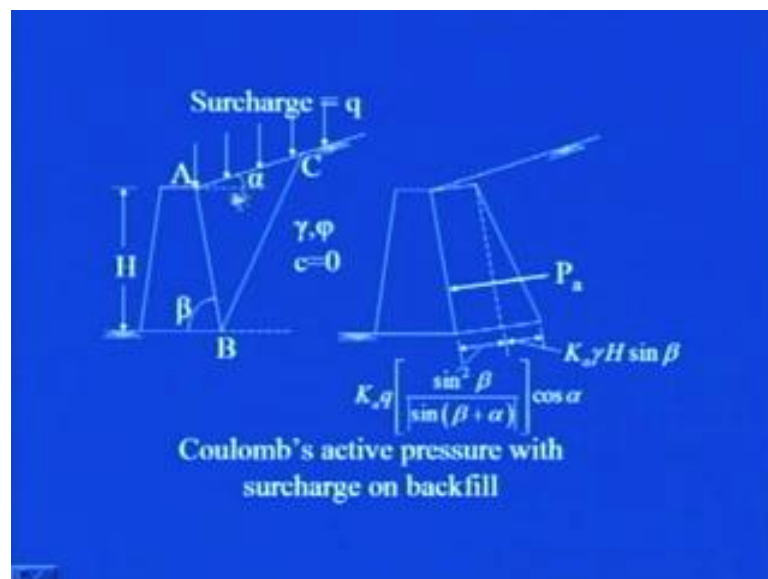
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Values of K_a for $\beta = 90^\circ$, $\alpha = 0^\circ$

Φ (deg)	δ (deg) \longrightarrow		
	0	5	10
28	0.3610	0.3448	0.3330
30	0.3333	0.3189	0.3085
32	0.3073	0.2945	0.2853
34	0.2827	0.2714	0.2633
36	0.2596	0.2497	0.2426
38	0.2379	0.2292	0.2230
40	0.2174	0.2098	0.2045
42	0.1982	0.1916	0.1870

This continues to give you these values also 0, 5, 10 and then correspondingly 28, 30, 32, 34, 36 and so on. So, you can simply pick the value of K_a from these standard tables, which are available in standard text books, now we have seen that first advantage of Coulomb's theory over Rankine's one is that Coulomb's theory can take into account the wall friction, which Rankine's theory could not. Now, let us try to see what is the next additional feature that Coulomb theory takes into account.

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You see here, in earlier case the backfill was inclined and there was no surcharge present on that. In this case the Coulomb's theory is able to take into account plus the any

surcharge, which is present on this inclined backfill, how it takes into account you can see here, that the backfill is cohesion less material that is c is equal to 0, it has unit weight γ , you have angle of internal friction to be equal to ϕ .

Then, the surcharge q as you have seen that how we calculate lateral earth pressure is that, we calculate the vertical stress at the any particular level, we multiply that by coefficient of earth pressure and simply we get the lateral earth pressure. So, exactly in the similar manner, since it is inclined if it would have been the horizontal one, then we would have simply said that K into q , but now since it is inclined. So, you see this is due to the surcharge, this is due to the surcharge part that is K_a into $q \sin^2 \beta$ by $\sin \beta + \alpha$ into \cos of α .

So, this is due to surcharge, now what about the backfill which is present over here, you see this triangular wedge which is being formed shown by this with the magnitude $K_a \gamma H \sin$ of β at Z is equal to H . So, this is going to be the difference with the addition of surcharge on the inclined backfill, so here again once you know once you have drawn the lateral earth pressure diagram, you simply take the area and you can get the magnitude of the total force per unit length of the wall.

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• Active force is calculated as

$$P_a = 0.5 K_a \gamma_{eq} H^2$$

where,

$$\gamma_{eq} = \gamma + \left[\frac{\sin \beta}{\sin(\beta + \alpha)} \right] \left(\frac{2q}{H} \right) \cos \alpha$$

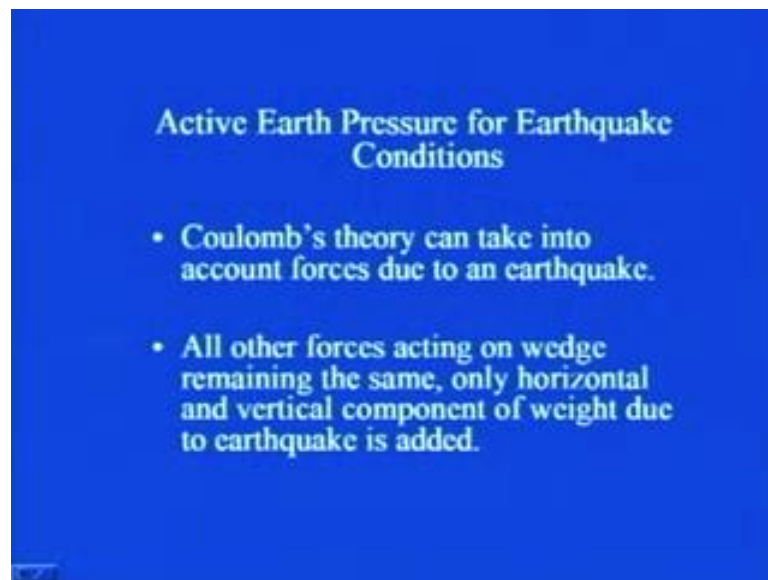
So, you see here the active force is calculated as $0.5 K_a \gamma_{eq} H^2$, that is equivalent unit weight into H square. Where, this equivalent unit weight can be determined by this expression that is γ plus this additional term, see if this q is 0 then this additional term will become 0. So, your γ_{eq} will become equal to γ , which will

be corresponding to the earlier case which we just now discussed, but in case this q is present that is the surcharge is present, your gamma value will be changing or you can say that for mathematical simplicity, we are representing this equation in this particular manner.

That is gamma has been replaced by gamma equivalent and given by this particular expression as $\gamma \cos \beta + \sin \beta \frac{2q}{H} \cos \alpha$. Where, alpha you know is the inclination of the backfill beta is the inclination of the back face of the wall from the horizontal, now these days this was all about in normal condition. But, these days you know that the earthquake in India we are I mean we are facing, so many earthquake these days and so usually in all the cases generally the analysis and as well as the design is carried out considering the earthquake conditions.

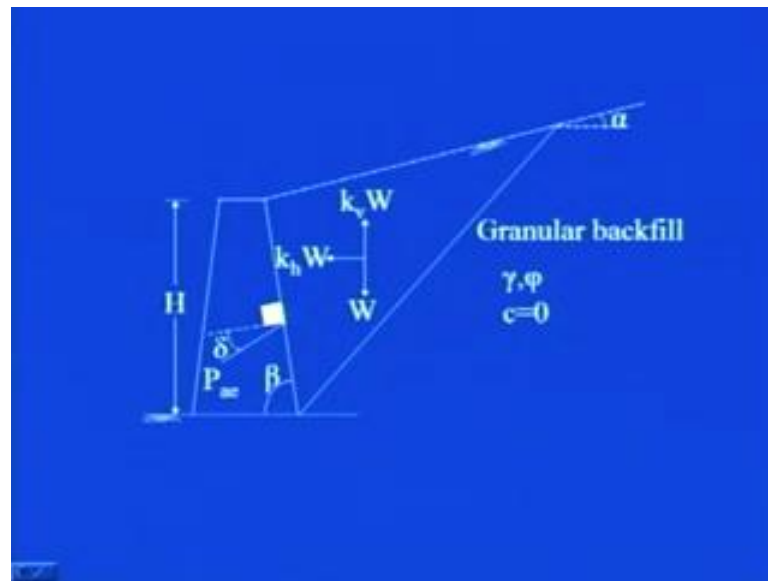
So, it becomes an essential part that you must know the little about that how you can go for analysis and design for earthquake conditions. So, you see that this is again another advantage of Coulomb's theory over Rankine's earth pressure theory, that it can take into account forces due to an earthquake.

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So, what is going to be the difference from the earthquake condition and the condition when the earthquake is not there. All the forces remaining same only horizontal and vertical component of weight due to earthquake will be added.

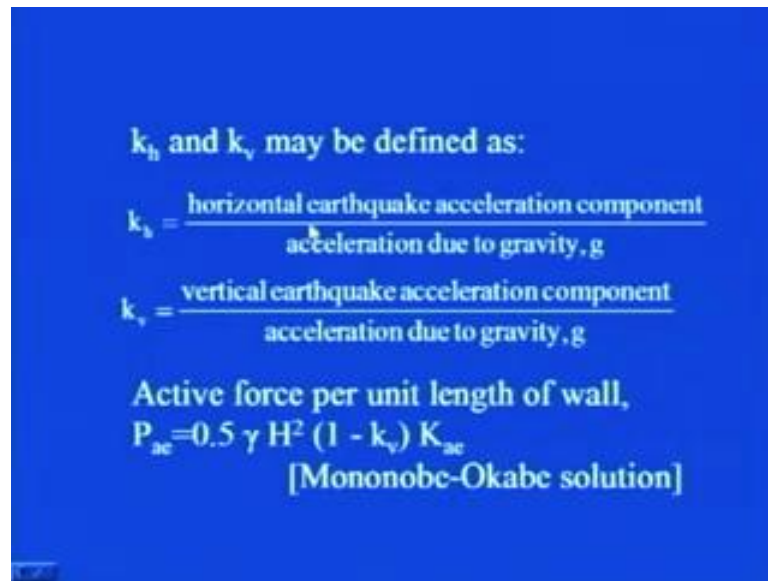
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Now, let us see that how it comes into picture, we have already seen earlier that the weight of the wedge was the only vertical force, which was present when we were analyzing any trial wedge. So, in this case when the earthquake force is considered in that case and opposite vertical force of amount of magnitude K_v into W is considered as shown in this figure. And then, an horizontal component of the weight W , which is K_h into W is considered in this direction, that is in horizontal direction.

You see, all the things remaining same P_a , P_a corresponds to active force due to earthquake condition, a stands for active, e for earth earthquake. Again, this is acting at an angle of δ which this line of action make from the normal to the back face of the wall granular backfill is use. So, c is equal to 0, γ is the unit weight of the soil, ϕ is the angle of internal friction, again in this case inclined backfill which makes an angle of α from the horizontal has been considered. Rest all things remaining same only two additional component of weight due to earthquake will come into picture.

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k_h and k_v may be defined as:

$$k_h = \frac{\text{horizontal earthquake acceleration component}}{\text{acceleration due to gravity, } g}$$
$$k_v = \frac{\text{vertical earthquake acceleration component}}{\text{acceleration due to gravity, } g}$$

Active force per unit length of wall,
 $P_{ac} = 0.5 \gamma H^2 (1 - k_v) K_{ac}$
[Mononobe-Okabe solution]

Now, these K_h and K_v these may be defined as your K_h is equal to horizontal earthquake acceleration component divided by acceleration due to gravity g , K_v is vertical earthquake acceleration component divided by acceleration due to gravity g . Then, in that case your active force per unit length of the wall P_{ac} will result to $0.5 \gamma H^2 (1 - k_v) K_{ac}$. It is beyond the scope of this course to know the derivation of this particular equation, but it can be done easily.

But, what is the main concern is that you must be aware of that how we can analyze and design the wall width or fall earthquake condition, this is a very well known equation and it is known as Mononobe Okabe solution.

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Active earth pressure coefficient, K_{ae} is

$$K_{ae} = \frac{\sin(\varphi + \beta - \theta')}{\cos \theta' \sin^2 \beta \sin(\beta - \theta' - \delta) \left[1 + \sqrt{\frac{\sin(\varphi + \delta) \sin(\varphi - \theta' - \alpha)}{\sin(\beta - \delta - \theta') \sin(\alpha + \beta)}} \right]}$$

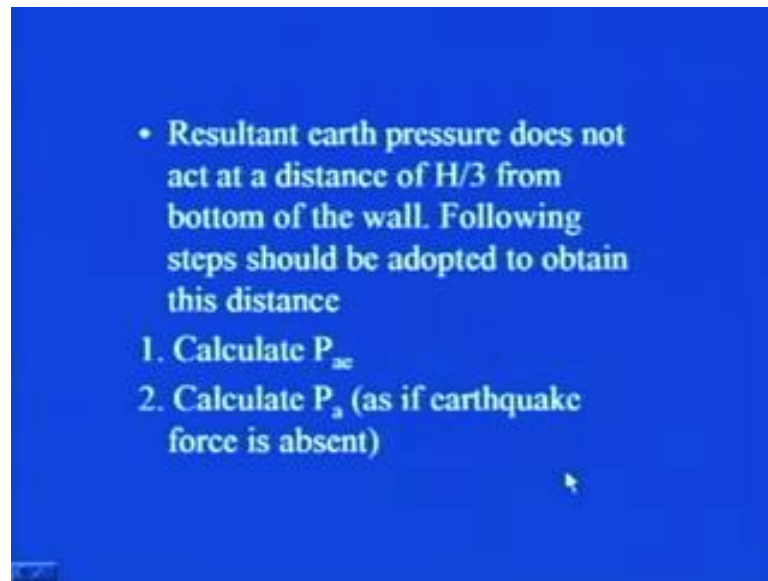
where, $\theta' = \tan^{-1} \left[\frac{k_h}{1 - k_v} \right]$

For no earthquake condition, $K_{ae} = K_a$

Then, in this case the active earth pressure coefficient that is K_{ae} is defined by this particular expression. Where, you know what is φ is angle of internal friction, β is angle of back face of the wall from horizontal and then δ you know angle of wall friction, θ' is define as $\tan^{-1} \left[\frac{k_h}{1 - k_v} \right]$. So, now you can say that from where we are calculating this k_h and k_v our code is giving some of the guideline related to this.

Now, if you see that let us say that what exactly happens, if I put here in this expression no earthquake condition. So, for no earthquake condition this θ' will not be there, so in that case if you compare the earlier expression as I already have discussed with you then in that case this K_{ae} will become equal to K_a .

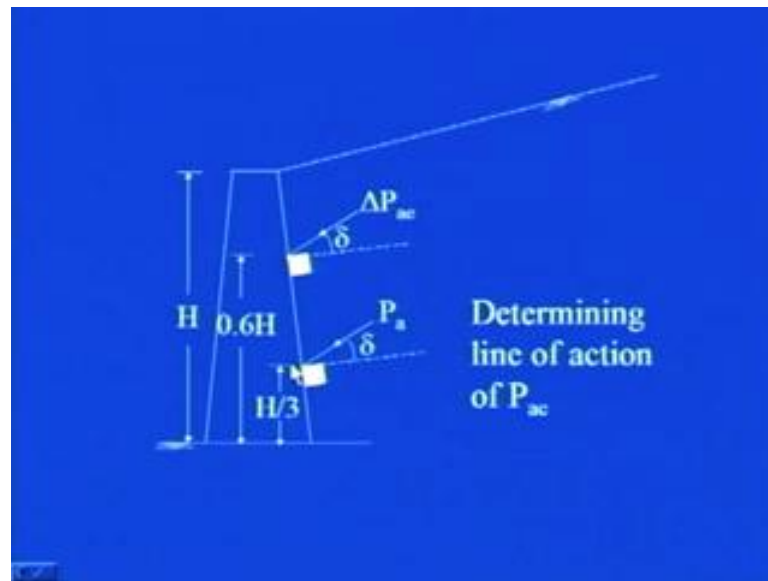
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Now, how we can get the active force that is total active force per unit length of the wall, in case the earthquake is present or in case we are carrying out the analysis for earthquake condition. In this case, the resultant earth pressure does not act at a distance of $H/3$ from the bottom of the wall which was there in earlier case, so we have to follow few steps to obtain the resultant earth force and its line of action, now what are those steps let us try to see one by one.

First of all we need to calculate P_{ae} , as I gave you the expression knowing the value of K_h , K_v , K_a you can calculate P_{ae} . Now, you assume that there is no earthquake present, that is all earthquake forces are absent and then in the simple manner you can obtain the value of P_a , as we did for any normal thing any normal condition.

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Then, what we do is we assume that this P_a is acting at an angle of δ and at a height of H by 3 from the bottom of the wall.

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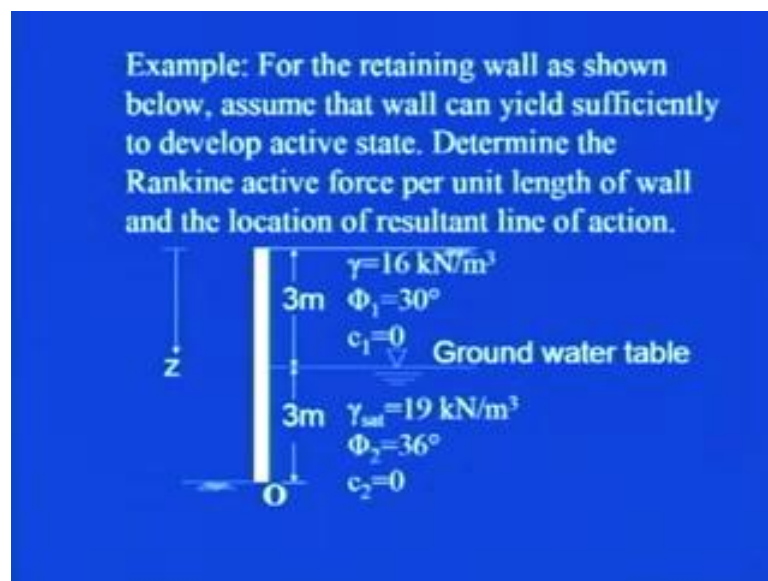
3. Calculate $\Delta P_{ac} = P_{ac} - P_a$
4. Assume that P_a acts at a distance of $H/3$ from bottom of wall.
5. Assume that ΔP_{ac} acts at a distance of $0.6H$ from bottom of wall
6. Calculate the location of resultant as
$$\bar{z} = \frac{(0.6H)(\Delta P_{ac}) + \left(\frac{H}{3}\right)(P_a)}{P_{ac}}$$

Now, what is the third step, third step is we calculate you have to calculate this ΔP_a which is the difference of passive force in earthquake condition to the passive force when the earthquake is or earthquake force they are absent. So, you can calculate this ΔP_a by this P_a minus P_a , then we have to make an assumption that assume that P_a acts at an a distance of H by 3 from the bottom of the wall. Another assumption

which is made is that P_a is assumed to act at a distance of 0.6 times H from the bottom of the wall ((Refer Time: 27:21)).

You see here P_a is acting at $H/3$ P_a is assumed to act as 0.6 times H , so once we know that where they are acting, we can determine the location of the resultant as that is $0.6 \times H \Delta P_a e$ plus $H/3 P_a$ divided by $P_a e$. See, if you take the moment from this particular point what will happen P_a into $H/3$ this moment will be in this manner plus $\Delta P_a e$ into 0.6 times H divided by your P_a that is the resultant force due to earthquake condition and it is line of action.

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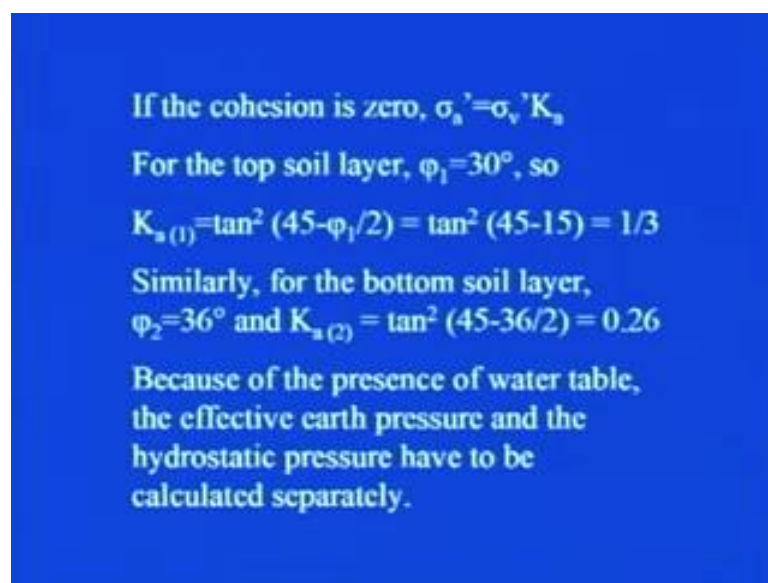
So, you will get the line of action of the force, which is acting due to your earthquake condition. So, we saw that Coulomb's theory can take into account inclined backfill, surcharge on that and it can also account for earthquake condition and angle of wall friction. So, this is what was all about your active earth pressure, now let us try to solve an example on this to give you better understanding that how we can analyze the wall and the total condition as far as active conditions are concerned.

So, I take an example that for a retaining wall as shown below assume that the wall can yield sufficiently to develop active state, determine Rankine active force per unit length of the wall and the location of resultant line of action. So, you see what is the wall, this is what is wall, the water table is present at depth of 3 meter from the existing ground surface, above the water table the soil is having unit weight of 16 kilo Newton per meter cube your angle of friction is 30 degree and the c is 0 means it is frictionless soil.

Below the ground water table see here, when we discuss the theory we did not we I told you that if the soil is same, but you see here the soil is also different. So, that will give you the feel that what will happen, in case the soils are different if a water table is present and all the things, that is the most realistic situation which you can come across in the field situation. Here, below the ground water table gamma the soil will be saturated, so you have to deal with gamma saturated.

This gamma saturated is 19 kilo Newton per meter cube, phi 2 that is 36 degree, c 2 is equal to 0, again the soil above and below the ground water table they are frictionless.

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If the cohesion is zero, $\sigma'_h = \sigma'_v K_a$
For the top soil layer, $\phi_1 = 30^\circ$, so
 $K_{a(1)} = \tan^2(45 - \phi_1/2) = \tan^2(45 - 15) = 1/3$
Similarly, for the bottom soil layer,
 $\phi_2 = 36^\circ$ and $K_{a(2)} = \tan^2(45 - 36/2) = 0.26$
Because of the presence of water table,
the effective earth pressure and the
hydrostatic pressure have to be
calculated separately.

See, before solving or before taking up this example what one thing that I would like you tell you is that, you should always keep into mind that first you determine that what, is the effective vertical stress at any particular depth Z. And as I told you, that if you multiply that by coefficient of earth pressure, that will simply give you the lateral earth pressure at that particular depth.

If you follow or if you remember this thing, you will never commit any mistake in drawing lateral earth pressure diagram. Because, you know that how important it is to draw exact and right lateral earth pressure diagram, if there is once you have drawn it rightly. Then, there is nothing left for that you simply take the area of that earth pressure diagram, you will be getting the total active force or may be the force at rest and it is line of action.

So, now, let us start the solution of this particular example, now in this case the soil is frictionless, that is the cohesion is 0. And you have seen that in the absence of cohesion your σ_a prime that reduces to σ_v prime into K_a , that is that $\frac{2c}{\sqrt{K_a}}$ term will be absent because, the cohesion is 0. Now, we need to know the K_a value of the top soil layer and the bottom soil layer, which is divided by the water table.

So, for the top soil layer it has been given that ϕ_1 is equal to 30 degree, so you know that what is the expression for K_a that is equal to $\tan^2(45 - \phi)$ in this case ϕ is equal to ϕ_1 . So, you see here, you can find out $K_{a1} = \tan^2(45 - \phi_1)$, ϕ_1 in this case is 30 degree, so that is $\tan^2(45 - 30)$, which is equal to 1/3. Similarly, for bottom layer if it would have been the same soil layer, then your K_{a2} would have become K_{a1} , but here it is with ϕ_2 is equal to 36 degree.

So, your K_{a2} will become $\tan^2(45 - 36)$ that is 18 degree and this will work out to be 0.26. Then, because of the presence of water table the effective earth pressure and the hydrostatic pressure have to be calculated separately, as I told you when we were discussing the effect of presence of water table, that the water exerts hydrostatic pressure. So, there is no need to multiply with the coefficient of lateral earth pressure to the vertical stress due to the water pressure because, whatever is the pressure due to water in vertical direction, exactly the same pressure will be there in horizontal direction.

Now, let us try to see what are the further steps, so at each depth first try to find out that what is the vertical effective stress. After that you simply multiply by the coefficient of lateral earth pressure that will give you the lateral earth pressure at that particular depth.

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• At $z = 0$, $\sigma_v' = 0$, $\sigma_a' = 0$

• At $z = 3$ m, $\sigma_v' = \gamma z = (16) (3) = 48$ kN/m²
At this depth, for the top soil layer,
 $\sigma_a' = K_{a(1)} \sigma_v' = (1/3) (48) = 16$ kN/m²

Similarly, for the bottom soil layer,
 $\sigma_a' = K_{a(2)} \sigma_v' = (0.26) (48) = 12.48$ kN/m²

• At $z = 6$ m, $\sigma_v' = (\gamma) (3) + (\gamma_{sat} - \gamma_w) (3)$
 $= (16) (3) + (19 - 9.81) (3)$
 $= 75.57$ kN/m²

So, you see here how I am taking the next step is that at z is equal to 0, you know that σ_v is γ into z , when z is equal to 0 σ_v prime will become 0. So, this will give you σ_a prime, which is equal to σ_v prime into K_a this will give you σ_a prime is equal to 0. Now, you see at 3 meter depth there is a presence of water table, so we will divide into two parts this problem will divide into two parts. Now at z is equal to 3, what is σ_v prime is γ times z , γ you have seen that it is 16 kilo Newton per meter cube into 3, which will result into 48 kilo Newton per meter square.

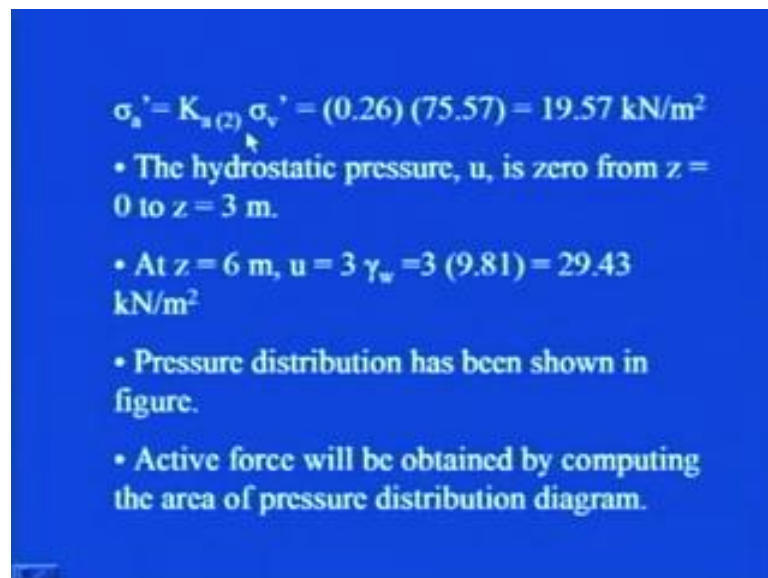
You see, just at the top of this location of water table, for that top soil layer σ_a prime will be $K_a 1$ into σ_v prime, σ_v prime we have found out to be 48 and $K_a 1$ we just found of that is 1 by 3 48 into 48, this will result as 16 kilo Newton per meter square. Now, see the water level is present at a depth of 3 meter below the ground surface, the soil above the water table and soil below the water table, they are different. And because of that add this interface, which is distinguished by the presence of your water table, I am treating another situation that is just below the top soil layer.

So, for bottom soil layer that will be although the depth is same, so σ_v prime is going to be same. But, since the bottom soil layer is different as compared to the top soil layer, so that will be multiplied by $K_a 2$ σ_v prime and $K_a 2$ we found out to be 0.26 into 48 eight that is equal to 12.48 kilo Newton per meter square. So, you see whenever there is any interface, whenever there is the change in the soil may be due to

the presence of either water table or may be there are layered soil system, there you have to split in into two parts that is one just above the interface and another just below the interface.

Now, you see at z is equal to 6 if we want to find out the σ_v' , that is effective vertical stress which will be equal to γ into 3 plus $\gamma_{\text{saturated}}$ minus γ_w into 3. See, we are finding out the effective vertical stress that is why, you have to subtract this unit weight of the water from the $\gamma_{\text{saturated}}$ that is saturated unit weight of the soil. So, you see here 16 into 3 plus 19 minus 9.81 into 3 and this will result in to 75.57 kilo Newton per meter square, so at z is equal to 6 meter your effective vertical stress is 75.57 kilo Newton per meter square.

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$\sigma_a' = K_a(z) \sigma_v' = (0.26) (75.57) = 19.57 \text{ kN/m}^2$

- The hydrostatic pressure, u , is zero from $z = 0$ to $z = 3 \text{ m}$.
- At $z = 6 \text{ m}$, $u = 3 \gamma_w = 3 (9.81) = 29.43 \text{ kN/m}^2$
- Pressure distribution has been shown in figure.
- Active force will be obtained by computing the area of pressure distribution diagram.

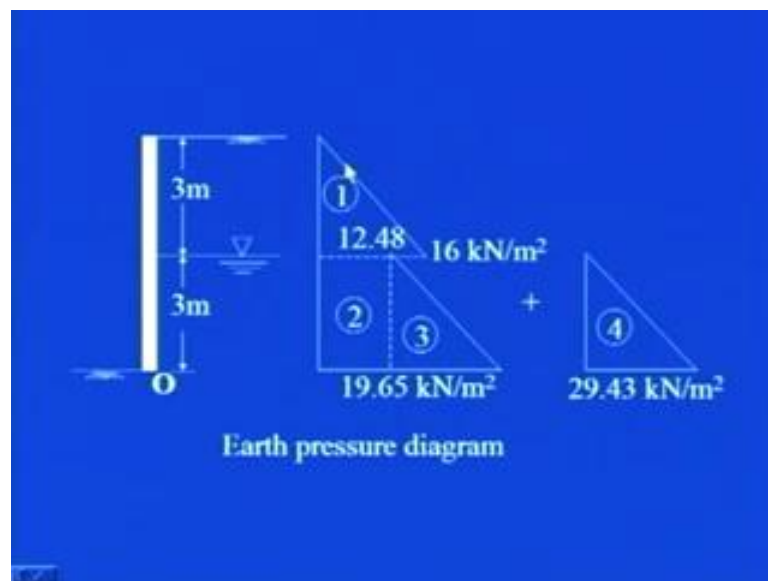
And since here there is no change in the soil strata, so we simply multiply by the K_a that is the coefficient of lateral earth pressure for the bottom soil layer to obtain the active pressure at z is equal to 6, that is the value is 0.26. So, if 0.26 into 75.57 this will result into 19.57 kilo Newton per meter square. Now, this was all due to the presence of backfill soil, now since there is water table present at a depth of 3 meter below the existing ground level.

So, from z is equal to 3 meter to z is equal to 6 meter, there will be the presence of pore water pressure, that is as I explained you earlier that the water exerts, hydrostatic pressure. So, the hydrostatic pressure u is 0 from z is equal to 0 to z is equal to 3 because, no water table is present over there, but at z is equal to 6 u will be 3 into γ_w and I

have taken γ_w to be equal to 9.81 kilo Newton per meter cube, which will result into 29.43 kilo Newton per meter square.

Now, these are the all forces are these are the all stresses, which will be acting upon the wall. Now, we have to draw the pressure distribution due to these pressures, which I will be showing you in the subsequent slide and then as you know that from once the pressure diagram is drawn, simply you take the area of that particular diagram and that will give you the total force per unit length of the wall.

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Now, you see here this due to this soil and this soil, since they are different the soil is also different and there is a presence of water table also. And we have found out that what is the pressure on top of this water table, what is the pressure in bottom of the this water table and then what is the pressure at the z is equal to 6. So, you see that from 0 to this z is equal to 3 your pressure was 16 kilo Newton per meter square, due to the soil which is present in the this top layer.

And then, just below this layer the pressure was this 12.48, so first it goes follows this line, then it gets reduced to 12.48. And then, at z is equal to 6 we found out that this lateral pressure or the horizontal pressure was 19.65 kilo Newton per meter square. So, from this point it will extend till this magnitude and so the area marked by this form line will give you the area of the lateral earth pressure diagram plus we are forgetting one thing, see this water which is present over here will cause the hydrostatic pressure.

And we have found out the magnitude of that and that was 29.43 kilo Newton per meter square. So, you simply have to add this particular area to this lateral earth pressure diagram, so you see the shape of the earth pressure diagram is from 0 till 16, then it get reduced to 12.48 and then it increases linearly with as 19.65 kilo Newton per meter square. Then, you have to incorporate this pore water pressure, which is 29.43 kilo Newton per meter square, so you see the if area which is formed by these form line include this earth pressure diagram.

Now, for the simplicity because, we once we draw this lateral earth pressure diagram we need to find out it is area to get the total force per unit length of the wall. So, that is why I have divided it into 4 parts, that is 4 areas area 1, which is this triangular area then area 2 this rectangular area, area 3, this triangular area plus area 4 which is due to pore water pressure.

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- The total force per unit length of wall

$$P_s = \text{Area 1} + \text{Area 2} + \text{Area 3} + \text{Area 4}$$

$$= \frac{1}{2} (3) (16) + (3) (12.48) + \frac{1}{2} (3) (19.65 - 12.48) + \frac{1}{2} (3) (29.43)$$

$$= 24 + 37.44 + 10.76 + 44.15 = 116.35 \text{ kN/m}$$

- The distance of the line of action of the resultant from bottom of wall (z_r) can be determined by taking moments about bottom of wall (point O in figure).

So, you see total force per unit length of the wall that is total active force is equal to area 1 plus area 2 plus area 3 plus area 4. Now, since it is simply triangle and rectangle, so it is very easy to find out the area, what is the area for this one area that will be simply half multiplied by 3 multiplied by this ordinate, which is 16 here in this case, ((Refer Time: 43:02)) for the second 1 it will be 3 into 12.48 for the third one it is half into 3 19.65 minus 12.48 plus this is the area due to pore water pressure that is half 3 into 29.43, this all if you add up it will result into 116.35 kilo Newton per meter.

Now, the distance of the line of action also we have to find out that where this resultant total force will be acting. And this can be found out by taking the moment of all the forces above this point O, you see if I take only one first area what will be the resultant of this particular force that is the area of this area one and that will be acting at a distance of H by 3; that means, 3 by 3 that is 1 meter from this particular level.

Now, where the force due to this area will act that will be acting at the center line because, it is rectangle. So, it will be acting at 1.5 meter above this one. Similarly, for the third one it will be acting 1 meter from the bottom of the wall, in this case also 1 meter from the bottom of the wall. So, we know all the forces and their line of action and combining all four of them together we can find out the resultant force and its line of action.

You see, the distance of the line of action of the resultant from bottom of the wall, let us say I call it to be Z_r can be determined by taking the moment of all the forces above bottom of the wall, which I showed you earlier that is point O.

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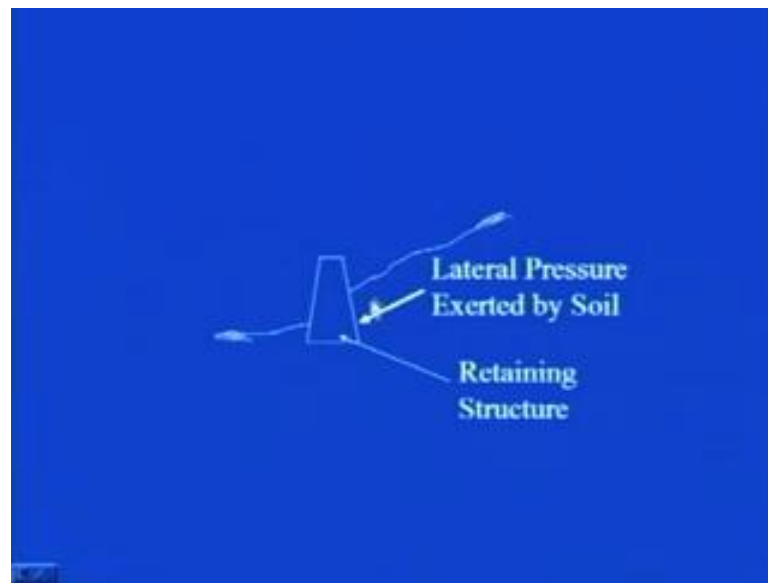
$$\begin{aligned}
 z_r &= \frac{(24)\left(3 + \frac{3}{3}\right) + (37.44)\left(\frac{3}{2}\right) + (10.76)\left(\frac{3}{3}\right) + (44.15)\left(\frac{3}{3}\right)}{116.35} \\
 &= \frac{96 + 56.16 + 10.76 + 44.15}{116.35} \\
 &= 1.78 \text{ m}
 \end{aligned}$$

So, you see that is 24 was a force for first one, so that is 3 plus 3 by 3 wherever it is acting and then divided by this total force, which will result into 1.78 meters. That means, that the resultant force of 116.35 kilo Newton per meter magnitude will be acting at a distance or at a height of 1.78 meter from the bottom of the wall. So, you have seen that in case the soil is different surcharge is present, water table is there, how you can

determine the magnitude of the total active force and its line of action, I hope that it has become clear to you with the help of this particular example.

Now, let us try to see this was all about your active earth pressure, then now let us come to the third one, that is when the soil when the wall is moving towards the soil. And I explained you earlier in the previous class, that this particular condition corresponds to passive earth pressure.

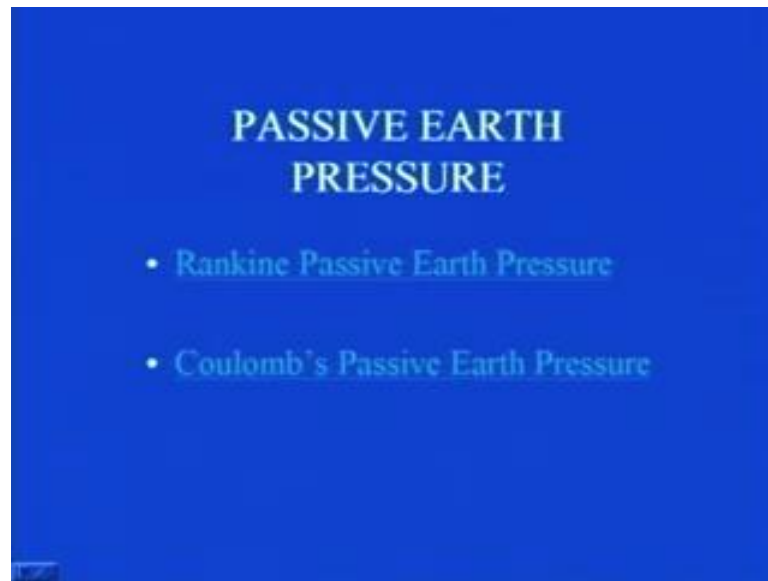
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Now, first let us try to see I mean from common sense you can find out this soil is exerting lateral earth pressure on the wall. So, wall is always having a tendency to move away from the soil, in what case it will be moving towards the soil, now to explain this case let us try to have a look on this figure once again. This is what is the lateral earth pressure, which is exerted by soil because of this pressure this wall will have the tendency to move in this particular direction.

So, here if you consider face of the wall here and the soil what type of condition is coming into picture is that, active condition. Because, this wall is moving away from this backfill soil, but what is happening here in the this process simultaneously, this wall is moving towards this soil. So, here the active condition is getting generated and simultaneously here passive condition is getting generated and that is why it is very necessary to estimate passive earth pressure properly to have proper analysis and design of this retaining wall.

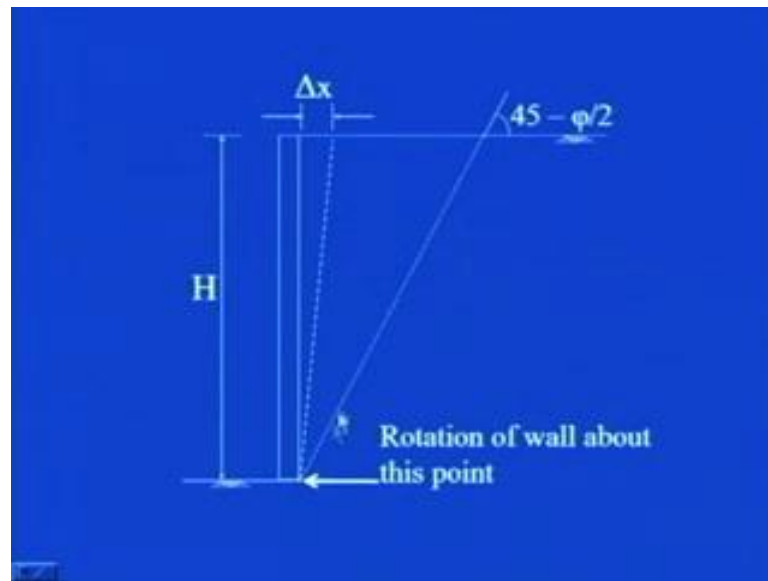
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Now, as in case of active earth pressure theory we had two active earth pressure theories that is Rankine active earth pressure theory and Coulomb's active earth pressure theory. Similarly, in case of passive earth pressure theories we have two theories, one is Rankine passive earth pressure theory, another is Coulomb's passive earth pressure theory. Exactly on the similar lines, as we discussed for active earth pressure we will be discussing for Rankine and for Coulomb passive earth pressure theories.

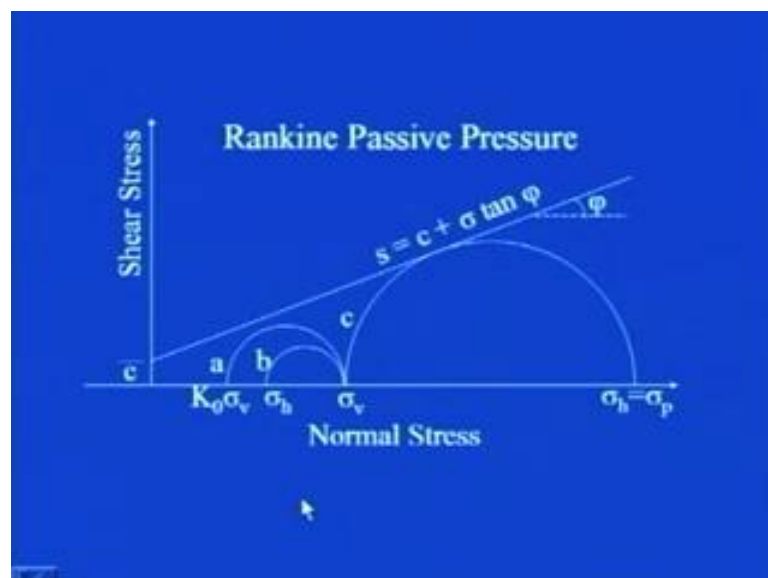
Again one has some advantage or some disadvantage over the other, let us try to discuss one by one the details or the process of analysis following either Rankine's passive earth pressure theory or Coulomb's passive earth pressure theory, first let us try to discuss Rankine's passive earth pressure theory.

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So, similarly as we discuss in case of active earth pressure here, that is 45 minus ϕ by 2 that is the failure wedge is making an angle of 45 minus ϕ by 2 from the horizontal surface. You see, in active case it was moving away from the soil and in this case the wall is moving towards the soil, you see it is pushing the soil by this Δx amount. Now, we are assuming that the rotation of the wall is taking place about this particular point, the height of the wall that is vertical height of the wall has been considered to be H .

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Now, as we discuss in case of active earth pressure, similarly in case of passive earth pressure we try to make or develop the understanding with the help of Mohr circle. You see, when the load is applied or when the soil is exerting lateral earth pressure, there will be initial condition that when the wall is not at all moving that is wall is at rest. So, you see, if I take this one, this Mohr circle which is corresponding to a, you see this one follow the pointer please this one that is σ_v is your vertical stress at any particular depth and correspondingly K_0 into σ_v will be the lateral earth pressure at rest.

You have seen that we denote that coefficient of lateral earth pressure by K_0 in this case. As the wall starts moving towards the soil, what happens is the wall movement that is wall displacement is increasing, what happens is in passive case the Mohr circle size of the Mohr circle goes on reducing. You see, earlier it was this at earth pressure at rest condition, then as the wall starts moving towards the soil it will be it goes on reducing.

And it takes any particular value that is σ_H , it goes on as the wall goes on displacing, the Mohr circle also goes on reducing in size and then started increasing in size on the other side, you see here on the other side of these particular Mohr circle. And the one situation occurs where the Mohr circle touches the Mohr failure envelope, this particular condition corresponds to the passive condition.

And the correspondingly the horizontal stress is known as your Rankine's passive pressure, you see this is Mohr failure envelope, which is making an angle ϕ from the horizontal it is equation is given by $s = c + \sigma \tan \phi$, c and ϕ are the shear strength parameters of the soil, c is cohesion and ϕ is angle of internal friction, s is shear stress where σ represents the normal.

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- Mohr's circle, a: corresponding to wall displacement = 0
- Mohr's circle, b: corresponding to case when wall pushed into soil mass > 0
- If wall moves farther inward, the corresponding Mohr's circle will touch Mohr-Coulomb failure envelope: Mohr's circle, c

So, let us try to put that figure into words, that is what exactly Mohr circle a b and c they correspond to the cases. If you see, Mohr circle a is corresponding to the wall displacement is equal to 0 that is at rest condition, Mohr circle b it corresponds to the case, when wall starts pushing the soil mass that is when it is greater than 0, the displacement of the wall is greater than 0. And if wall moves farther inward the corresponding Mohr circle will touch Mohr Coulomb failure envelope and this corresponds to Mohr circle c.

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- Mohr's circle, c: failure condition in soil mass; horizontal stress = σ_p ; referred to as the Rankine passive pressure.
- Major principal stress = σ_p
- Minor principal stress = σ_v

Mohr circle c, that is an important one and it is the failure condition in soil mass and in that case, the horizontal stress is equal to σ_p , which is referred to as Rankine's passive pressure. And if you see, that figure as I explained you earlier that in that case your major principal stress is equal to your σ_p , while the minor principal stress is equal to σ_v . So, how you can find out an expression for the passive earth pressure, subsequently passive force its line of action, all these things we will be discussing in the next class.

So, today what we saw was that, with the help of an example we discussed that how you can estimate Rankine's active earth pressure, along with the theory behind Coulomb's active earth pressure theory. And then, we discuss the various aspects of passive earth pressure, that how exactly it is getting generated and then we started with the Rankine active earth pressure theory, rest all things we will be covering in the next class related to Rankine's active earth pressure theory, along with Coulomb's passive earth pressure theory.