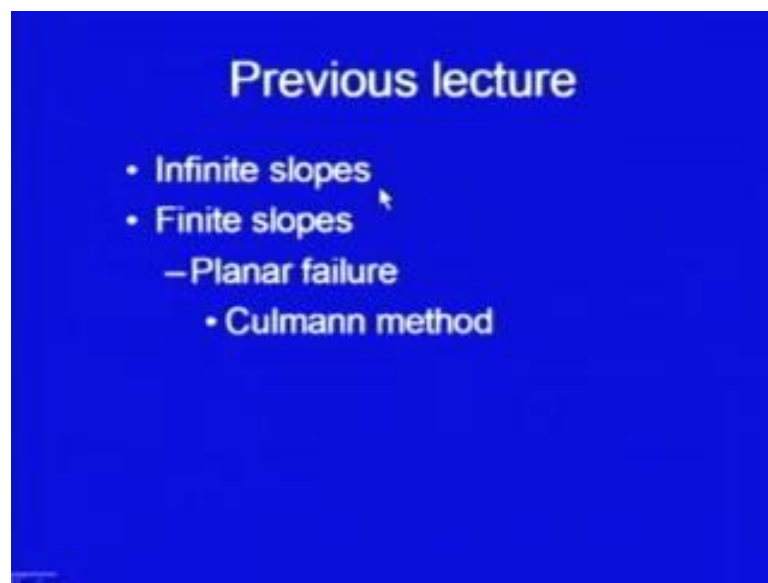


Foundation Engineering
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Module - 03
Lecture - 09
Stability of Slopes

Welcome back to the classes of on this Stability of Slopes, we have been discussing this chapter for last few lectures.

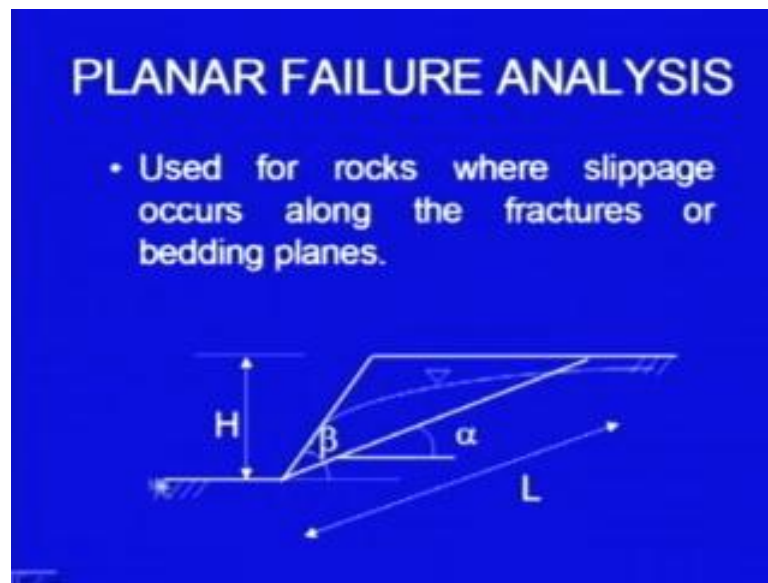
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We started discussing the infinite slopes where the extent of the slope is infinite and there we considered several cases of the seepage conditions. We took the water table to be present there or it was completely dry slope and water table was parallel to the ground surface. Secondly, we considered the plane of rupture parallel to the ground surface.

After that, we had started discussing about the finite slopes, generally finite slopes means those slopes which are having small extent they are man made structures and we were discussing the planar failure. We have already discussed the Culmann's method and we are proceeding with the same topic now planar failure analysis.

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Now, this analysis as we did it last time, it is used for rocks where slippage occurs along the fractures or the bedding planes. Here, it is a slope and here it is a geological medium rock and this is the fracture or it is a bedding plane. This is a water table and height of the slope is let us say H , the slope angle is β and this fracture or the dip or this particular discontinuity the bedding plane is making angle α . This is the apparent dip in this direction, so when you see the section here the apparent dip is α degree.

The analysis is simple as we did for the other cases, we are going to consider this particular mass. And in present case, it is not an isotropic medium it is an unisotropic medium and if this is the weakest link and it is going to fail over this our assumption is that sliding is going to take place on this plane.

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$$\begin{aligned} \text{Weight of wedge / m length} \\ &= W = f(\beta, \alpha, H, \gamma) \\ \tau &= (W/L) \sin \alpha \\ \sigma' &= (W/L) \cos \alpha \\ \tau_f &= c' + \sigma' \tan \phi' \\ &= c' + \{(W \cos \alpha)/L - u\} \tan \phi' \\ u &= \text{water pressure} \\ F_s &= \frac{c'L + [W \cos \alpha - \psi L] \tan \phi'}{W \sin \alpha} \end{aligned}$$

So, to do the analysis, we will find out the weight of the wedge, the triangular wedge and perpendicular to the plane of paper again we are taking 1 meter length. So, the weight you can find in terms of as a function of slope angle beta and dip angle alpha height of the slope and the unit weight of the material.

Then, we have to resolve these forces the shear stress. (Refer Slide Time: 01:40) So, here it is the weight of the wedge it is acting in this direction, vertical direction we are going to resolve it in two components as usual. So, one component will be perpendicular to the plane of this plane of sliding, another component will be parallel to the plane of sliding, this component which is parallel to the plane of sliding is going to induce instability and that component which is perpendicular to the plane of sliding that is going to enhance this shear strength.

So, this sliding component in terms of shear stress will be W into $\sin \alpha$, this is the component and L , L is the dimension of this particular length, this is L and perpendicular we are taking one, so this area becomes L into 1. So, shear stress becomes $W \sin \alpha$ upon L into 1, so that is nothing but W upon L into \sin of α .

The normal stress normal component will be $W \cos \alpha$ and again it has to be divided by the area along which the sliding is taking place and that area is L into 1. So, the normal stress becomes W upon $L \cos$ of α .

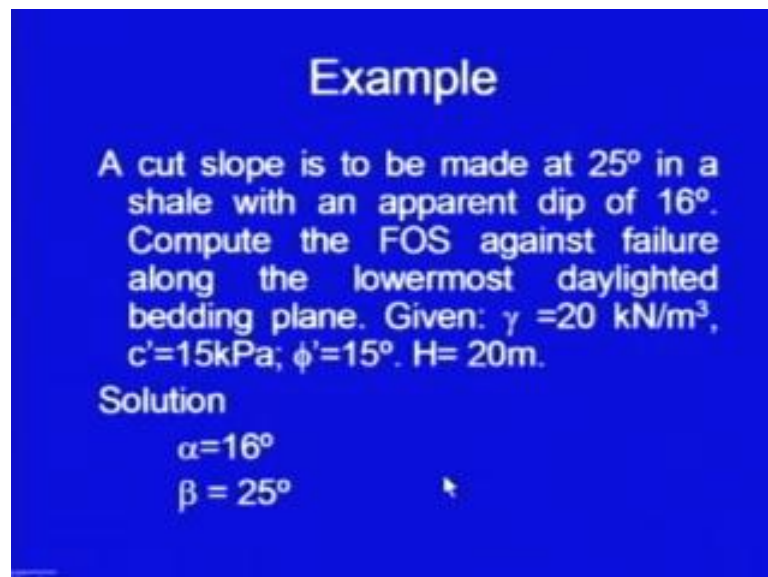
The shear strength that is the maximum stress that can occur along the joint plane will be equal to let us say these are the shear strength parameters. Please note these, C dash and

ϕ' are the shear strength parameters for the fracture or the bedding plane or the joint plane where we are assuming the sliding to occur. So, τ_f will be equal to the same equation we are using the linear molecular criterion, so $c' + \sigma' \tan \phi'$ and I put this values σ' , so this becomes $c' + W \cos \alpha$ upon L and here I have subtracted u is bore water pressure.

So, it has to be weak we are doing the analysis in terms of effective stresses, so if water pressure is there it has to be subtracted. So, that you get the effective stress and multiplied by $\tan \phi'$, so σ' here I did not show it here, but here I have taken it, so water pressure is u .

So, finally, we can have the expression for the factor of safety, the available strength is this much $c' + W \cos \alpha$ upon L minus u into $\tan \phi'$ and divided by the applied shear stress is this much W upon $L \sin \alpha$ and when you simplify this the L will cancel and it will come here. So, final expression we are going to get for the factor of safety against shear strength is $c' L + W \cos \alpha - u L$ into $\tan \phi'$ divided by $W \sin \alpha$.

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Example

A cut slope is to be made at 25° in a shale with an apparent dip of 16° . Compute the FOS against failure along the lowermost daylighted bedding plane. Given: $\gamma = 20 \text{ kN/m}^3$, $c' = 15 \text{ kPa}$; $\phi' = 15^\circ$. $H = 20 \text{ m}$.

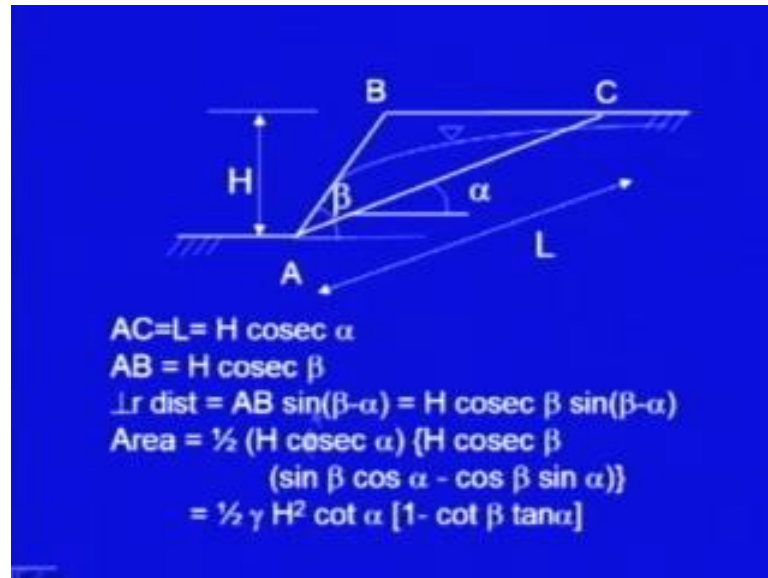
Solution

$\alpha = 16^\circ$
 $\beta = 25^\circ$

Let us have an example, it is given that there is a cut slope it is to be made at 25 degree in a shale with an apparent dip of 16 degree compute the factor of safety against failure along the lowermost daylighted bedding plane.

And, the parameters are given as unit weight is 20 kilo Newton per meter cube C dash along the bedding plane is 15 kPa and phi dash along the bedding plane is 15 degree height of the slope is given as 20.

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So here in this case, dip alpha is 16 and slope angle is 25, so here is the fail this is the be slope this is the height. So, we have to calculate its weight, to calculate the weight let us find out the area of this wedge.

So, I can find out the area in terms of in this dimension AC, if I know AC and half into AC into this perpendicular distance that is going to give me area. Then, I will take 1 meter length perpendicular to the plane of paper that will give me that volume and then multiply it by the unit weight we will be getting the weight of the wedge.

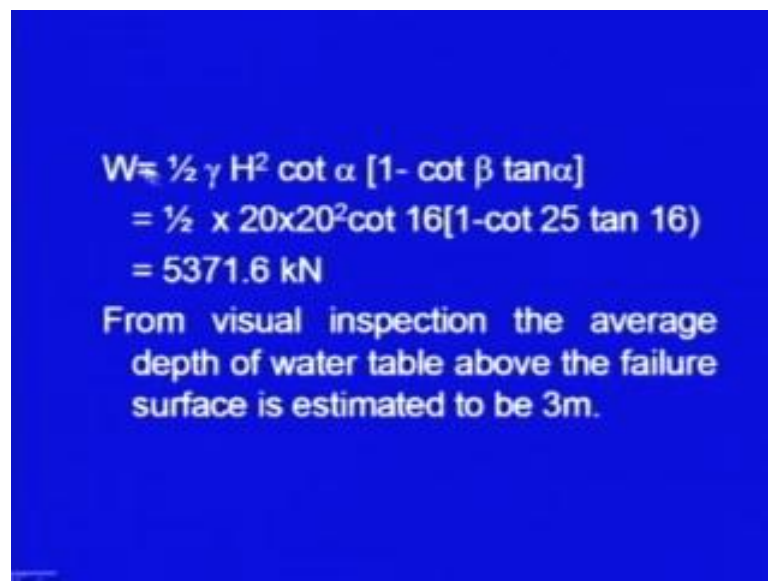
So, AC is equal to here, in terms of alpha you can get it, this height is H and this angle is alpha. So, from this right angled triangle I can find out AC will be equal to H into cosec of alpha and also you can find out AB will be equal to this angle is beta, so AB will be equal to H cosec beta.

And, this perpendicular distance which I need I drop up a perpendicular from B on AC. So, this perpendicular distance will be nothing but from this right angled triangle from point B to this point to point A from this triangle I am going to get this perpendicular distance. So, perpendicular distance will be equal to AB this into sin of this angle and this angle is nothing but beta minus alpha.

So, perpendicular distance will be equal to AB sin of beta minus alpha put value of AB. So, it becomes H cosec beta into sin of beta minus alpha. So, I get the area equal to half into AC, AC is H cosec alpha into this distance perpendicular distance H cosec beta and sin beta cos alpha minus cos beta sin alpha.

And finally, when I simplify it I get a very simple expression, area is equal to this is the weight in fact, you have to multiply it by gamma also. So, weight is equal to half gamma H square cot alpha one minus cot beta tan alpha.

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$$W = \frac{1}{2} \gamma H^2 \cot \alpha [1 - \cot \beta \tan \alpha]$$
$$= \frac{1}{2} \times 20 \times 20^2 \cot 16 [1 - \cot 25 \tan 16]$$
$$= 5371.6 \text{ kN}$$

From visual inspection the average depth of water table above the failure surface is estimated to be 3m.

So, finally, this is the expression for the weight of the wedge and now I put it in the expression. I put all the values and the weight of the wedge is obtained to be 5371.6 kilo Newton.

(Refer Slide Time: 07:59) Now from this given figure you have to make some assumption you have to make some estimation of the water pressure also. You can check how much what the variations are, how the water table depth is varying above this failure surface and take some average value.

So, from visual inspection the water table above the failure surface has been estimated to be 3 meter in this case. So, there can be some bias in this you can also reset it accurately you can get the accurate average value.

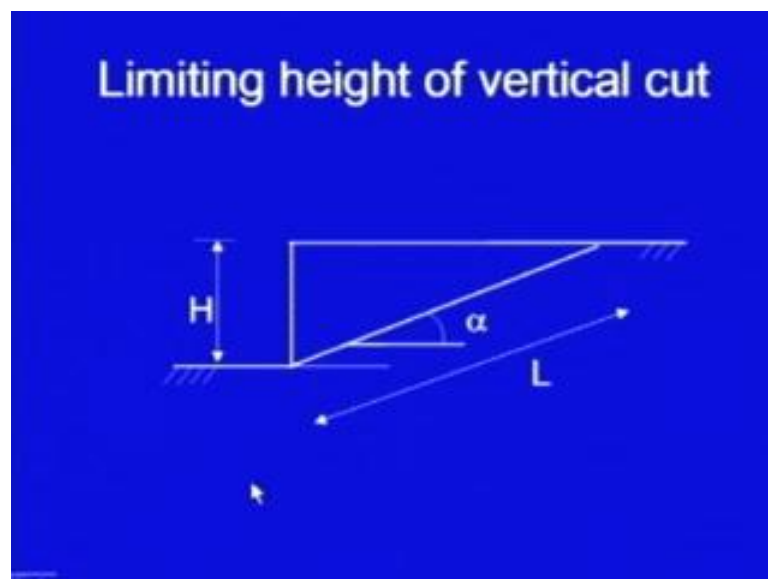
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$$\begin{aligned} \Rightarrow u &= \gamma_w z = 9.81 \times 3 = 29.43 \text{ kPa.} \\ \text{also } L &= 20 \cot 16 = 69.74 \text{ m} \\ F_s &= \frac{c' L + [W \cos \alpha - uL] \tan \phi'}{W \sin \alpha} \\ F_s &= \frac{15 \times 69.74 + [5371.6 \cos 16 - 29.43 \times 69.74] \tan 20}{5371.6 \sin 16} \\ &= 1.47 \end{aligned}$$

So, u will be equal to $\gamma_w z$, z is 3 meter which was estimated. So, pore water pressure comes out to be this much. Also the length L is $20 \cot$ of 16 equal to this much,

This was the expression for the factor of safety F_s is equal to $C \text{ dash } L \text{ plus } W \cos \alpha \text{ minus } u \text{ into } L, L \text{ is the base length of the wedge, } \tan \phi \text{ dash upon } W \sin \alpha$. So, all the parameters are available with me I put them in this equation and finally, the factor of safety comes out to be 1.47.

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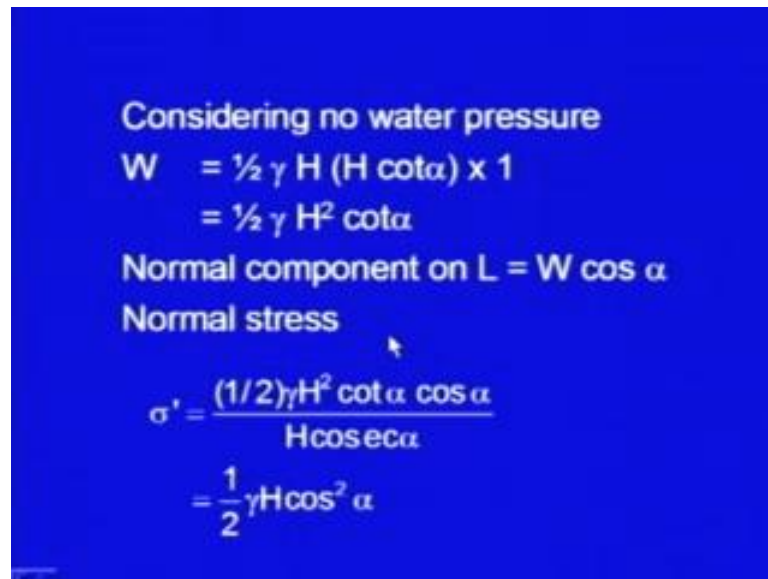


Let me, now go to another case again it is the same problem which we did last time, but I am now trying to find out, what can be the limiting height of a vertical case, so here

again this is the upper surface and this is the ground level and here it is slope and this is a discontinuity or bedding plane at an angle alpha.

The analysis is philosophically mechanistically same, this is the L is the length of the base. Here we are assuming that the sliding is taking place on that is this plane and we will be finding out H again using the same mechanics same principles.

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Considering no water pressure

$$W = \frac{1}{2} \gamma H (H \cot \alpha) \times 1$$

$$= \frac{1}{2} \gamma H^2 \cot \alpha$$

Normal component on L = $W \cos \alpha$

Normal stress

$$\sigma' = \frac{(1/2) \gamma H^2 \cot \alpha \cos \alpha}{H \operatorname{cosec} \alpha}$$

$$= \frac{1}{2} \gamma H \cos^2 \alpha$$

We are not considering water pressure at present, so the weight of this wedge will be equal to half gamma H into H cot alpha into 1.

(Refer Slide Time: 12:34) So here, it is nothing its half this dimension, this dimension is H and this dimension this angle is alpha, this angle will also be alpha, so this dimension will become H cot alpha. So, the area of this wedge will be half into H into H cot alpha and from that you can get the weight, weight will be volume and we are using we are assuming one meter distance perpendicular to the plane of paper.

So, weight will be equal to half gamma H square cot of alpha and again we will resolve this force W it is acting in vertical direction, we will resolve in normal to the plane as well as. So, here it is W, W is acting in this direction again we are resolving it normal to the plane and parallel to the plane.

So here, normal component on L will be equal W into cos of alpha and the normal stress now will be equal to you can find it out, half gamma H square cot alpha this is W. This value into cos alpha this divided by the area, area over which the sliding is taking place,

so from here this is H this is the dimension L that becomes H cosec alpha and perpendicular direction it is one. So, the area becomes H cosec alpha into 1.

So, sigma dash the normal stress will be equal to this much and when I simplify it this is the expression which is obtained sigma dash is equal to half gamma H cos square alpha.

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Shear stress $\tau = (W \sin \alpha)/L$

$$\tau = \frac{(1/2)\gamma H^2 \cot \alpha \sin \alpha}{H \operatorname{cosec} \alpha}$$

$$= (1/2)\gamma H \cos \alpha \sin \alpha$$

Shear strength $\tau_f = c' + \sigma' \tan \phi'$

$$= c' + (1/2) \gamma H \cos^2 \alpha \tan \phi'$$

For stable slope $\tau \leq \tau_f$

Also the component which is creating instability is the shear stress along the plane. So, that will be W into sin of alpha, it is the component which is along the tangential direction divided by area, area is L into 1. So, the shear stress put the value of W and solve it the shear stress is half gamma H cos alpha sin alpha.

Now, to get the limiting height the maximum height which the slope can have shear strength should be equal to the shear stress applied. So, let us calculate the shear strength first, so shear strength the maximum stress which can occur along this plane is C dash plus sigma dash tan of phi dash. So, C dash plus put the value of sigma dash previously we have obtained it half gamma H cos square alpha, so put it here and you get this expression.

And now, for stable slope tau will be equal to tau f and for limiting case, for stable slope tau should be less than less than or equal to tau f and for limiting case these two quantities will be same.

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$$\Rightarrow (1/2)\gamma H \cos\alpha \sin\alpha \leq c' + (1/2)\gamma H \cos^2\alpha \tan\phi'$$
$$H \leq \frac{2c'}{\gamma \cos^2\alpha (\tan\alpha - \tan\phi')}$$

Note:

- i. Height obtained above should also satisfy the condition $\gamma H \leq q_c$
- ii. For $\alpha < \phi'$, negative value will be obtained \Rightarrow get H from UCS.

So, this is tau and this is tau f I put these expressions here this is the value of tau, tau is less than tau f. So, for stable slope take this quantity H outside. And then, solve this expression simplify it and you are going to get H less than equal to 2 C dash gamma cos square alpha bracket tan alpha minus tan of phi dash. So, this is this expression is going to give you the limiting value of H, so maximum value will be when we put H is equal to this much.

Now, there are certain some important notes I have included here, first note is that height obtained above should also satisfy the condition gamma H is less than q c. So, remember q c is the uni-action compressive strength of the material and there may certain case when this value will turn towards infinity, in fact if you put alpha equal to phi dash suppose phi dash is equal to 35 and you take alpha also 35, then this value becomes 0.

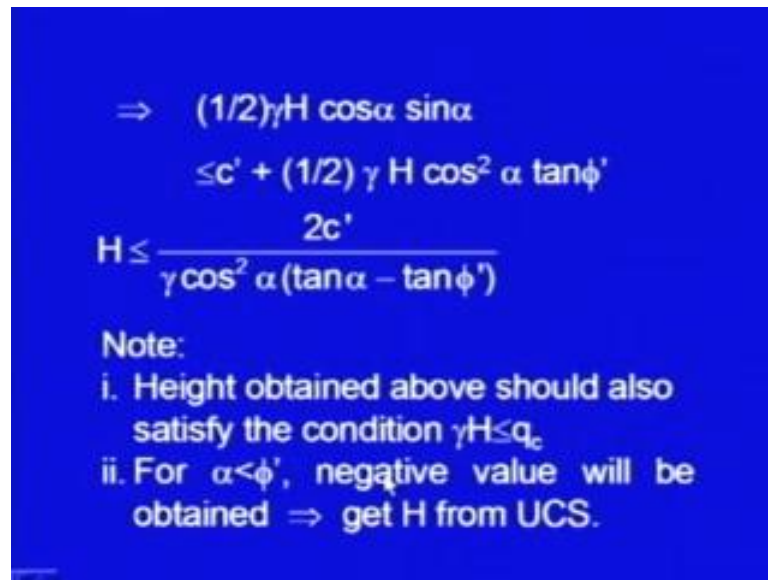
So, denominator is becoming 0, it is tending towards 0; that means, H will tend towards infinity. So, as per this expression this is a expression which we have got which we have obtained from mechanics, so for the equilibrium to exist infinite height will be possible, but it will not be possible practically why, because at the toe the pressure as you know if the unit weight of a material is gamma and its depth is H or the height is H, then at depth H the pressure is given by gamma into H.

So, if this particular pressure it becomes too high if H value becomes higher and higher this pressure will be become higher and higher. This should not exceed the uni-action

compressive strength otherwise it will fail, so the pressure the maximum pressure is which it can take under unconfined state is equal to its uni-action compressive strength.

So, this is the limiting condition which we will always check and second thing is that for certain values for alpha less than phi dash.

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$$\Rightarrow (1/2)\gamma H \cos\alpha \sin\alpha$$
$$\leq c' + (1/2) \gamma H \cos^2 \alpha \tan\phi'$$
$$H \leq \frac{2c'}{\gamma \cos^2 \alpha (\tan\alpha - \tan\phi')}$$

Note:

- i. Height obtained above should also satisfy the condition $\gamma H \leq q_c$
- ii. For $\alpha < \phi'$, negative value will be obtained \Rightarrow get H from UCS.

If I put alpha here let us say 20 and phi dash is 40, so tan alpha will be less than tan of phi dash.

So, you are going to get negative value here. So, this expression is going to give you negative answer, but that is not feasible that is not possible, what it means is that, you for making it for keeping it in limit equilibrium you have to apply negative load. So, physically it is not possible.

So, if you are getting alpha less than phi dash; that means, again you have to keep you can take this H value equal to the limiting value which we calculated using this first criterion that is gamma H is less than equal to q c. So, you have to get in that case also you have get height from the UCS of the material.

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Example

Following properties are given for a bedded rock along the bedding planes: $c' = 1000$ kPa; $\phi = 25^\circ$ also $\gamma = 20$ kN/m³; $q_c = 50$ MPa. Get:

- i. Limiting height of slope if bedding planes dip at 40° ;
- ii. Show variation of limiting height as a function of dip angle α .

To explain this particular phenomena, this particular case let us have an example again, the following properties are given for a bedded rock along the bedding planes C dash is 1000 kPa ϕ dash is 25 degree γ is 20 kilo Newton per meter cube and UCS is 50 MPa. And, it has been asked to find out limiting height of slope if bedding planes dip at 40 degree and also show variation of this limiting height as a function of dip angle α .

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Solution

i. Given: $c' = 1000$ kPa; $\phi = 25^\circ$; $\gamma = 20$ kN/m³; $\alpha = 40^\circ$;

Limiting height of slope:

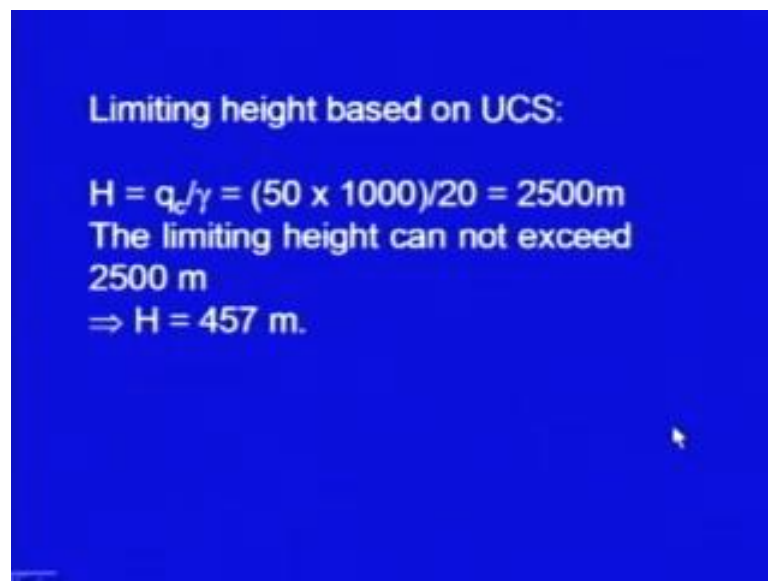
$$H = \frac{2c'}{\gamma \cos^2 \alpha (\tan \alpha - \tan \phi')}$$
$$= \frac{2 \times 1000}{20 \cos^2 40 (\tan 40 - \tan 25)} = 457\text{m}$$

So, this is the first the solution of the first part, these are the given parameters and limiting height of the slope from the expression H should be less than equal to $2 C$ dash upon $\gamma \cos$ square $\alpha \tan$ α minus \tan ϕ dash.

Maximum value will replace this less than equal to sin by equal to sin, so this is the maximum height each it can have. In fact; that means, this will be having factor of safety equal to 1 for this particular height the sliding plane will be having factor of safety equal to 1.

Shearing stresses which are being applied there and the shearing strength shear strength they will become equal. It will be the limiting case, so from this computation I get H is equal to 457 meter.

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Limiting height based on UCS:

$$H = q_c / \gamma = (50 \times 1000) / 20 = 2500 \text{ m}$$

The limiting height can not exceed 2500 m

$$\Rightarrow H = 457 \text{ m.}$$

Now, I should check it, whether the pressure at the toe of the slope is exceeding the UCS of or not. So, H is equal to q_c upon γ I used this expression, the pressure is given as γ into H.

So, H is equal to q_c upon γ , the q_c value was given as 50 MPa. So, 50 MPa means 50 into 1000 kPa and γ is 20 kilo Newton per meter square and when I solve the answer is 2500 meter. So, this means if the slope is failing, because of the uni action compressive strength being exceeded by the pressure, then it should have height more than 2500 meter.

Up to the height of 2500 meter the pressure at the toe is not going to exceed the UCS. So, we obtained in the in the first part we obtained H is equal to 457, so this is the $c F$ value. So, limiting height will be 457 meter.

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ii. To get variation of limiting H, vary α from 0 to 90° and get H. At each step if computed H is negative or more than 2500m it is limited to 2500m. Values obtained are as follows:

$\alpha^\circ =$	0	10	20	30	40	50
H,m =	2500	2500	2500	1200	457	334
$\alpha^\circ =$	60	70	80	90		
H,m =	316	375	639	2500		

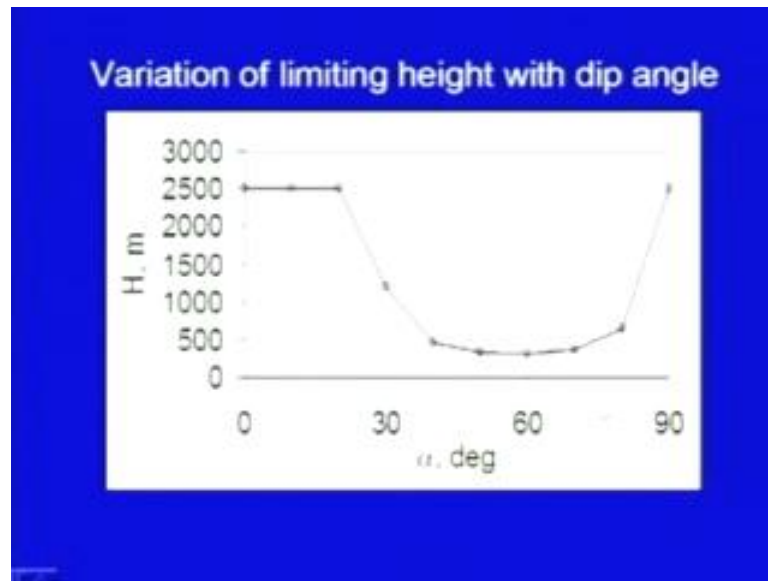
Let me, go the second part of the question, we were asked to find out, how the limiting height H is varying with alpha and alpha has been varied from 0 to 90 degree. So, it is a simple calculation, you have to put different values of alpha in to the expression and then get H values.

And, at each step if computed H is negative those two criteria I have to remember, number one if it is negative then also and if it is more than 2500, then also we have to limit the H value to 2500 meter. So, I am not showing the calculations and the final values are shown here.

This is alpha, 0 10 I have taken it at 10 degree interval 20 30 40 50 and so on up to 90 and you can see here, the final answer is 2500. So, I might have got negative value or more than 2500, so here also it is 2500, here also it is 2500. So, up to here in calculation procedure I might have got values more than 2500 or negative.

And here, it is 1200, so it is 457 334 316 375 639 and 2500. So, again here probably we might have got infinity value.

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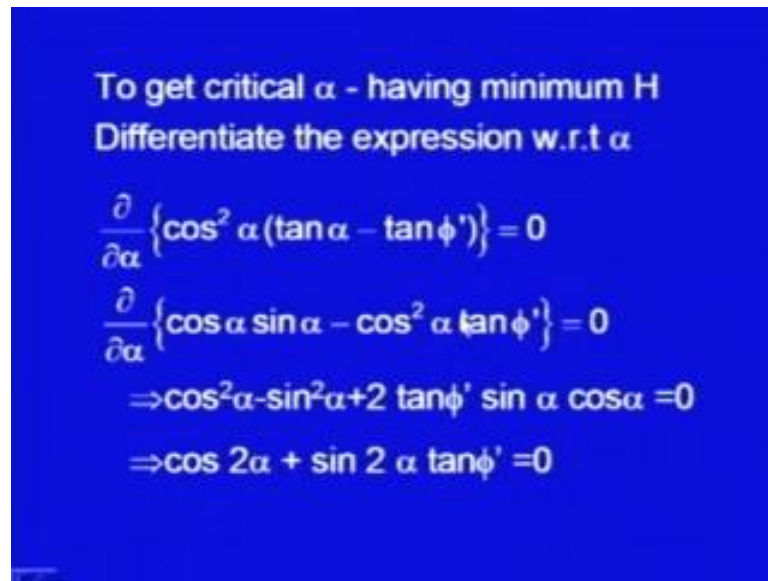
Let me, show the variation here on x axis we have drawn the angle alpha on y axis, the limiting height and you can see how it is varying. So, this is the curve which we are going to get from that expression.

And it was moving like this it should be a smooth curve it will be moving like this it goes to it starts from infinity here, comes here. Then goes to some certain minimum value at 60 degree and then it starts increasing and then again at 90 degree that value from expression will turn towards infinity. But, this is the limiting value which we have taken the final height which is being suggested is this much.

So 2500, this remains constant up to here and then it goes to a minimum value, then maximum. In fact, I had join this point right up to here, theoretically if you take the points at 1 degree interval at a very small interval it should come somewhere here. Then it should tend it should proceed towards infinity and here again you will be getting that straight line limiting portion.

Now, let us proceed further with the same problem, let us get critical alpha which is having minimum H, though in the previous problem we have solved this numerical problem and we have got the minimum value somewhere here.

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To get critical α - having minimum H
Differentiate the expression w.r.t α

$$\frac{\partial}{\partial \alpha} \{ \cos^2 \alpha (\tan \alpha - \tan \phi') \} = 0$$
$$\frac{\partial}{\partial \alpha} \{ \cos \alpha \sin \alpha - \cos^2 \alpha \tan \phi' \} = 0$$
$$\Rightarrow \cos^2 \alpha - \sin^2 \alpha + 2 \tan \phi' \sin \alpha \cos \alpha = 0$$
$$\Rightarrow \cos 2\alpha + \sin 2\alpha \tan \phi' = 0$$

Let us get it theoretically also, so we have the expression for the minimum H, what we are going to do is, we are going to differentiate that expression with respect to a angle alpha and make it equal to 0.

So, del y del alpha cos square alpha tan alpha minus tan phi dash this we are going to put equal to 0. So, let me multiply this cos square alpha into tan alpha it becomes cos alpha into sin alpha and here cos square alpha into tan of phi dash.

So, now I differentiate them with respect to alpha, so first function differentiation of second. So, it becomes cos alpha into differentiation of sin alpha is cos alpha, so cos square alpha plus second function sin of alpha differentiation of first that becomes minus sin alpha, so sin alpha into minus sin alpha is minus sin square alpha.

And then, phi dash is constant, so tan of phi dash here and differentiation of cos square alpha will be equal to 2 cos alpha into minus sin alpha, so minus minus becomes plus. So, finally, we have this expression cos square alpha minus sin square alpha plus 2 tan phi dash sin alpha cos alpha equal to 0.

I can now simplify it cos 2 alpha will be equal to cos square alpha minus sin square alpha. So, replace this term by cos of 2 alpha and here 2 sin alpha cos alpha is again sin of 2 alpha into tan of phi dash is equal to 0.

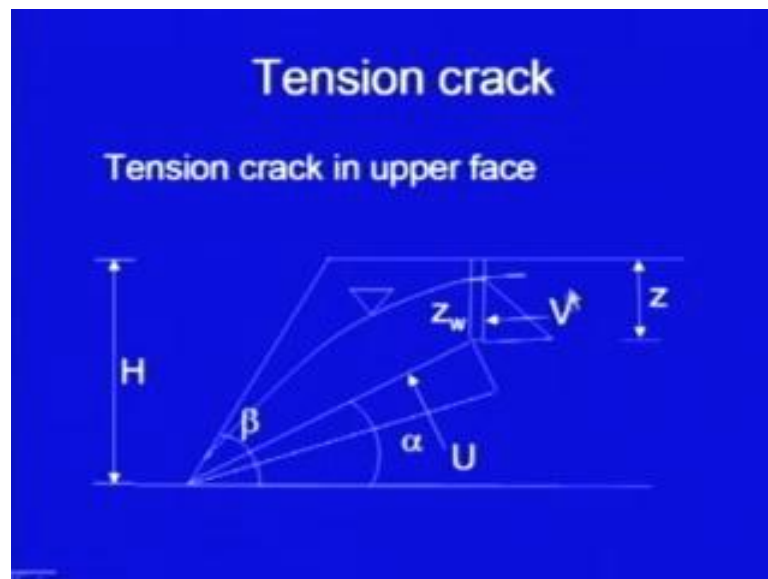
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$$\begin{aligned} \Rightarrow \tan 2\alpha &= -\cot \phi' \\ \Rightarrow \tan 2\alpha &= \tan (90 + \phi') \\ \Rightarrow \alpha_{\text{critical}} &= 45^\circ + \phi'/2 \quad (\text{For vertical cut}) \end{aligned}$$

And, when I take this term on right hand side and simplify it I get tan of 2 alpha equal to minus cot of phi dash, then I convert it into this form tan of 90 plus phi dash. So, tan of 2 alpha is equal to tan of 90 plus phi dash, so, 2 alpha is equal to 90 plus phi dash and finally, I get the critical value of alpha is equal to 45 degree plus phi dash by 2.

Please remember, this we have solved for vertical cut, so for vertical cut you can find out the critical value of alpha will be 45 plus phi dash by 2.

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Now, in the previous analysis we considered this wedge extending right up to this place, what is observed in the field is, that it is found that the tension cracks developed, so if the

tension crack develops, then the analysis will be little bit different. So, the tension crack tension crack can develop here, this is the case which I have shown here the tension crack in upper face, so this is the upper face and a tension crack has developed here.

The depth of the tension crack is Z and we have assumed the water table also water is also seeping, so this is the water table and in the tension crack the depth of water is equal to Z w. You can find out the pressure of the distribution of the water pressure, so water pressure distribution diagram will be triangular here in the tension crack. So, this V is let us say the pressure due to water.

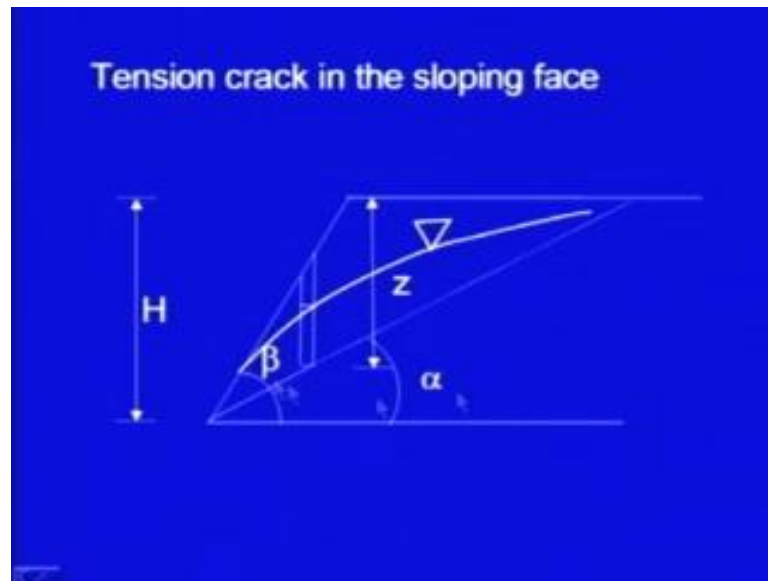
This ordinate it will be 0 here and γw into $Z w$ at this place and the area of this triangle; that means, half into γw into this height into this ordinate that is going to give you the water pressure which is acting at 1 meter length of this tension crack.

And then, this joint plane or refracture plane is extending from toe to this point and our assumption is that here, the variation of the water pressure is linear it may be non-linear also.

So, we have taken a triangular variation of the water pressure at this point the ordinate of the water pressure diagram will be same as this one, so this ordinate verse γw into $Z w$, so this ordinate will also be γw into $Z w$. And then, you can find out total pressure acting over this surface its value will be equal to half into this length into this ordinate.

So, this ordinate is γw into $Z w$, so that into this into half; that means, area of this triangular diagram is going to give you the water pressure acting at this particular surface and we have denoted it by letter capital U . And, other parameters are same, the fracture plane is inclined at an angle α and slope angle is β height is H . The depth of the tension crack is being measured from upper surface and it is equal to Z .

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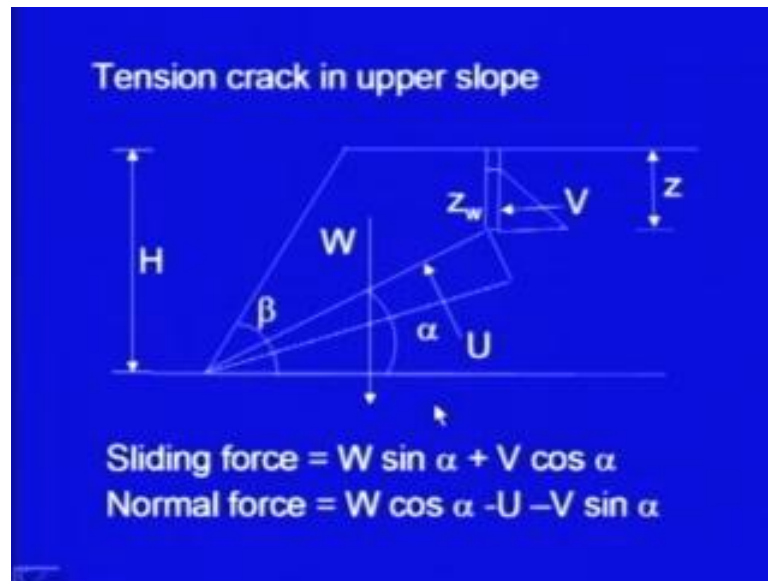


The tension crack can also occur in the sloping face here, so this is the second case of the tension crack. This is the tension crack, this is the sloping face and this is the upper surface and this is the joint plane or the bedding plane again at an angle α . Here is the slope face at an angle β and water table is there.

In the tension crack, this depth this will be $Z w$ and again you will be able to find out you can find out, what will be the pressure, which will be acting on this face. You can also find out using the same principle as we discussed in the previous case that here also pressure distribution diagram can be taken triangular.

At this particular point, the pressure from this means this $1 \gamma w$ into $Z w$ that will be same. For the wall of the tension crack and here for the fracture plane, the same value will be taken and then from here to here you can take the triangular variation for the stability and analysis.

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Let us go to the analysis spot., so here it is the first case when the tension crack is in upper slope. So, here somewhere here depth of the tension crack is Z , I have already explained the different forces which we have to take into account. This is the wedge now which is going to fail, which we are assuming that it is sliding. Its weight is acting in downward direction and our principles will be same.

We will be finding out the forces, which are trying to create instability means all the forces which are in this tangential direction, tangential to the fracture plane and the forces which are trying to enhance the shear strengths by increasing normal stress. So, we will be finding out the algebraic sum of all the forces acting normal to the joint plane of the fracture plane.

V is the force which water is exerting on the wall of the tension crack, you from your mechanics you can find out where should be its center of gravity. It will be $1/3$ of height is Z_w , Z_w is the depth of water here and this is the pressure U which is acting at the fracture plane in normal direction normal to the plane of fracture plane.

So, we can now resolve the forces the sliding force will be equal to $W \sin \alpha$ W is acting here in this direction. So, this component will be $W \cos \alpha$, this component will be $W \sin \alpha$., so $W \sin \alpha$ plus one component of this horizontal force V , so V is acting in horizontal direction we have to find out its component which is acting in this direction.

Now, this angle is alpha, so it is the angle between the direction of V and this line will also be alpha. So, this component along the sliding direction will be V cos alpha. So, sliding force will be W sin alpha plus V cos alpha, W we have to find out using the geometry of this wedge.

The normal force that is in this direction will be W cos alpha and here additional component this U is coming. So, it is acting in this direction, so minus U and minus one component of this the component which is acting in this direction perpendicular to the plane. So, it will be also it will also be in negative direction.

So, minus V sin alpha, so you can appreciate here that, what this presence of water is going to do. You look at this place the sliding force is being increased by an amount V cos alpha, this is the amount by which the sliding force is being increased. And, the normal force which is trying to help in stability is being decreased by an amount this much U and V sin alpha, so this is the way the water pressure is going to act and its effect, in fact is very substantial.

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$$\begin{aligned} \tau_f &= c' + \sigma' \tan \phi' \\ \text{Resisting force} &= \tau_f A \\ &= c'A + (W \cos \alpha - U - V \sin \alpha) \tan \phi' \\ F_s &= \frac{c'A + (W \cos \alpha - U - V \sin \alpha) \tan \phi'}{W \sin \alpha + V \cos \alpha} \\ A &= (H-z) \operatorname{cosec} \alpha \\ V &= \frac{1}{2} \gamma_w z_w^2 \\ U &= \frac{1}{2} \gamma_w z_w (H-z) \operatorname{cosec} \alpha \end{aligned}$$

Let us come to the analysis, so as usual we find out the shear strength C dash plus sigma dash tan of phi dash.

The resisting force will be equal to we are talking here in terms of the forces tau f into area A. So, I put this value here, so C dash into A plus sigma dash into A into tan of phi dash and sigma dash into A is nothing but this force which I calculated here normal force.

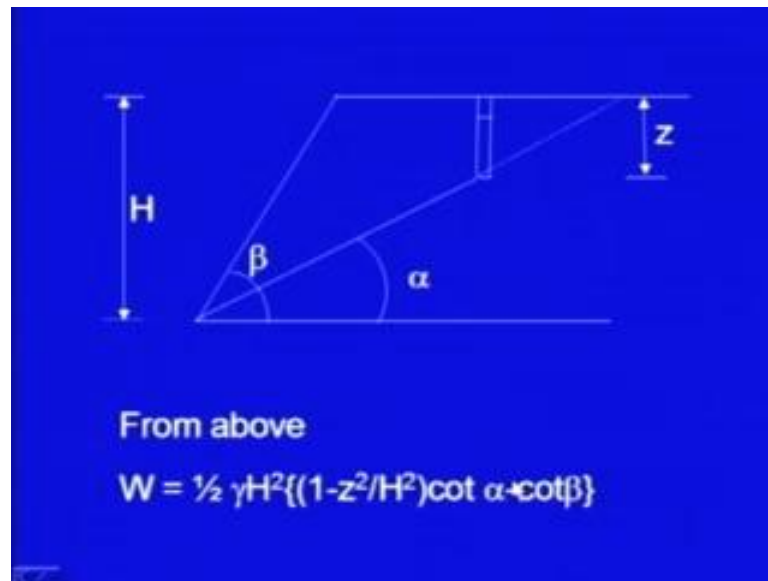
So, normal force will be $W \cos \alpha$ minus $U \sin \alpha$ plus $V \cos \alpha$ and this is the resisting force and the sliding force was $W \sin \alpha$ plus $V \sin \alpha$ we had already calculated it. So, the factor of safety expression is F_s is equal to $C \text{ dash } A$ plus weight of the wedge into $\cos \alpha$ minus the pressure water pressure $U \sin \alpha$ plus $V \cos \alpha$ is again another pressure. This is because of the water in the tension crack into $\tan \phi \text{ dash}$ divided by $W \sin \alpha$ plus $V \cos \alpha$, so this is the expression for the factor of safety.

And, this area A , you can find out from here area A is this dimension into one, so let us find out this dimension, you can extend it up to this place. So, this total will be equal to from this triangle you can find it out, this is H this angle is α , so from here to here it will become $H \operatorname{cosec} \alpha$.

And from this triangle, you can find out this hypotenuse, this will be equal to $Z \operatorname{cosec} \alpha$. So, area A will be equal to $H \text{ minus } Z \operatorname{cosec} \alpha$ into 1 , so that is equal to $H \text{ minus } Z \operatorname{cosec} \alpha$, similarly pressure V is half γw into $Z w$ square.

(Refer Slide Time: 34:45) So, here it is half into this ordinate, this is $Z w$ this is equal to γw into $Z w$. So, area of this diagram gives you this half γw into $Z w$ square and U will be equal to half γw into $Z w$ into $H \text{ minus } Z \operatorname{cosec} \alpha$ it is nothing but area of this triangle. (Refer Slide Time: 34:45) This ordinate is equal to γw into $Z w$ and this distance we have already got $H \text{ minus } Z \operatorname{cosec} \alpha$ and half into this into this. So, finally, you get U equal to half $\gamma w Z w H \text{ minus } Z \operatorname{cosec} \alpha$.

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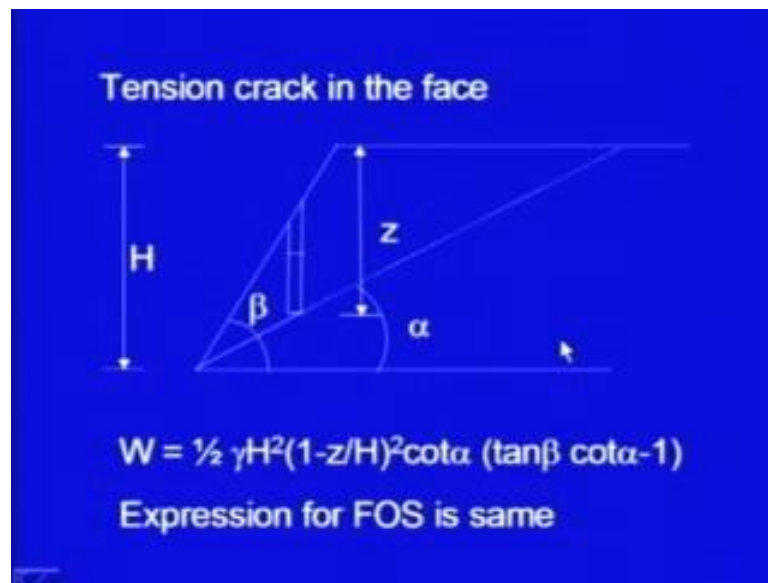


The value of W also you can find out I am not showing the entire computation it is very easy again you have to use geometry. This dimension I have already told you it will be $H \operatorname{cosec} \alpha$.

This dimension will be equal to $H \operatorname{cosec} \beta$, you can get this perpendicular distance you can get entire area of this wedge and then you also can get area of this wedge. And, you can subtract from the major area you can subtract this smaller wedge and then you can multiply by the value γ and perpendicular to the plane of paper again we are taking unit dimension.

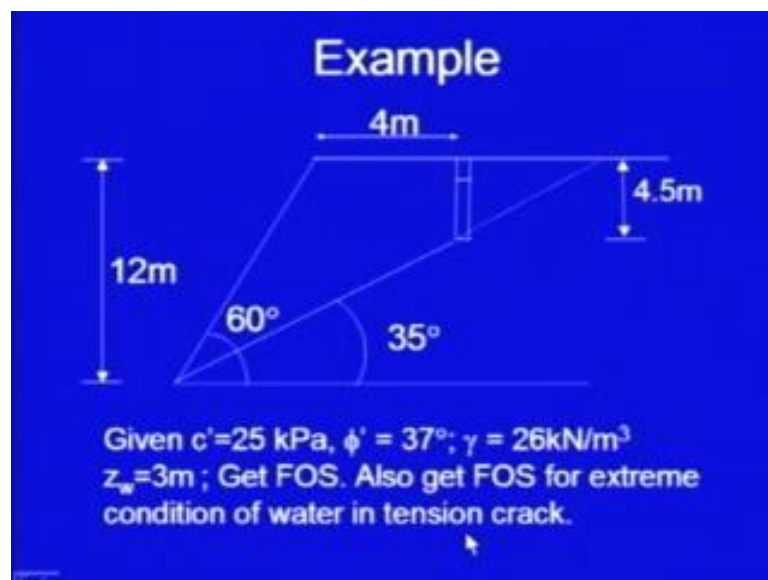
So, this is the final expression which we are going to get for the weight W half $\gamma H^2 \left(1 - \frac{z^2}{H^2} \right) \cot \alpha - \cot \beta$.

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Similarly, for tension crack in the lower face, in the sloping face also you are going to get this expression W equal to half gamma H square 1 minus Z upon H whole square \cot alpha into \tan beta \cot alpha minus 1 . So, these are simple geometrical expressions which you can get from the geometry and expression for factor of safety is same.

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Let us, again have an example, here it is given that there is a slope this is the fracture plane slope angle is given as 60 degree. The dip of the fracture plane is 35 degree height of the slope is 12 meter.

Tension crack is 4 meter away from this point and the depth of tension crack is 4.5 meter also the properties are given C dash 25 kPa and phi dash 37 degree. Please note down it is these are not the properties of the entire material, these are the properties of base surface and gamma is given for this material entire mass gamma is 26 kilo Newton per meter cube and it is given that depth of water in the tension crack is 3 meter.

So, here 3 meter is from here to here, please remember depth of water we are measuring from the bottom of the tension crack whereas, the depth of tension crack itself we measure from the upper surface from here. So, and we have to find out factor of safety and also get factor of safety for extreme conditions of water in tension crack.

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Solution

$$F_s = \frac{c' A + (W \cos \alpha - U - V \sin \alpha) \tan \phi'}{W \sin \alpha + V \cos \alpha}$$

A = (H-z) cosec α
= (12-4.5) cosec 35 = 13.08 m²

V = $\frac{1}{2} \gamma_w z_w^2 = \frac{1}{2} \times 9.81 \times 3.5^2$
= 44.15 kN

U = $\frac{1}{2} \gamma_w z_w (H-z) \text{ cosec } \alpha$
= $\frac{1}{2} \times 9.81 \times 3 \times (12-4.5) \text{ cosec } 35$
= 192.41 kN

So, let us solve this problem, the expression of the factor of safety includes C dash that is available A area on which the sliding is occurring that we will calculate then weight, then alpha, then water pressure U water pressure V phi dash and sin alpha cos alpha these terms.

So, let us calculate all these terms 1 by 1, A is equal to H minus Z cosec alpha, I have already explained it. In this is the area on which sliding is going to take place. So, this total is H cosec alpha and up to here this is Z cosec alpha.

So, you can get this expression and then you can get the area equal to 12 minus 4.5 cosec of value of alpha and then I am getting area equal to 13.08 meter square. Then, pressure V pressure V is acting here, so it will be half into gamma into Z w square, so half gamma w gamma w is 9.81 and Z w square. So, this is equal to this much.

And, another pressure $U = \frac{1}{2} \gamma_w Z_w \text{ into } H \text{ minus } Z \text{ cosec } \alpha$, U is acting here in this direction on it is acting on the fracture plane and its direction is normal to the fracture plane. So, it is half $9.81 \text{ into } 3$ value of H is 12 Z is 4.5 cosec of 35 and I get U equal to this much 192.41 kilo Newton.

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Putting values:
 $F_s = 1.24$

For completely dry condition ($z_w=0$)
 $U = V = 0$
 $F_s = 1.54$

For tension crack completely filled with water
 $z_w = 4.5\text{m}; V = 99.32 \text{ kN}; U = 288.6\text{kN}$
 $F_s = 1.04$

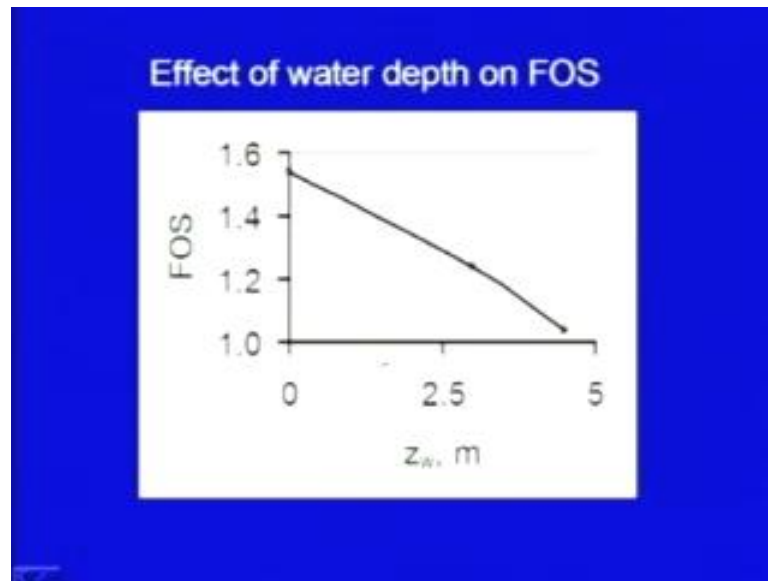
And, when I put these values in the expression for the factor of safety, I get factor of safety 1.24 , so this is the answer of the first part.

Now secondly, he is asking to find out the factor of safety for extreme conditions, so let us say the first extreme condition is completely dry condition, so when it is completely dry Z_w will be equal to 0 , U and V both will be 0 . So, I am not showing the expressions again and the value you are going to get 1.54 .

You can see here, the depth of water table was there and factor of safety was 1.24 and it is it has now increased to 1.54 . The other extreme condition is when the tension crack is completely filled with water, it may happen after the rainfall the tension crack may be filled completely it may be completely filled with water.

So, that is the extreme condition, so Z_w will be 4.5 and these values I have calculated using the same expressions which we discussed in the previous slide. So, V comes out to be 99.32 , U comes out to be 288 and when we put it in the factor of safety expression factor of safety comes to as low as 1.04 . So, there is a variation of 1.54 to 1.04 there is extreme this is substantial variation in factor of safety.

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For the sake of comparison we have shown here, the variation of the factor of safety with the water depth in the tension crack on x axis it is the depth of water in tension crack. So, 02.5 and some where it is 4.5 the maximum depth and you can see the factor of safety is varying from one point five something. So, from this much high value and it goes on reducing and becomes very small somewhere here.

So, there is a substantial effect of the water table of water on the stability. So, you can do that parametric analysis, you can you can suggest that if the heavy rainfall is there, what is going to happen, if the proper drainage is not there or suppose there is the drainage has choked and the seepage is not taking freely. Then, what is going to happen, the factor of safety goes to a very low value and there can be danger of failure of the slope.

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Critical tension crack depth

- The tension crack may not be visible . The probable depth and location may be obtained for dry condition.
- FOS is given as:

$$F_s = \frac{c'A + (W \cos \alpha - U - V \sin \alpha) \tan \phi'}{W \sin \alpha + V \cos \alpha}$$

So in the previous analysis, we had discussed we have discussed the tension crack also and sometimes this tension crack may not be visible we have to asses it. So, the critical tension crack depth can also be found out, the probable depth and location can be obtained for dry condition.

So, this is the factor of safety expression F_s in terms of these parameters and for dry case F_s becomes $c \text{ dash } A \text{ plus } W \cos \alpha \tan \phi \text{ dash upon } W \sin \alpha$.

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For dry state

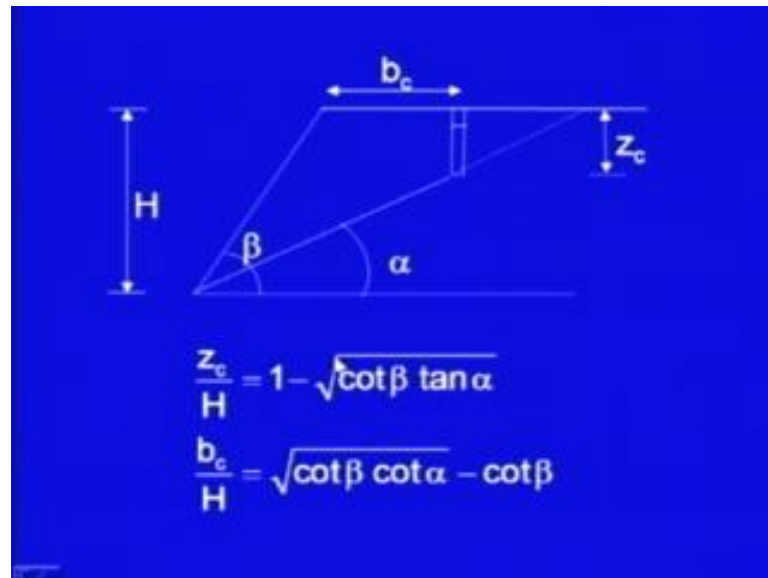
$$F_s = \frac{c'A + (W \cos \alpha) \tan \phi'}{W \sin \alpha}$$
$$= \frac{c'A}{W \sin \alpha} + \cot \alpha \tan \phi'$$

For crack in upper surface
differentiate RHS w.r.t z/H and equate to zero.

So, all the U and V component become zero. So, finally, this is the expression which is available at the end. And now, you can differentiate this expression and by

differentiating and putting it equal to 0, we and simplifying it finally, we will be getting these two expressions for the critical depth of the tension crack and also the location.

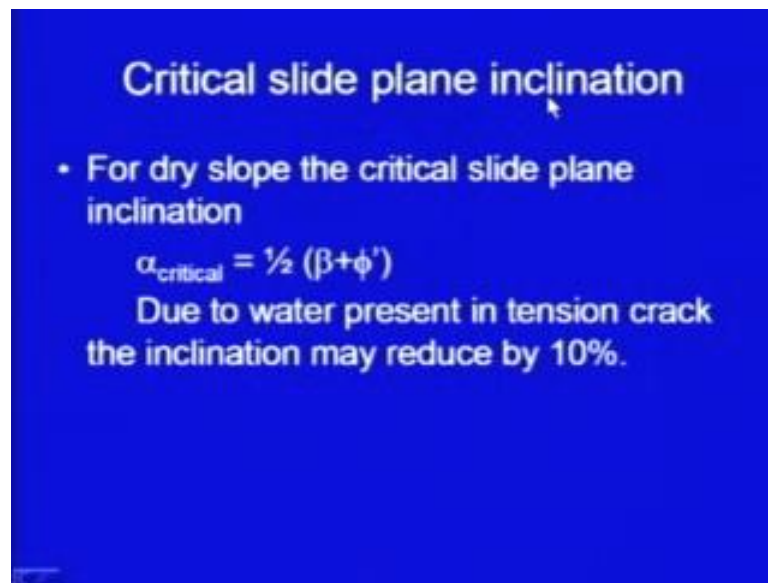
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So, Z_c upon H is equal to 1 minus under the root cot beta tan alpha using this expression you can estimate that the optimal value, the critical value of the tension crack will be Z_c and where it is going to occur.

This distance b_c you can calculate from this expression, b_c upon H is equal to root cot beta cot alpha minus cot of beta. So, using these expressions you can estimate where is the location of tension crack and what is the depth of the tension crack for dry case. Also we can find out the inclination critical slide plane inclination.

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Critical slide plane inclination

- For dry slope the critical slide plane inclination
$$\alpha_{\text{critical}} = \frac{1}{2} (\beta + \phi')$$

Due to water present in tension crack the inclination may reduce by 10%.

For dry slope, the critical slide plane inclination comes out to be again we have to do the differentiation I am not showing the computations here. So, alpha critical comes out to be half of beta plus phi dash, this is the case for dry case and roughly if it is the water is there then this inclination may reduce by 10 percent

So, friends we are continuing our discussion on the finite slopes, we have considered the planer failure case. We have discussed the limiting, how to get the limiting height, then we have also discussed about the tension cracks, we have discussed about their critical orientation and location and we have discussed about the factor of safety, how to find out factor of safety in different cases.

In the next lecture we will continue with the same finite slopes with failure plane as far as the planer surfaces and then we are going to take the circular failure surfaces.

Thank you.