Foundation Engineering
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Module - 02
Lecture - 01
Lateral Earth Pressure Theories and Retaining Walls – 1

Good morning friends, today we are going to discuss the Lateral Earth Pressure Theories and then after that Retaining Walls. Before starting our discussion, let us first try to understand what exactly to be mean by lateral earth pressure and then what are the various theories; which are developed to estimate the lateral earth pressure. Now, first of all let us try to understand, that what do we mean by lateral earth pressure, how it occurs, so for that, I will just show you this figure.

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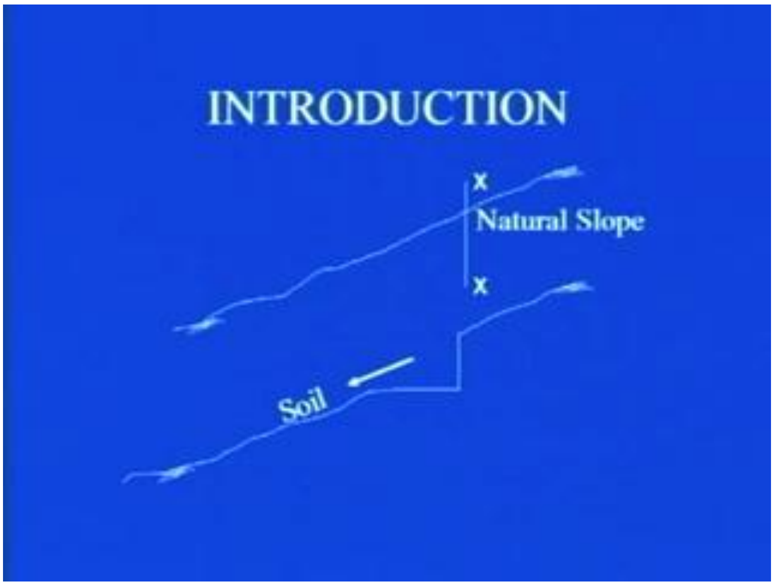

You can see that, there is a natural slope and if I cut it by a vertical plane x x as shown in the figure, it will result into this kind of situation and you can see that this soil above this part, will have the tendency to slide over. So, in case if you are planning to expand a road, then what will happen, this soil will slide over this road, which is an undesirable situation.

So, for all means, we need to protect this soil to slide down, now how can we do that, either we have to provide some retaining structure or we have to provide some measure for that. Now, see this soil which is sliding down, it will be sliding with some pressure and that pressure is called the lateral earth pressure earth is soil, so that is lateral earth pressure.

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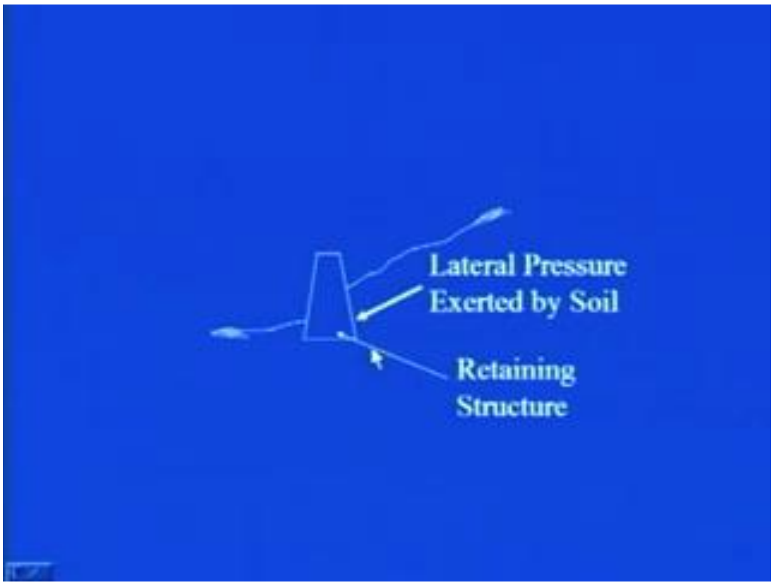

Now, let us see this, if I give there a retaining structure of this kind, then what is happening, the soil which was there about in this part. This is creating a lateral pressure, which will be resisted by this retaining structure and that is why it is called a retaining structure. So, you see that this retaining structure will function properly in the case, when this lateral earth pressure is estimated properly.

Because, let us say that lateral earth pressure is very nominal, then we do not need to require such a big cross section of a retaining structure and accordingly if it is magnitude is higher we have to go for larger cross section. So, that the retaining structure will be able to take, that much of the load without failing, the stability is the only criteria, which has to be taken into account. So, that is why the estimation of this lateral earth pressure, which is exerted by the soil, becomes very essential.

Now, you see the soil, I will be using some of the technical terms from now onwards, so you must know that, what exactly do I mean by those words, I will be using a term backfill. So, the soil which has to be retain, that is the soil which is lying in this part by this retaining structure, I will be calling that as backfill material or backfill soil.

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Now, as we saw that, what is lateral earth pressure and why it is so important to estimate that properly. So, you can see to summarize that, if I say that a proper design of retaining walls, cantilever sheet-pile walls, braced cuts and other similar structures require proper estimation of lateral earth pressure. So, these are retaining walls or cantilever, sheet pile walls, brace cut, they all are different types of retaining structures.

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Now, before estimating, we must know that on what parameters, these lateral earth pressure will vary, so this lateral earth pressure is basically a function of type and amount of wall movement. Now, let see you have seen that figure, the soil was trying to slide down, which will cause the pressure on the wall, wall will have the tendency to move either towards the soil or from or away from the wall.

So, the type and the amount of the wall movement, which is taking place will affect the lateral earth pressure, then shear strength parameters of soil, what do we mean by shear strength parameters is that the cohesion and angle of internal fiction. So, depending on the property of the soil, the lateral earth pressure will be varying, now for here, what do we mean by soil is as I explained you, is the backfill soil. Then, the unit weight of the soil and then drainage conditions in the backfill.

Drainage conditions, means you know that there are basically two types of soil, cohesive soil and non cohesive soil, then in cohesive soil the drainage conditions are not that good, but in case of non cohesive soil drainage conditions are very good. So, depending on the type of the soil and subsequently drainage condition of that, the lateral earth pressure will be different.

That is the lateral earth pressure, in case of cohesive backfill will be different and the lateral earth pressure, in case of cohesion less backfill is will be different.

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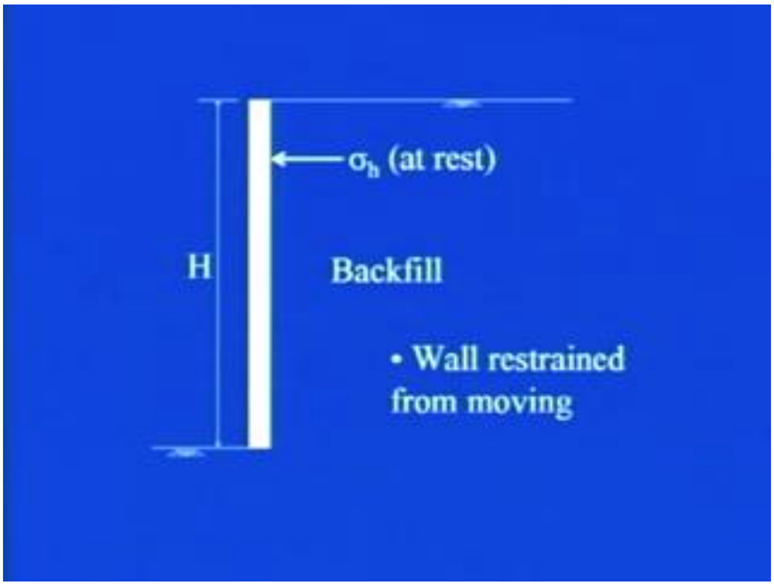

Now, as I told you that wall will have tendency to move upon the action of this lateral earth pressure, which will be exerted on the retaining structure by the soil. Now, the thing is that there can be three conditions either the wall is not at all moving or second thing wall can move away from the soil, that is away from the backfill or the wall can move towards the backfill. So, let us try to discuss 1 by 1, all these three conditions.

So, first condition that, I am going to discuss is that, when the wall is at rest that is wall is not at all moving. So, you can see here in this figure; that this is a kind of retaining structure, any retaining structure and here the soil which has to be retained is called as backfill. And then, if the height of this structure is H and wall is restrained from moving, that is wall is in it is position and then the sigma H is the pressure, which is exerted by the soil on this retaining structure.

You have to keep in mind; that the wall is not at all moving, wall has been restrained from moving and then this horizontal pressure or a stress, which we are calling is sigma H, which is at rest.

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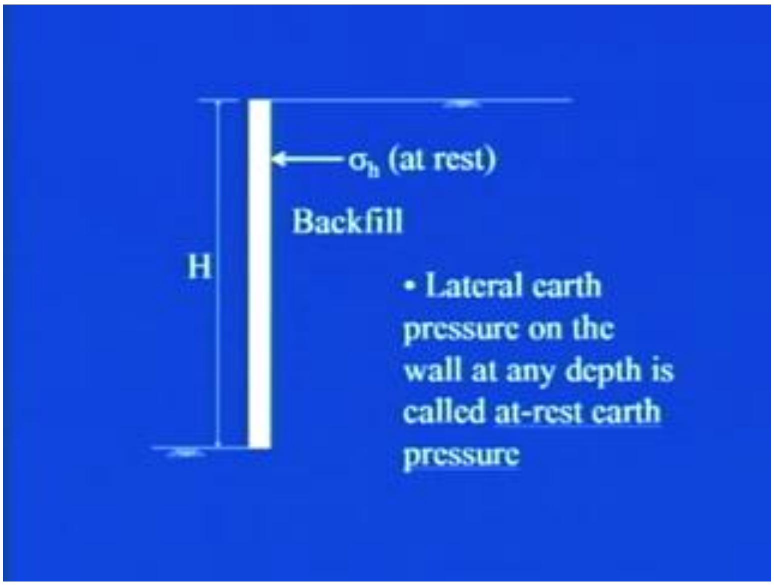

Now, you see that the lateral earth pressure on the wall at any depth, in this position of the wall is called at rest earth pressure. So, is this clear, in this one, the wall is not at all moving, now let us try to understand, that what happens if the wall moves away from the soil.

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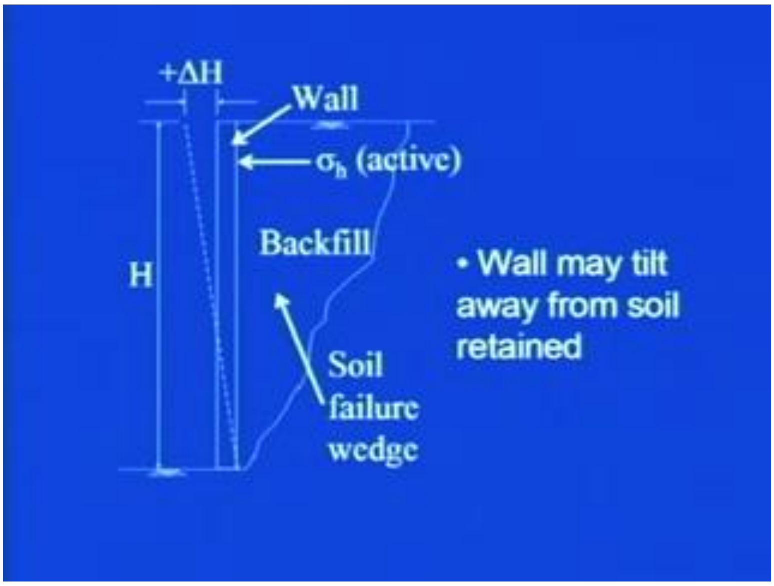

So, you can see here, that this is a kind of wall or retaining structure, then it is moving away from the soil by this delta H amount. Then, it will cause the soil to fail along this say soil failure, I mean it is not a very hard and fast that soil will fail along this failure wedge only. It can take any other shape, it can fail like this or it can fail like this, but let us say for tentative thing, we assume that is this is the wedge of the soil which is failing, so here wall may tilt, away from the soil which has to be retained.

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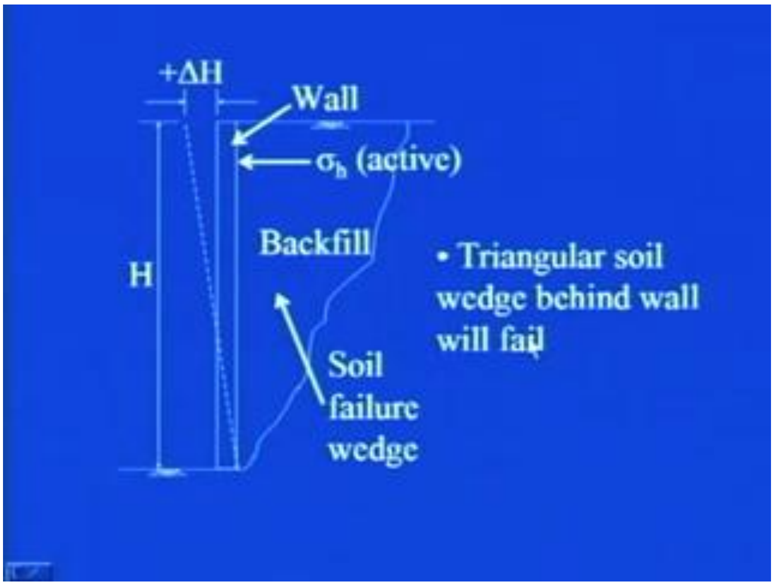

This condition is called active condition, you see a triangular soil wedge, behind the wall will fail.

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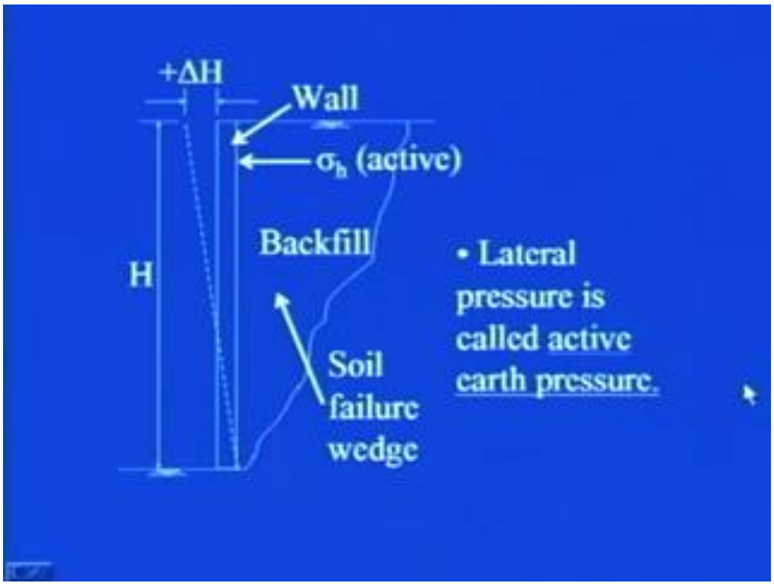

And the lateral pressure, in this condition is known as active earth pressure, now this one, we discussed, that when the soil is at rest, the second thing when the soil is moving, away from the soil.

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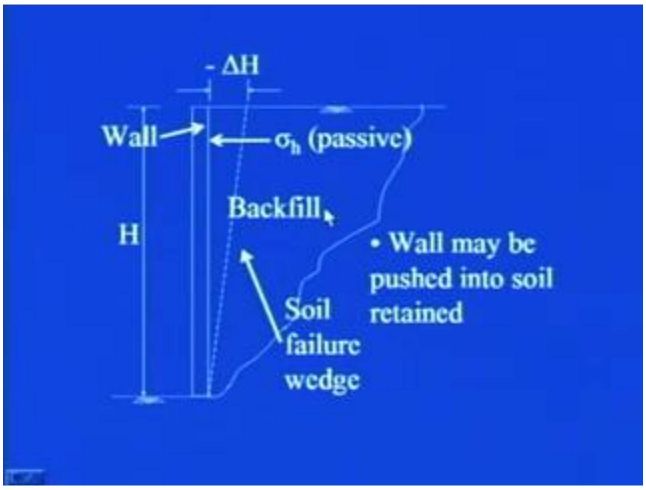

Now, the third one is when the wall is moving towards the soil, so you can see in this in earlier figure, the wall was moving away from the soil, in this case the wall is moving towards the soil. So, the wall may be pushed into the soil retain, there may be cases, I will explain you in the subsequent slides all those things. So, in this case the wall is moving towards the backfill soil and again I assumed that, this is the soil wedge failure; that is the triangular wedge is found.

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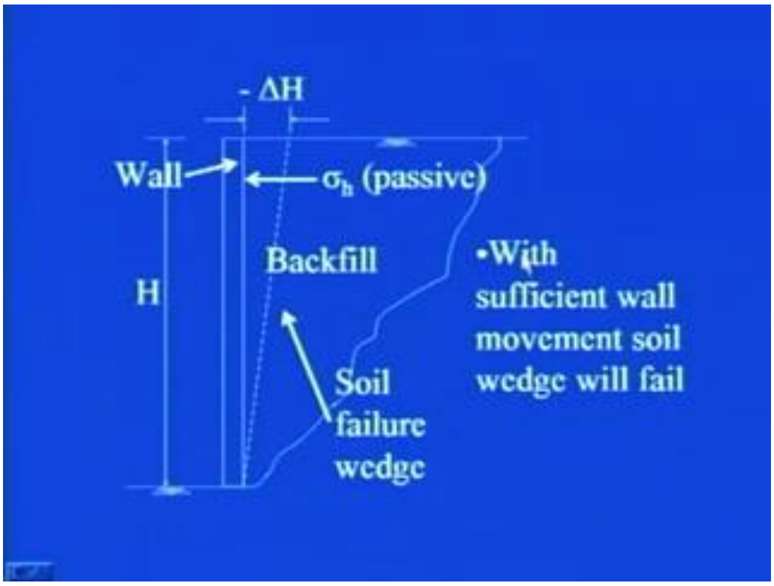

So, you see with sufficient wall movement the soil wedge will fail and in this case you can see here, this is the lateral pressure, which is acting upon the wall are retaining structure by the soil is called as passive earth pressure.

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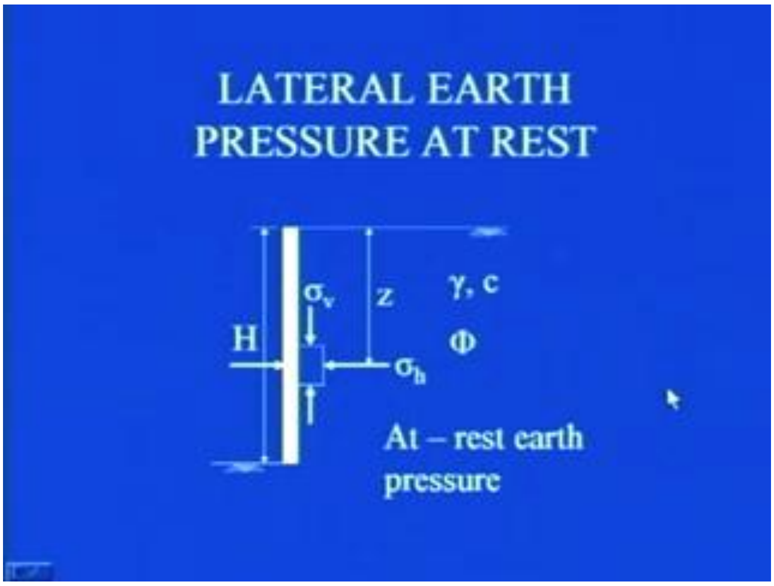

Now, after knowing all these three types of lateral earth pressure which can occur or which can act on a retaining structure, due to the presence of back fill. Let us try to analyze the case 1 by 1, that how exactly we can estimate the lateral earth pressure in these three different conditions. So, first let us try to have a look on the estimation of lateral earth pressure at rest condition, so you see, here is the wall, since it is at rest condition, it is not at all moving.

If, I take any soil element at a depth z, from the existing ground surface, you see this is existing round surface; here is the backfill, if I take any soil element which is represented by this rectangular part at a depth z. Then, what will happen that due to the over burden of this soil, there will be vertical stress, which will be acting on the soil element. And then, correspondingly on that soil element, this horizontal stress will be acting, so this is lateral earth pressure.

Now, how to estimate it, first of all, we fix the properties of this soil, so you see c and phi those are shear strength parameters of the soil, as I told you earlier that the amount of lateral earth pressure, it depends on the shear strength properties of the soil. So, you see here, these are coming into picture, along with the unit weight of the soil, so the height of the wall is H in this case.

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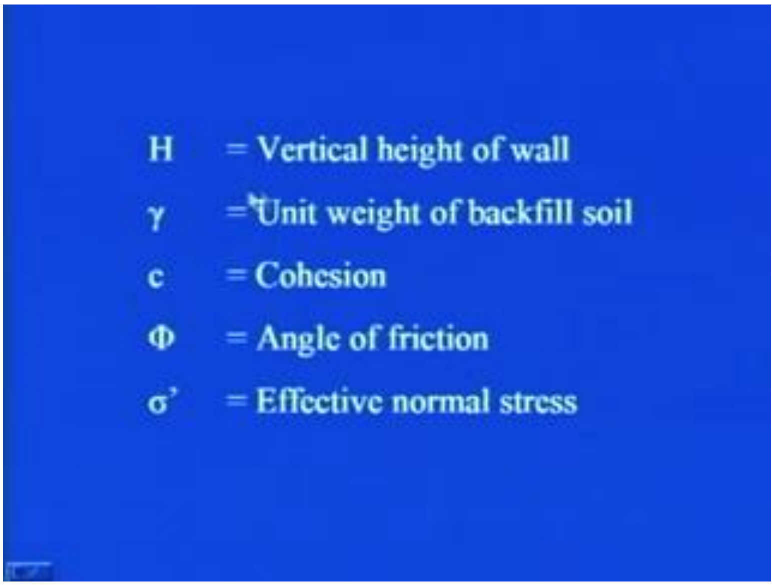

So, H is your vertical height of wall, gamma is your unit weight of backfill soil, c is cohesion phi is angle of internal friction and sigma prime, if I call that as effective normal stress.

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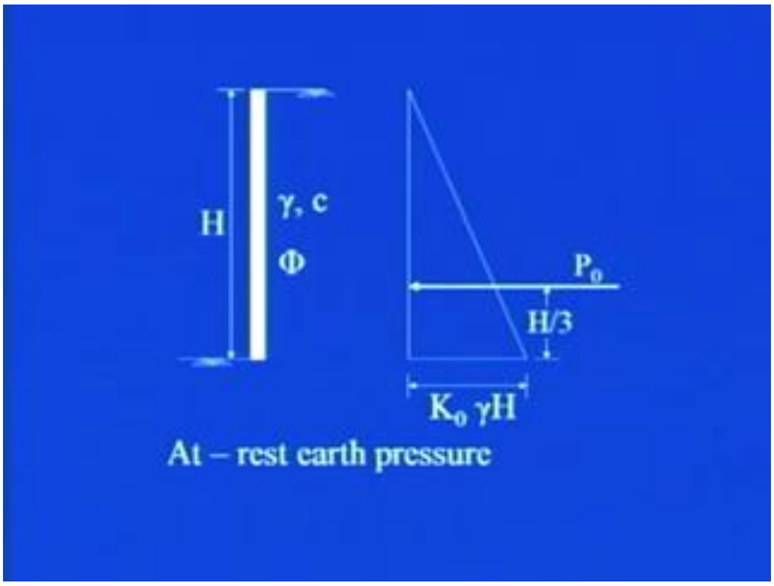

Then, the lateral earth pressure diagram at any particular depth will be this much you see I can draw it here that, this is a K naught times gamma into H. You see at any particular depth z, what will be the vertical stress, you know it, that it is gamma into that particular depth z. So, here at depth H, this is gamma H and if you multiply it by K naught, this becomes your sigma H and the resultant of this triangular force will be given by this P naught, which is lateral earth force.

So, this is what is pressure diagram and the total area of it will give you the total earth force, which will be acting, you know that the this will be acting at, since it is triangular. So, it is resultant will be acting at H by 3, from the base of the wall, that is from this one.

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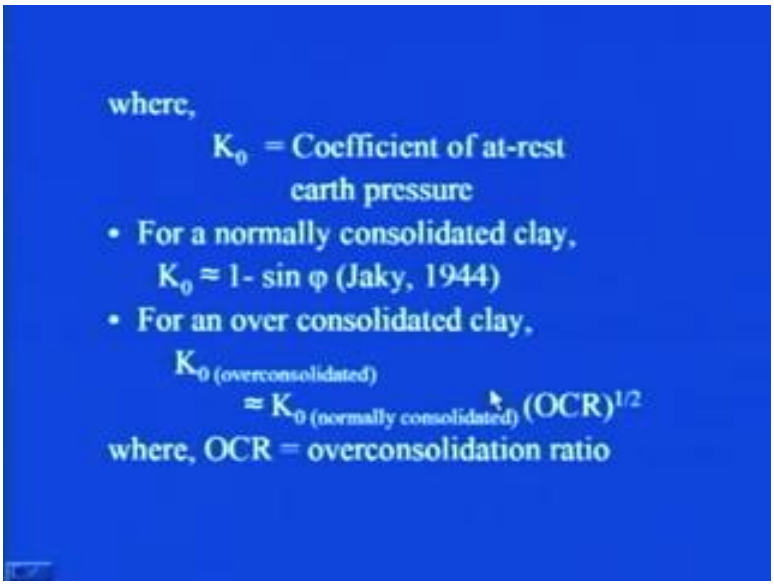

Now, let us try to see what exactly is this K naught, K naught is called the coefficient of at rest earth pressure, many research workers have found out these values of K naught based on the experiments and their engineering, judgment and their experience. So, you see that Jaky in 1944 gave an expression for K naught, which is equal to 1 minus sin phi is angle of internal friction.

Now, this is normal consolidated clay and for over consolidated clay, K naught over consolidated is approximately equal to K naught, normally consolidated into square root of over consolidation ratio. Now, I hope that, you know that what exactly the normal consolidated clay and over consolidated clay,, but to give you rough idea about them is that normally consolidated clay is the clay, which is subjected to the maximum pressure from it is history.

And the over consolidated clay is; that means, that this clay has been subjected to more pressure as compared to it is present condition in it is history. And that is over consolidation ratio is the K naught over consolidated by K naught, normally consolidated whole square it from this relation.

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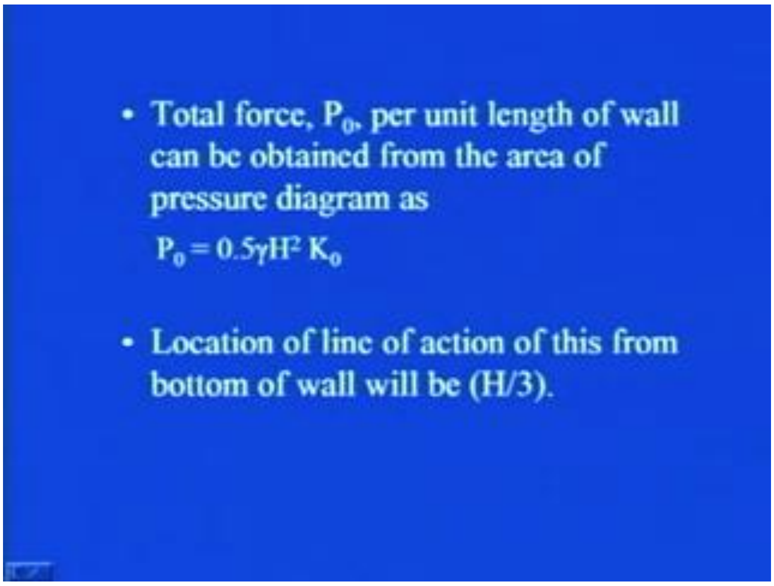

Now, how can we estimate the total force, we have already drawn the pressure diagram, as I told you that if you take the area of that pressure diagram, you will be able to get the total force. So, you see here, we have the total force which is P naught and that will be per unit length of the wall. It can be obtained from the area of pressure diagram, which is this expression, that is P naught is equal to 0.5 gamma H square into K naught and as I explained you earlier.

The location of line of action of this from the bottom of the wall will be H by 3, where H is the vertical height of the wall.

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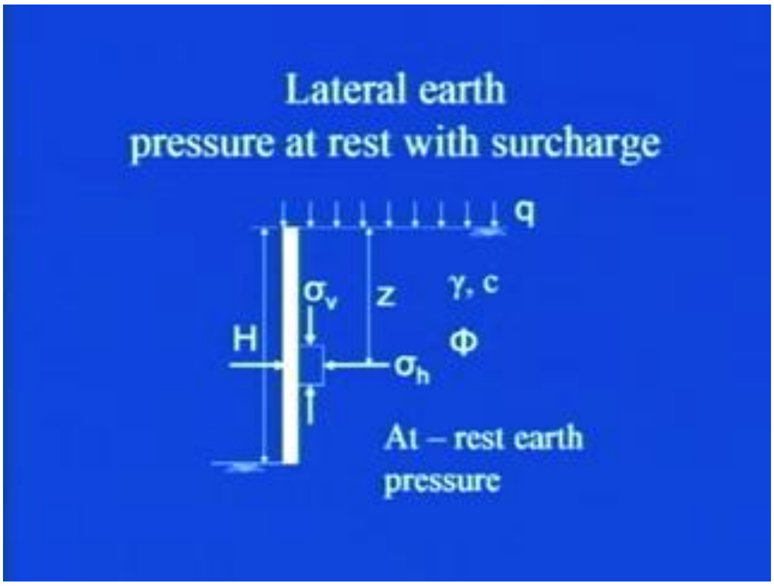

Now, we took a very simple case to make you understand, that what exactly do we mean by lateral earth pressure at rest. Many a times, it may happen that the backfill soil is subjected to some surcharge; it may be due to the existence of some structure some residential building or something like that. So, here I discuss a case, where lateral earth pressure with the presence of surcharge.

So, you see here the surcharge which is q has been applied, exactly the similar condition, all the things remaining same, only an additional parameter, that is the surcharge has been added to the previous condition. Now, let us see that what will be it is effect on the lateral earth pressure diagram and then subsequently the estimation of lateral force. You see, if I take any soil element at a depth z from the existing ground surface, then similarly in this case your sigma v is acting on the soil, then sigma H is acting.

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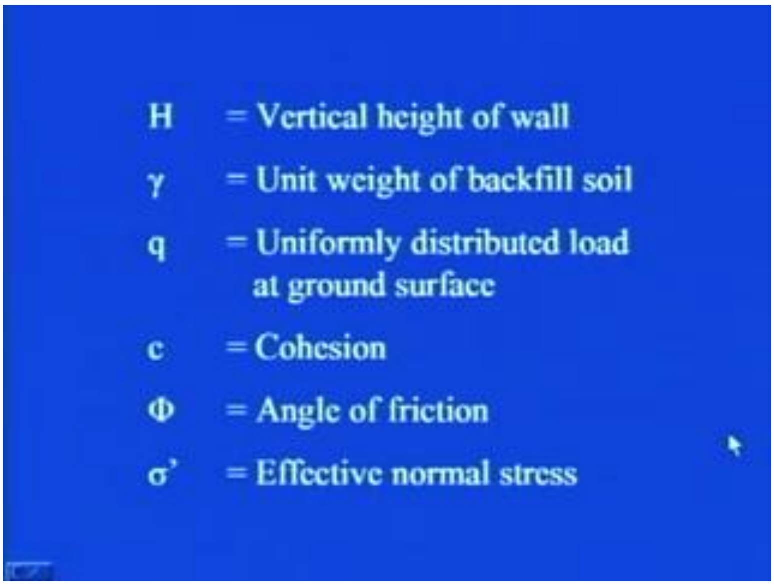

The properties of the soils are c and phi, c is cohesion, phi is angle of internal friction and gamma is you see here, H is vertical height of wall. Gamma is unit weight of backfill soil q is uniformly distributed load at ground surface, c is cohesion, phi is angle of internal friction and sigma prime is effective normal stress.

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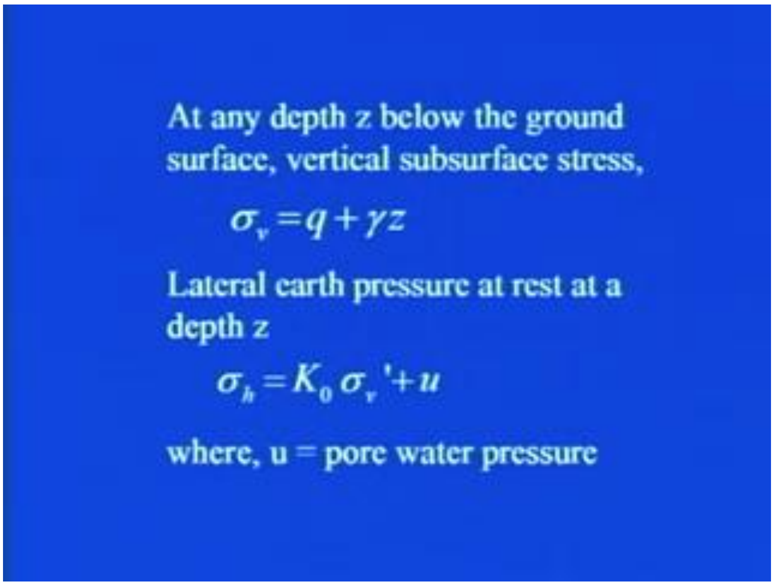

Now, first of all before finding that lateral earth pressure at any particular depth, we need to find out, what is the vertical effective pressure at that particular depth. Because, if you simply multiply the vertical effective stress by your coefficient of lateral earth pressure, then it will result into the lateral earth pressure. So, first let us try to see that what is the vertical sub surface stress at any particular depth z.

So, you see sigma v is equal to q plus gamma z and then the lateral earth pressure at rest at that particular depth will be that K naught into sigma v prime plus u. Now, from where this u is coming, here I have not taken them into account the presence of water table, in case of any water table, what will happen your effective stress, your total stress will be the sum of effective stress plus the pore water pressure. So, this u is representing that particular case, which is pore water pressure.

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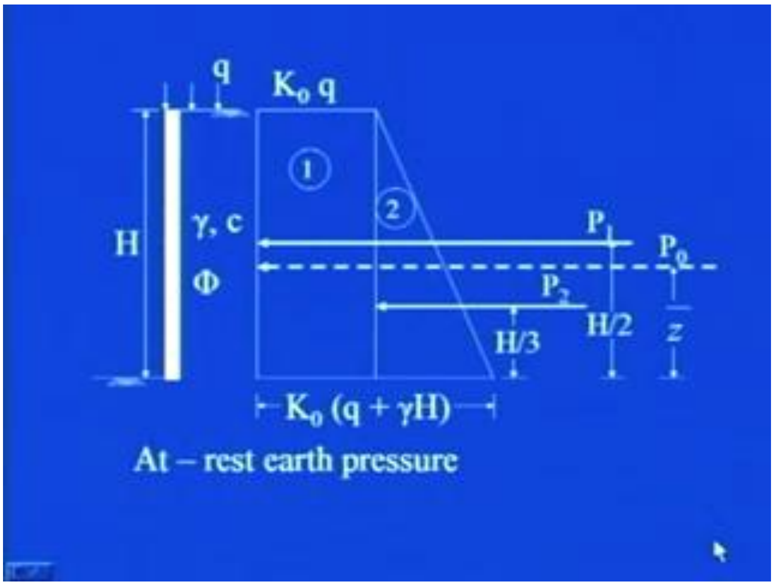

So, you see in K in the absence of any water table, just the surcharge has been added to the previous case as discussed earlier, what is going to be the difference, you see at the this particular depth, that is at z is equal to 0, your vertical stress is q. Now, if the vertical stress is q, wall is at rest position, what will be horizontal stress, it will be K naught into q,. So, here at depth z is equal to 0, we plot it to be K naught into q.

Now, at depth z you see at depth z is equal to H, what will be this, again due to this surcharge, the vertical stress at this particular point, that is at z is equal to H is going to remain same, that is q only. So, the horizontal stress is also going to remain same, which is K naught into q,, but due to this backfill soil, this additional part, you see this triangular part, which is denoted here by area two will come into picture.

And then at this particular depth, your vertical stress is gamma H and if you multiply that by K naught; that will give you lateral pressure,. So, the total lateral pressure will work out to be K naught into q plus gamma times h. So, you see, if I take this first 1, this denoted by the first 1, that is this rectangular area,. So, where the resultant will act, it will be acting at the height H by 2. So, you see here I am denoting it by P 1, P 1 is what the force due to this rectangular area, which is denoted by 1 and this will be acting at a depth H by 2.

Now, due to this triangular portion, you have already seen, that this triangular portion how it is coming and where this resultant force, what will be the line of action of this resulting force. That is let us say P 2 and this will be acting at a distance of H by 3 from the bottom of the wall. Now, here these are two different areas,, but what is happening in the reality, they are not two separate thing.

They are one entity and. So, after knowing this lateral earth pressure diagram, we need to find out, that what will be the total resultant force and what will be it is line of action. So, we have seen that separately for figure 1, it is P 1 which is acting at a depth H by 2 from the bottom of the wall. And then, for the second area, it is P 2 which is acting at H by 3 distances from the bottom of the wall.

Now, we must find out the resultant of this P 1 and P 2 in magnitude as well as it is line of action,. So, let us try to see that, how we can find that, let us say that this resultant of P 1 and P 2 is P naught and acting at a distance z bar from the bottom of the wall.

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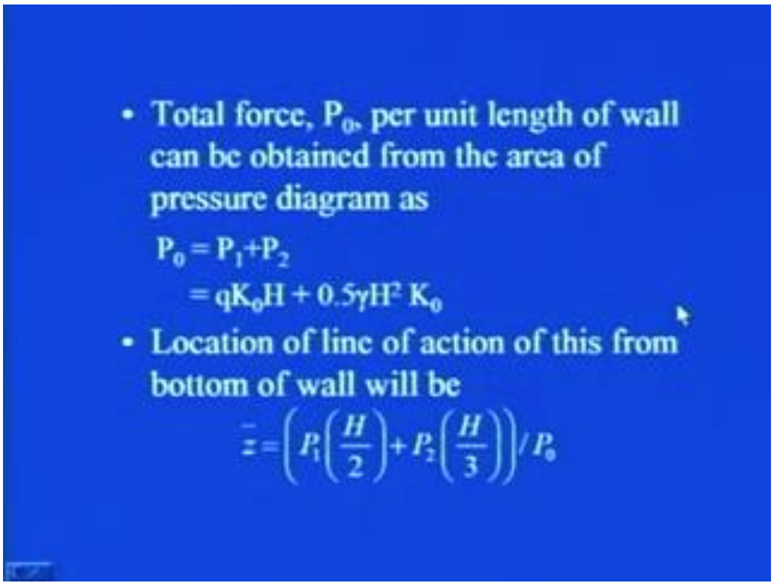

Now, let us try to see, that what is the procedure to estimate this, as you can see exactly in the similar manner, you have to find out the total area and that will give you the magnitude of the total force, which will be acting and per unit length of the wall. So, you see, I am writing here as P naught is equal to P 1 plus P 2 and if you see what will be the area of this K naught into q into this distance h,. So, that will be equal to P 1.

So, you see K naught into q into H plus in the similar manner as we discussed, when the surcharge was not there, that is plus point five gamma H square into K naught and the location of the line of action of this wall will be z bar is equal to. If you take the moment of all the three forces, about this point, what you will get is P 1 into H by 2 plus P 2 into H by 3 that is equal to P naught into z bar. So, from there, this expression for z bar, you will be able to get.

So, this way you can find out, that what is the total force, P naught per unit length of the wall and it is location also.

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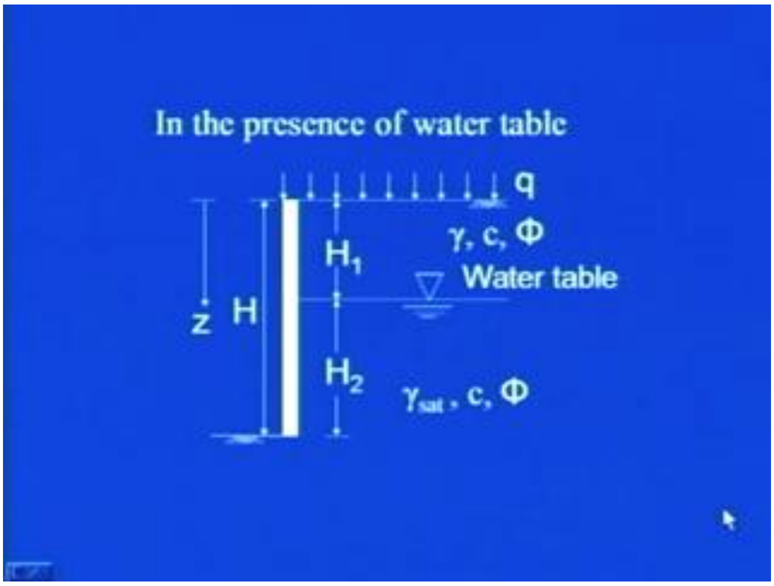

Now, I further complicate the situation that let us say that the water table is also present along with the surcharge. So, you see we are going step by step, first we saw that in K in the absence of any surcharge, only the wall was there and the backfill soil was there and then we saw that how we can determine the lateral earth pressure at rest condition. Then, we incorporate it, the surcharge on that, now it is the third step that, I incorporate the presence of water table.

So, you see in this figure, this figure shows you that, this is the position of your water table, which is at a depth z 1 from the ground surface and then the remaining is going to be your H 2. So, what is the value of H 2 is H minus H 1 and z, I am measuring from here. So, you know that when the water table is present, the soil below that becomes saturated.

So, that becomes the unit weight of the soil becomes gamma saturated in this case here below the water table, while above the water table it remains gamma only and I assumed that this backfill soil is same above and below the water table. Now, let us try to see that how it affects the methodology in the estimation of your lateral earth pressure or what will be the effect of this water table on the magnitude of lateral earth pressure.

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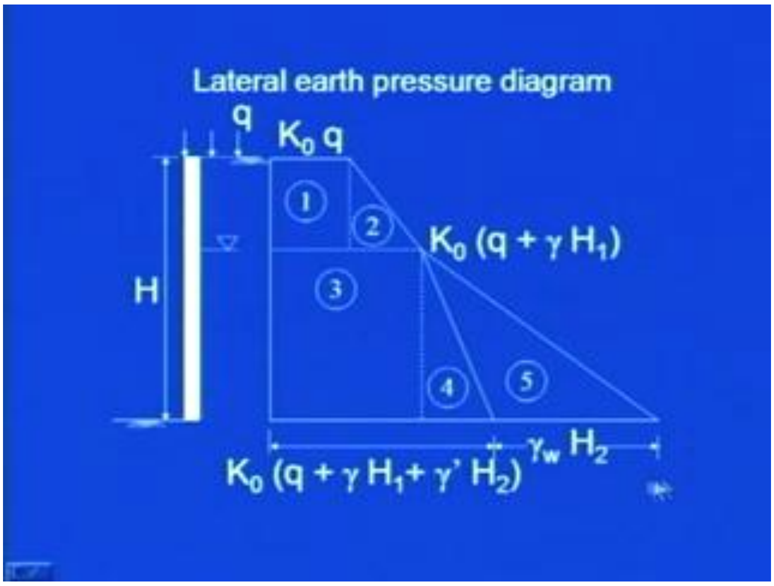

So, you can see, here I am showing you the earth pressure diagram in which surcharge is present and the water table is also present. So, you see first of all, I take a case above the water table, what is the vertical stress at z is equal to 0 is q only, so horizontal stress will be K naught into q. So, see I am showing it here, what will happen and z is equal to H 1, in this one, this is H 1.

So, you see what will happen at H 1, due to this q, it will remain same, so it is coming as it is, but due to this particular backfill, this extra triangular portion is coming into picture, that is the second one. So, what will be this total earth pressure, lateral earth pressure, this will be K naught into q plus gamma times H 1. Now, on this particular level, this will be acting as surcharge.

So, this will come as it is till this particular point and then the additional one, due to the soil below the water table; that is of depth H 2, this fourth part is coming into picture. And here, we are taking into account this gamma prime, this gamma prime is the gamma submerged, because it is below water table. So, this will become the total amount will become K naught q plus gamma H 1 plus gamma prime into H 2 and then due to this water pressure, there will be some additional lateral earth pressure on the wall.

As you know that water always gives you the hydrostatic pressure, so whatever is the pressure in vertical direction, due to the water. That will remain the same in horizontal direction also and that is why in that pressure, we do not multiply with this factor K naught. Always, remember this thing, that whenever you consider any pore water pressure or the lateral pressure for the presence of water in that, that is not multiplied by the factor K naught, it will simply taken to be gamma w into H 2.

And then, you can see here that, we have divided into five parts, z 1 which is due to the surcharge, over the depth H 1, second due to backfill over the depth H 1, third due to the surcharge. This total over the depth H 2 forth due to this backfill and fifth due to the pore water pressure which is present over there.

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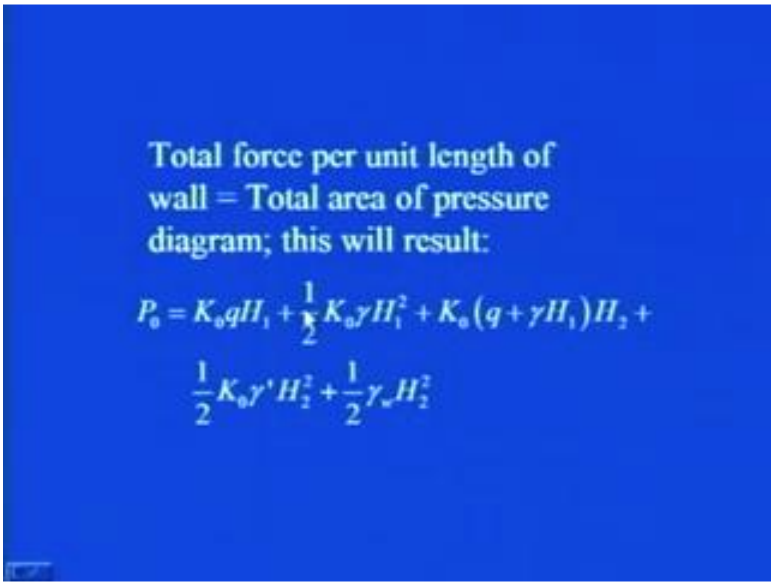

So, in the similar manner, the total force per unit length of the wall, it will be equal to the total area of the pressure diagram and you can see that it is simply triangular or rectangular area. So, very easily we can find out their area and if we sum them up, we get this expression and so we get the total force per unit length of the wall. See, this is the area of the first part, that is one, what will be the area of this one, that is half into distance into this distance, so that is here.

Then, again for the third 1, this is K naught into q plus gamma H 1 into this height, that is H 2. So, you see here I am writing this, in the similar manner I get the two triangular area for 4 and 5 respectively over here.

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Now, this was all about your lateral earth pressure in or at rest condition, now as we discuss that there are three conditions, at rest, active earth pressure and then passive earth pressure. So, now let us try to have a look on active earth pressure, that how we can estimate active earth pressure, you must remember, when this active earth pressure will come into picture. Whenever, the retaining wall is pushed towards the backfill soil and then your active earth pressure will come into picture.

Now, various research workers have given different theories, to estimate this active earth pressure, mainly two are used that is they are Rankine active earth pressure and Coulombs active earth pressure. Both have some advantages or disadvantage, one has some limitation, over the other, we will be discussing, all these aspects 1 by 1. So, first let us try to see that or what do we mean by Rankine active earth pressure.

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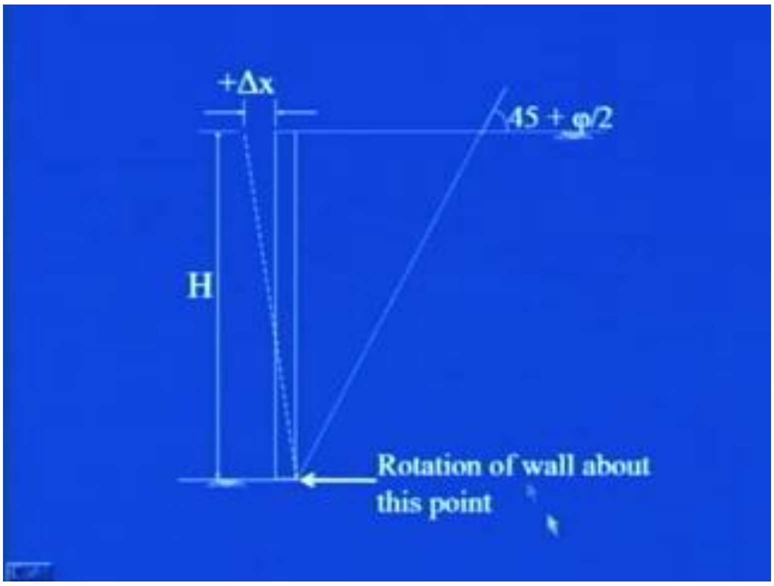

So, as we say that it is active pressure, you see wall is moving away from the backfill, Rankine assumed that the rotation of the wall, takes place along this point, which is at the bottom of the soil and the failure takes place along this failure plane, which makes an angle of 45 plus phi by 2 with the horizontal. So, you see the failure wedge which is getting formed is triangular in nature, that is this one and then delta x is the wall movement away from the soil.

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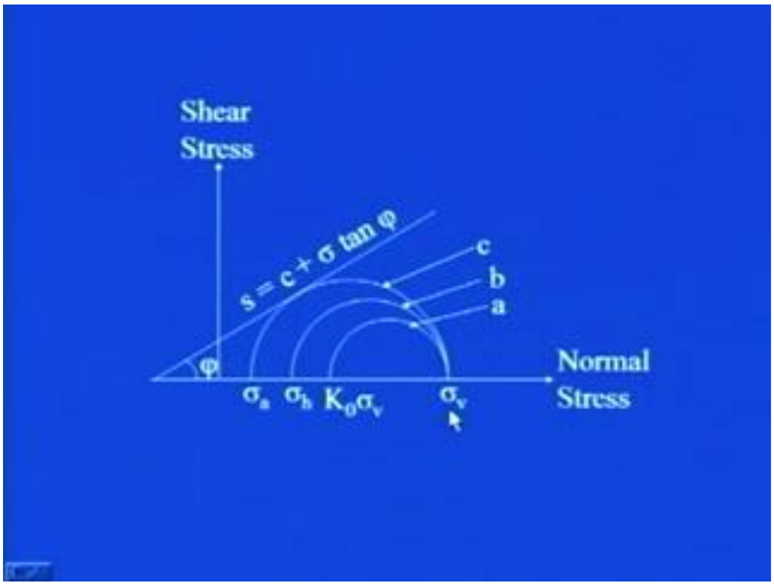

So, if we plot, let us say normal stress, because you have seen that, so you see here you know, you have already studied in your soil mechanics course, that what you mean by Mohr circle. Now, if I plot that Mohr circle; that is the variation of normal stress and the shear stress, which we plot. So, you see if I draw for different conditions, the normal stress versus shear stress, we get a circle kind of thing.

So, at any particular depth, if sigma v is the vertical stress K naught sigma H, I am talking of here active earth pressure, you see the wall was moving away from the soil. So, at any particular depth z, if sigma v is the vertical stress, what will happen that as the wall, let us say in first case, the wall will not start moving spontaneously or instantaneously, so for the first case it will be at rest.

So, in that case what will be your horizontal stress, that will be simply you have to multiply the vertical stress at that particular depth by K naught. So, you see this Mohr circle a, which is the smallest one is there which is corresponding to the Mohr circle at rest condition, that the wall is not at all moving. Now, when the lateral earth pressure started exerting on the wall by the backfill, what will happen the wall will slowly start tilting away from the soil.

So, you see at any particular, say at any particular tilt delta x, I say that this is the second Mohr circle, that the wall has started moving away from the soil, if sigma v is the vertical stress, vertical stress is not going to change at that that particular depth. So, accordingly corresponding to that wall movement, your sigma H will be changing. So, now sigma H will become, say any particular typical value and that will be different from the sigma H at rest condition.

So, Mohr circle will start moving in this particular direction and as the movement of the wall increases, there will be one situation, where the Mohr circle will touch this Mohr envelope; that means, that as soon as is it touches the Mohr envelope, that is the soil has failed. The equation of the Mohr envelope, you know it already is s is equal to c plus sigma tan phi, where what is s is your shear stress, c is your cohesion, sigma is normal stress, phi is angle of internal friction.

So, when this Mohr circle, that is when the wall movement has become that much that the Mohr circle corresponding to that condition, touches the failure envelope, that condition is called the active condition. And you can see here, that the horizontal pressure corresponding to that is called your sigma a, that is active earth pressure, so you see earlier, it was at rest, when the wall movement went on increasing.

So, the Mohr circle will go on shifting, here in this particular direction and it attains a maximum value of horizontal stress, just before failure, that is what is called active earth pressure.

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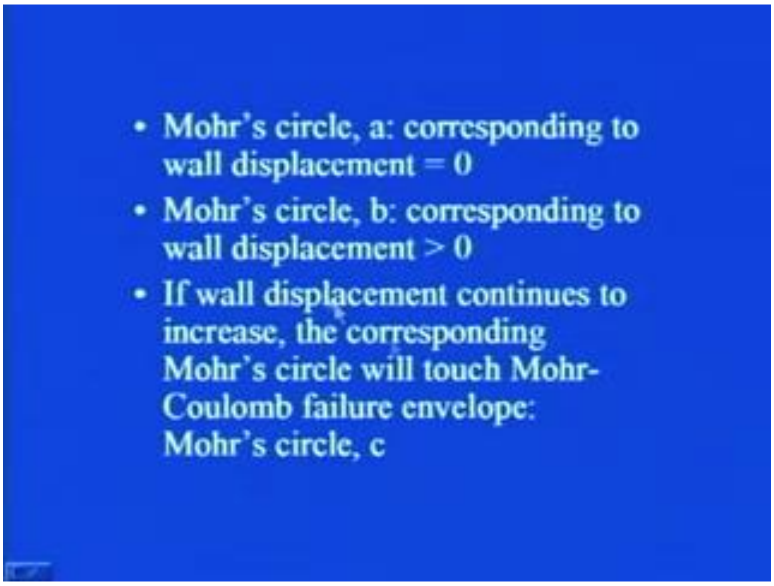

So, you see here Mohr circle a as I explained you, Mohr circle a corresponds to wall movement equal to 0,. So, you see corresponding to wall displacement is equal to 0, now as the wall displacement, goes on increasing it corresponds to Mohr circle b. So, Mohr circle b is corresponding to the wall displacement, which is greater than 0. Now, as I explained you, that if the wall movement goes on increasing, the corresponding Mohr circle will touch Mohr coulomb failure envelope and this corresponds to this Mohr circle c in this particular figure.

So, I hope that it is clear, please make sure that this mechanism is very much clear to you, otherwise you will get all type of confusion. I will repeat it again; that the first Mohr circle corresponding to sigma v and K naught sigma v is the lateral earth pressure at rest condition that is when the wall displacement is 0, as the wall displacement increases. That is the wall is moving away from the soil, what will happen is that, it will go on shifting in this particular manner.

And a condition will come, that corresponding to that wall movement, the more corresponding and Mohr coulomb will touch the failure envelope. And that condition will correspond to the failure condition and the corresponding lateral earth pressure will be known as active earth pressure.

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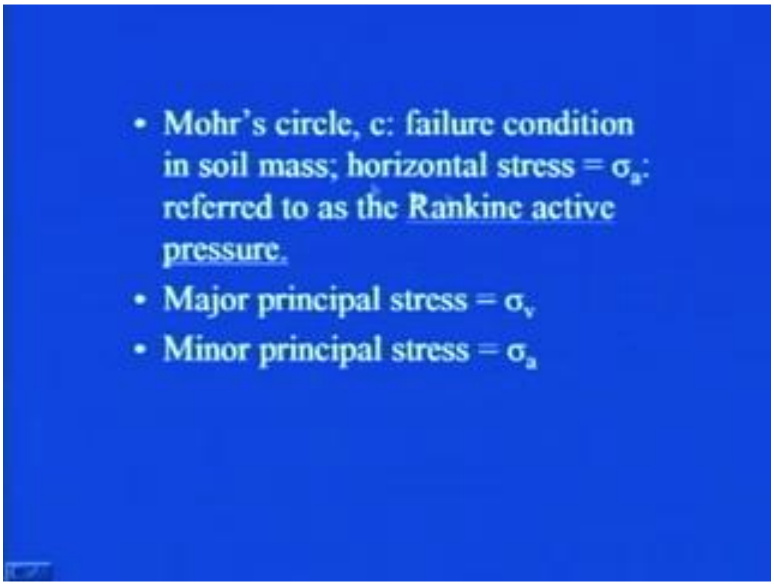

So, you see Mohr circle c, that is the failure condition in soil mass, corresponding horizontal stress is sigma a, a stands for active condition, this is referred to as Rankine active pressure. Now, you know in Mohr circle two extremes 1 extreme correspond to major principal stress, another stream corresponds to minor principal stress.

So, in case of this active condition your major principal stress is sigma v and the minor principal stress is sigma a, that is earth pressure corresponding to the active condition. So, I write it here as major principal stress, which is equal to sigma v and the minor principal stress sigma a.

Now, as you have done, I think first year level that theory of elasticity, you know the relation between the major principal stress and the minor principal stress.

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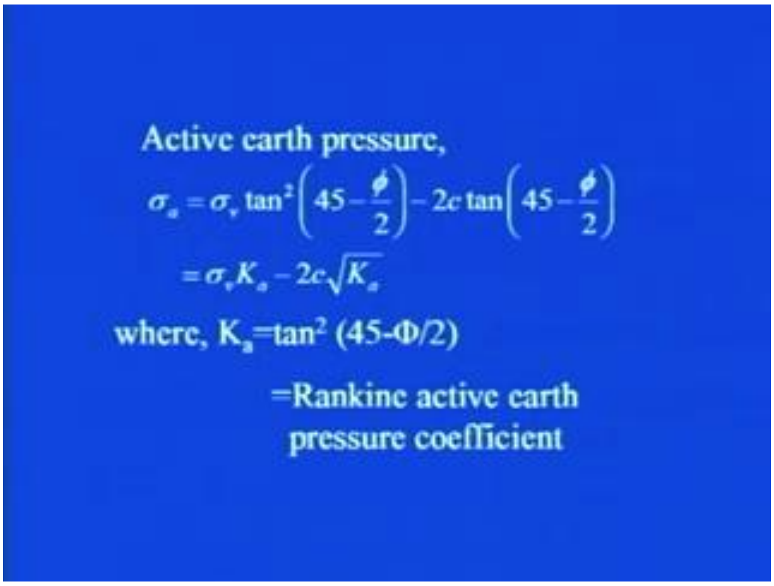

So, from the principal of the theory of elasticity, we can get the expression for sigma a, as sigma v tan square 45 minus phi by 2 minus 2 c tan 45 minus phi by 2, which I further to simplify this expression. I write as sigma v into ka minus 2 c square route of ka, where your ka is equal to tan square 45 minus phi by 2, which is known as Rankine active earth pressure coefficient, you have seen that in the earth pressure at rest condition we name that as K naught.

That was lateral earth pressure coefficient at rest condition here this ka corresponds to active earth pressure coefficient, since we are discussing the Rankines theory. So, this is corresponding to Rankine active earth pressure coefficient.

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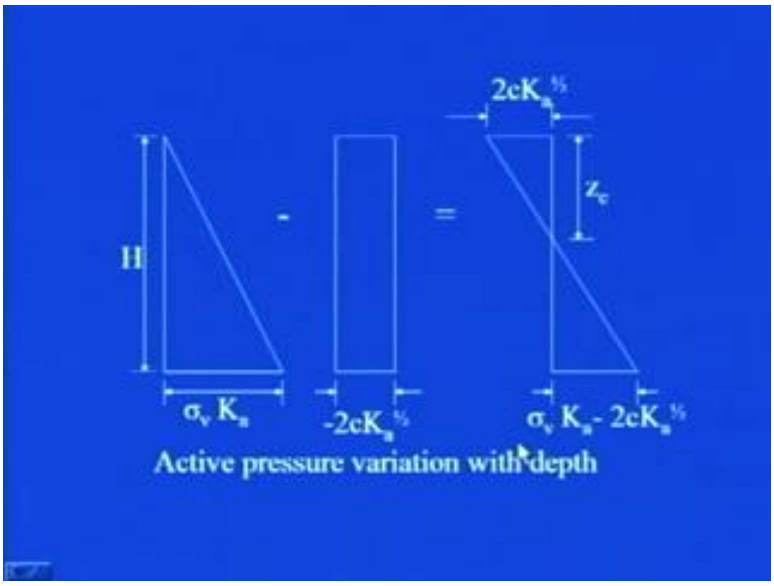

Now, how we can draw the lateral earth pressure diagram, because or it is always better to draw first earth pressure diagram, because after that your problems become very simple. That simply you have to take the area of that earth pressure diagram and then you will know the total force, per unit length of the wall and once the area is known. You can find out that, where are the different forces, which are acting and then you can find out the line of action of the resultant force.

So, we are trying to develop the active earth pressure variation with depth in case of Rankines active earth pressure. So, you see here, what does this expression give you, that at any particular depth, if sigma v is your vertical stress, then a part of this active earth pressure is sigma v K a and a part is minus 2 c square route of K. The c is coming, because in case, the soil is cohesive soil, in case the soil is cohesion less, then what will be this value of c, it will become 0 and then simply your sigma a will become sigma v into K naught.

So, I am talking in general terms, that is let us say the backfill is any c phi soil, that is any general kind of soil, but for any particular case you can simplify it accordingly. If it is purely cohesive soil, you can provide phi to be equal to 0 or if it is purely cohesion less soil you can put simply c 2 be equal to 0. So, now if you see in this figure the first part is your at any particular depth z, it will have the triangular variation and which is equal to your sigma v into K a.

Now, you see this is the second term with negative sign, so here I have this rectangular part, which why it is rectangular, because whatever was the value at z is equal to 0, that will be same as for all z values that is from 0 to H. So, you see here for the first one, it is this is varying with respect to z in a triangular fashion and it is equal to sigma v into K a and this is equal to minus 2 c times K a.

And if you combine these two, what will be the result, you can see it is very interesting, here it is 0, here it is minus 2 c square route of k, so it will be towards the opposite site. So, you see 2 c square root of K a, now and here at depth z is equal to see this was at depth z is equal to 0. Now, at z depth z is equal to H, what will be the case, if you combine sigma v K a minus 2 c square root of K simply.

So, one situation will come, where this lateral earth pressure will become 0, so you can see here at this point the lateral earth pressure is becoming 0, now let us called that at depth z c, this situation is arising. Now, what is the physical interpretation of this particular depth z, that how and why this is becoming 0, lateral earth pressure is becoming 0 is explained in the subsequent slides.

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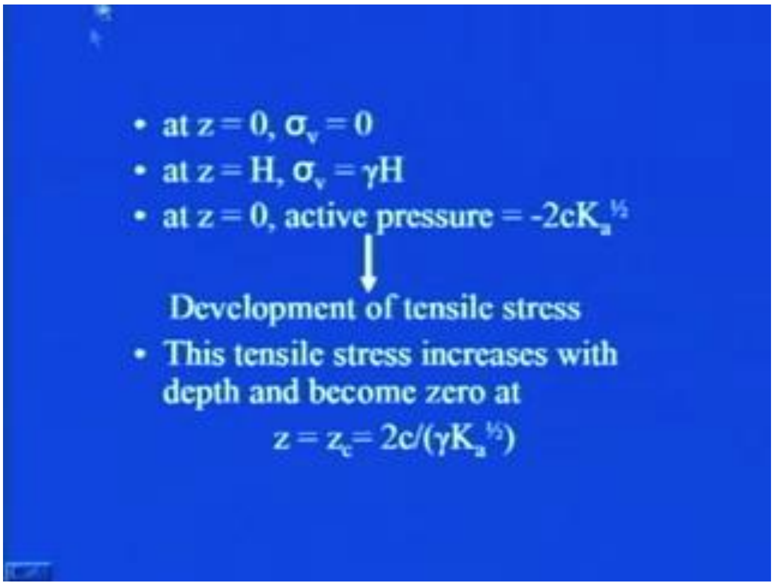

So, you see as I explained you, that at z is equal to 0, your sigma v is equal to 0 and so your sigma H is equal to 0, in that case at z is equal to H, your sigma v is gamma H at z is equal to 0, what is active pressure, you see here 0 minus this. So, that will be minus 2 c square root of K a, since it is with minus sign and you know that in soil is very good in taking compression,, but soil cannot take any tension, so minus sign signifies the presence of some tensile stresses. So, as soon as your active pressure becomes negative, it develops the tensile stress.

So, this corresponds to the condition of tensile stress that is development of tensile stress, this tensile stress increases with depth; you can see here, that as it was it is minus, so any quantity going from minus to 0 means, it is increasing. So, you see at z is equal to 0, it is 2 c square root of K and here it is becoming 0, so this tensile stress increases with depth and becomes 0 at z is equal to z c.

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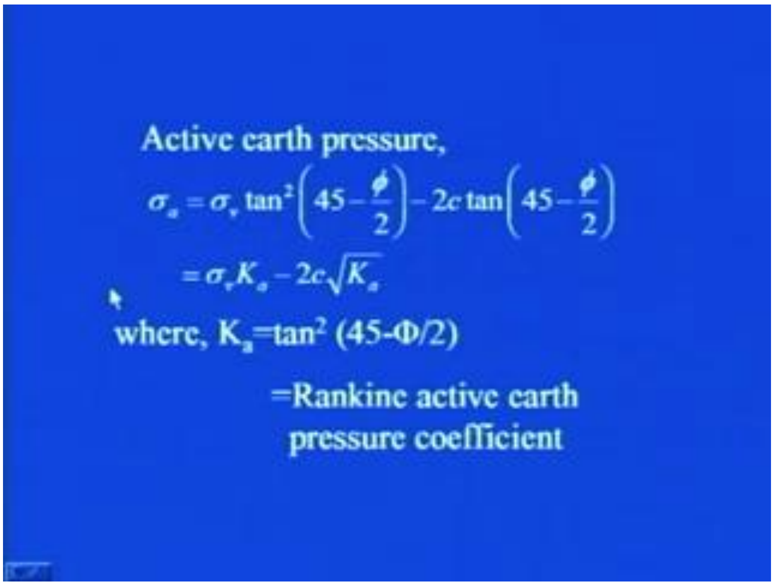

Now, how I am writing this z c is equal to 2 c by gamma square root of K a, you can see here that, your sigma a is equal to sigma v, K a minus 2 c square root of K a. If you substitute this to be equal to 0 and sigma v at z is equal to z c will be your gamma into z c and from there. You will be able to get this expression, for z is equal to z c is equal to 2 c by gamma square root of K a.

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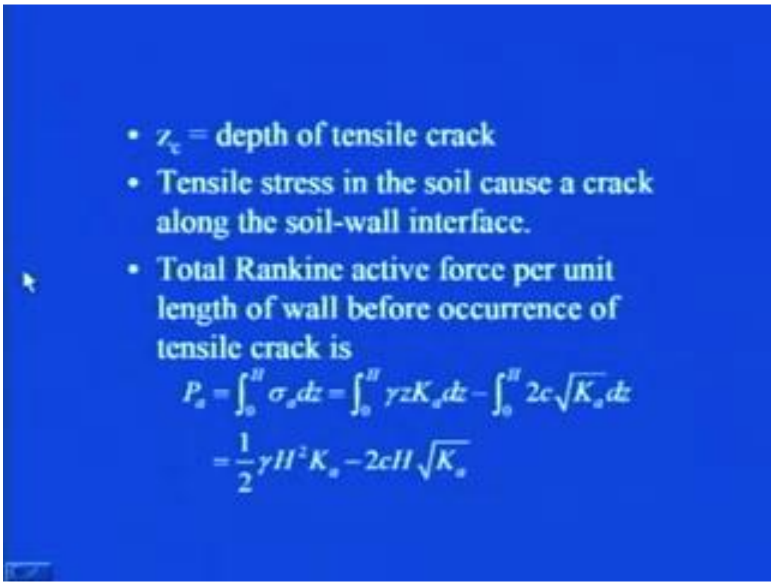

Where this z c is known as your depth of tensile crack, tensile stress in the soil, cause a crack along soil wall interface. So, you must understand that; what is the physical interpretation of active earth pressure; becoming 0 at any particular depth, as soon as it is becoming 0; that means, above that level. Then, there is a development of tensile stress and so there will be an occurrence of crack on soil and wall interface, that is, what do we mean by the occurrence of tensile crack.

Then, the total see there can be two kind of situation, in one situation just before the occurrence of the tensile crack and in another situation after the occurrence of tensile crack. So, first I discuss the Rankine force per unit length of the wall, before the occurrence of tensile crack, you can see here we were taking the area of the earth pressure diagram to find out the forces.

If, let us say the area is not well defined or it is the function of any particular depth, what do we do, we integrate it. So, that is what we are doing over here in this case, if I integrate this over the length, over the depth, that is 0 to H sigma a into d z is equal to. You can see that sigma, this 1 is gamma into H into ka minus 2 c square root of K a, you see here gamma z, K a ,d z minus the second term, that is 2 c square route of K a d z, so you see that gamma and K a, they are constant.

So, they can come out of the integration sign and if you integrate only z, d z over the depth 0 to H, what will that result to be H square by 2, so that is what it is coming here.

So, the first term will result in to half gamma H square K a minus, you see here again this whole term is constant which will come out of the integration sign and 0 to H d z will correspond to H..

You see, do not forget this thing, that how we are finding out the occurrence, the Rankine force per unit length of the wall, before the occurrence of the tensile crack.

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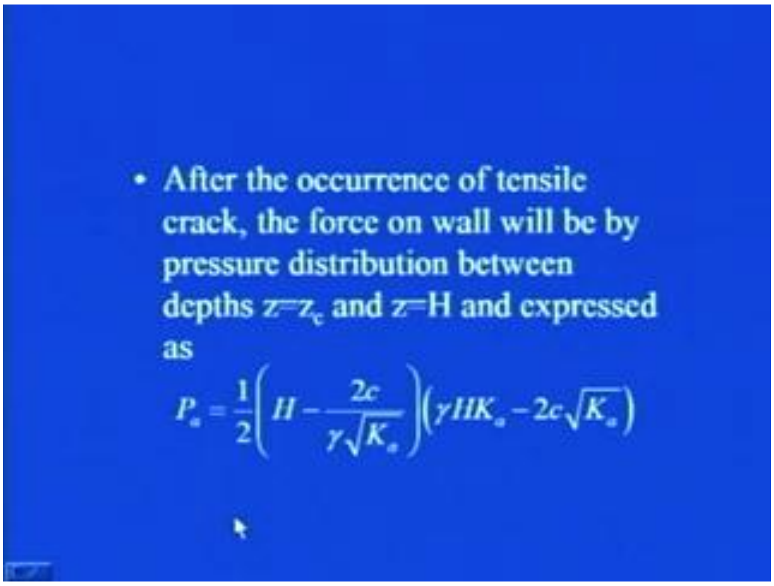

Now, the second situation when the tensile crack has already occurred, what will happen, as the soil is unable to take any tensile stress, whatever is the depth of your tensile stress. Along that depth, there will not be any pressure, because soil is unable to take any tensile stress, so the only force which will be acting on the wall will be between the depths, you see here z c and H. Because, this portion is in tension as the soil cannot take any tension, it will not exert any load on the wall or any retaining structure.

So, whatever is the stress, after the occurrence of tensile crack in this situation will be equal to the pressure, which is acting on the wall between the depth z is equal to z c and z is equal to H. So, when we will be finding out the Rankine force per unit length of the wall, after the occurrence of tensile crack, we just have to take in to account the diagram, that is earth pressure diagram between the depth z is equal to z c and z is equal to H.

So, how we can do that, this is simply expressed as P a is H minus 2 c upon gamma square root K into gamma H, K a minus 2 c K a. Now, you must be thinking that from, where it is coming, it is as simple as that, I have just taken the area of this part. You see half of this length, which is H minus z c into this base width of this triangular wedge, which is sigma v K a minus 2 c square route of K a. What is sigma v at this particular level, it is gamma into h, so that is gamma H K a minus 2 c square root of K a.

Exactly the similar thing H, minus z c, z c I have shown you, that how you can find out, that is H minus z c, that is 2 c by gamma square root of K a into gamma H K a minus 2 c square root of K a. Now, this is how we can find out the active force, due to the movement of the wall, away from the backfill soil, this is what is given by the Rankine. Now, this theory can take in to account, in case your backfill surface is not horizontal that is it is inclined with the horizontal.

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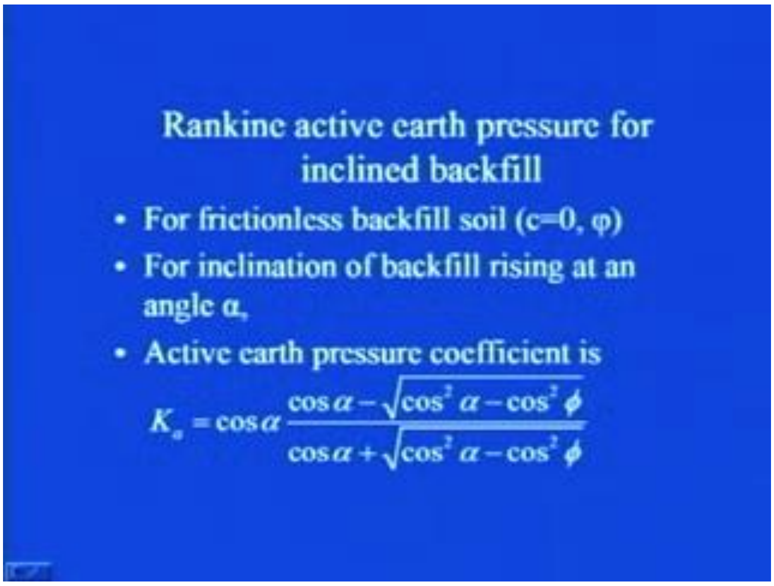

So, what will happen in that case, let us try to have a look on that, so your Rankine active earth pressure for inclined backfill, I assume that, usually in practice it is adopted that your backfill soil is frictionless. So, in case of frictionless soil, that is purely frictionless soil, your c becomes 0, it is just phi, which comes into picture, as for as shear strength properties of the soil is concerned. So, you see for frictionless backfill soil, your c become 0 only, phi remains there.

For inclination of backfill rising at an angle alpha, the only change which will occur in the expression, is that your active earth pressure. Coefficient will become cos alpha into cos alpha minus square root of cos square alpha minus cos square phi, divided by cos alpha plus square root of cos square alpha minus cos square phi.

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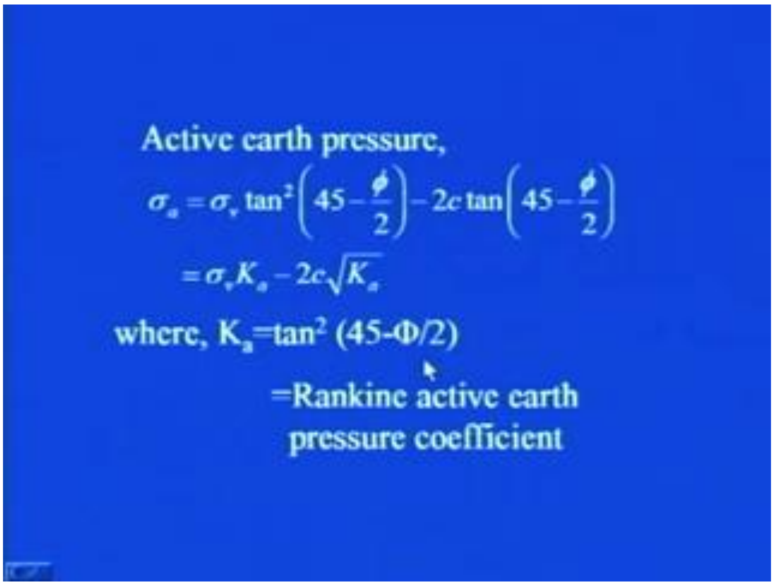

Earlier, what was that, that was simply tan square 45 minus phi by 2, just see here earlier it was tan square 45 minus phi by 2. In case of horizontal backfill, but now when the backfill is inclined at an angle alpha, your K a expression will become this.

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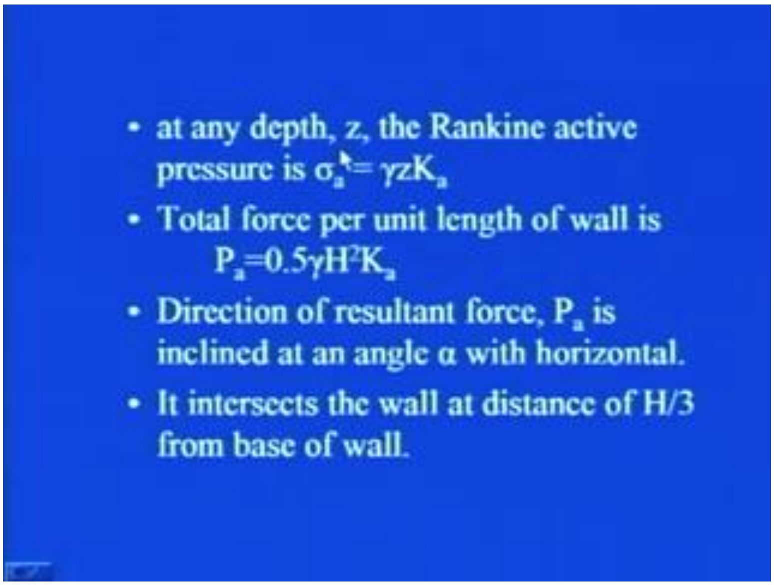

And, how we can find out the Rankine pressure in this case is that at any depth z, the Rankine active earth pressure will be gamma z K a. Now, you must be thinking that, why this minus 2 c K a square root K term is not coming, see here I am talking of frictionless backfill soil, where the c is 0, so the second term is not at all there in the picture.

So, my total force per unit length of the wall will become simply the area of that triangular part, that is triangular part of lateral earth pressure diagram, which will be point five gamma H square K a. Now, direction of this resultant force P a will be inclined at an angle alpha with horizontal and where it will intersect this, that it intersects the wall at a distance of H by 3 from the base of the wall.

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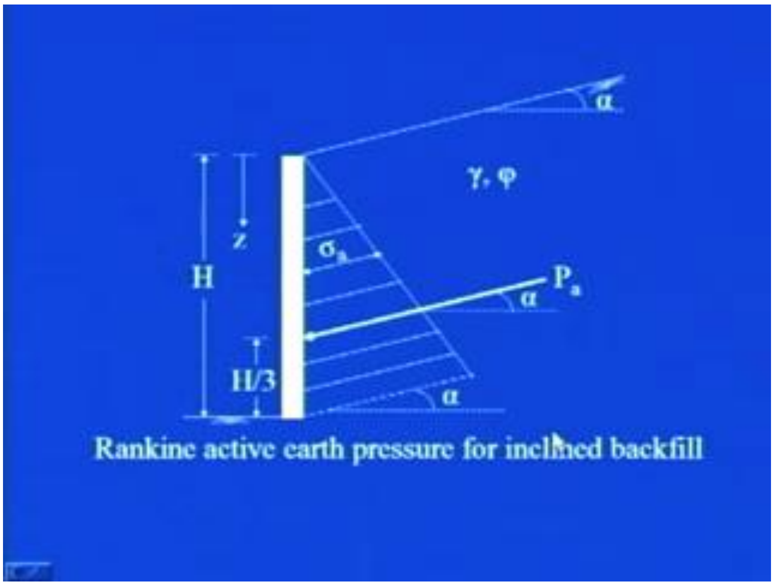

So, as you can see from this figure as I told you earlier, that this P a, that is total force, in case of an inclined backfill is acting at a distance of H by 3 from the base of the wall and it is parallel to the inclination of your backfill. So, you see at with depth, how it is changing and what is the value, that it will attain at this z is equal to H, along with this property of the soil as gamma and phi.

Now, you see here, this is the very tedious kind of expression, so standard tables are available in standard text books, which gives the value of K a for different values of alpha and phi. I will show you some of the typical values in a tabular form; you can just have a look.

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Values of active earth pressure coefficient, K_a

α (deg) ↓	Φ (deg) →		
	28	30	32
0	0.361	0.333	0.307
5	0.366	0.337	0.311
10	0.380	0.350	0.321
15	0.409	0.373	0.341
20	0.461	0.414	0.374
25	0.573	0.494	0.434

That for different values of alpha, that is 0, 5, 10, 15, 20, 25 angle of internal friction to be 28, 30, 32, how this K a is taking the value. It is nothing, but simply, what is exactly is the value, which you are obtaining from the expression, as I showed you earlier.

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Values of active earth pressure coefficient, K_a

α (deg) ↓	Φ (deg) →			
	34	36	38	40
0	0.283	0.260	0.238	0.217
5	0.286	0.262	0.240	0.219
10	0.294	0.270	0.246	0.225
15	0.311	0.283	0.258	0.235
20	0.338	0.306	0.277	0.250
25	0.385	0.343	0.307	0.275

Again, this is the further some more values corresponding to phi is equal to 34, 36, 38 and 40 for different values of alpha, all these are in degrees and since this is coefficient it is an non dimensional parameter. So, if you know the value of alpha; that you will know, because it is the incrimination of the backfill and you know the property of the soil, which is phi, in this case because c is equal to 0, as we have assumed this soil to be frictionless.

So, in that case, you can simply pick the value of K a, from these standard tables and putting into the expression of the total force per unit length of the wall. You can get the magnitude of the Rankine active, force per unit length of the wall, which will be acting at a depth of H by 3 from the bottom of the wall.

So, today what we saw was that, what exactly do we mean by lateral earth pressure, how it occurs, how it affects nearby structures, why should we need to estimate it properly, how it is going to affect the retaining structure design. And then, we saw that, there are three conditions that is wall at rest, wall moving away from the soil, wall move moving towards the soil that is active and passive conditions respectively.

And we saw that, how the analysis is being carried out for the determination of lateral earth pressure, at rest condition and then we started with the estimation of lateral earth pressure in active condition. I mentioned you, that there are two types of theories, which are commonly adopted, that one is Rankine active earth pressure, second is Coulombs act active earth pressure condition. When, we discussed with the Rankine active pressure conditions with horizontal back fill and then with inclined backfill.

You have seen that, how they are different, how they can be incorporated for different type of backfill, whether it is frictionless or whether it is cohesive backfill. So, next time we will be starting with the coulombs lateral earth pressure theory and then, we will try to see, we will try to solve some of the examples, based on that to make all these things more clear to you.

Thank you.