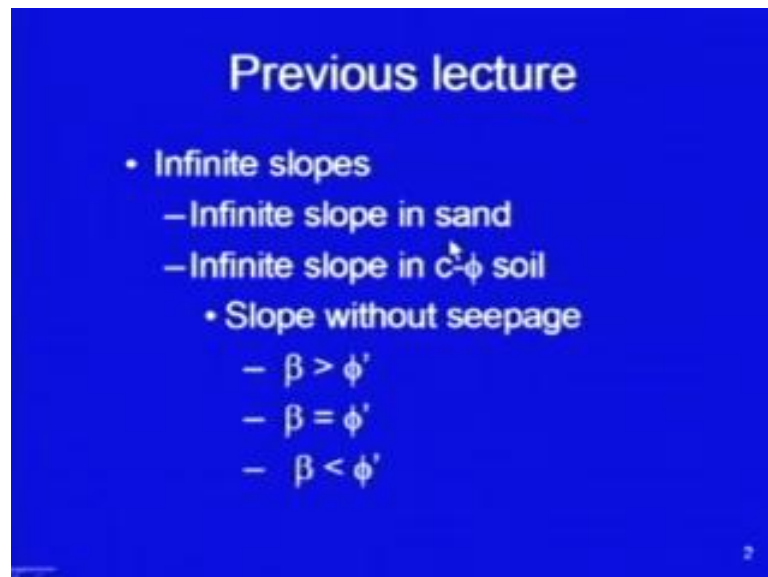


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Module - 03
Lecture - 08
Stability of Slopes

Welcome viewers, we were discussing the Stability of Slopes and in our previous lecture we had started the infinite slopes.

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In infinite slopes we started with slope in sand, here we had assumed the plane of ruptured to be parallel to the ground surface. And then, we had found out the stability and it was concluded that if the slope is steeper than phi it is going to be unstable. Then next case, we took infinite slope in c phi soil, so now additional c component was also there and there we found out the expression for the stability and then different cases we had considered.

So, slope without seepage, in that case we have discussed three cases beta is the angle of slope, so first case we took was when the angle of slope is more than phi dash. So here, if the angle of slope is more than phi dash means if the slope is steeper than phi dash what happens. And we saw that because of see there was some additional strength in the slope

and it was possible to make it stable for some height for some depth, which we called as critical depth.

Second case, we took beta equal to phi dash and third case we took beta less than phi dash and in both the cases we saw that the slope was stable for all the depths.

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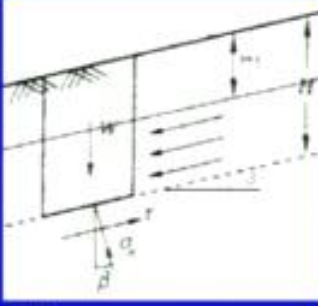
Then, we took the case of infinite slope with seepage and seepage was occurring throughout the entire mass; that means, up to the ground surface the seepage was there and we had considered its expression for the slope stability analysis. Now the next case, now we have to going to take is partial seepage case and then we will be discussing completely submerged slope and then we will move on to the finite slopes.

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B: Infinite slope with partial seepage

Normal stress at the shearing plane:
$$\sigma' = (\gamma_t H_1 + \gamma_{sub} H_2) \cos^2 \beta$$
$$H_2 = H - H_1$$

Shear stress
$$\tau = (\gamma_t H_1 + \gamma_{sat} H_2) \cos \beta \sin \beta$$



So this is the second case, infinite slope with partial seepage, here it is the ground surface and this is the assumed rupture plane. The rupture plane may coincide with the hard stratum, if some hard stratum is there we take up the rupture plane to be there and up to here for this depth H, the soil is same. And, seepage is taking place not through the entire mass this time, but the water table is at a depth H1 below the ground surface. And, water table is parallel to the ground surface that is the assumption we have used in this analysis.

Like our previous cases, again we consider one slice here and now this slice is being acted upon several forces, the equilibrium of this rupture plane is being considered at this particular point. So, again the weight of this slice is acting in downward direction and the shearing resistance the mobilized shearing resistance will be acting in upward direction and normal stress here will be acting normal to the plane of rupture.

And here, we have shown the seepage is taking place in the downward direction, so to do the analysis we assume again the limit equilibrium method. The normal stress at the shearing plane is obtained, the process is same, I have shown here final expression. You will be getting the vertical stress, vertical stress by taking the weight of this slice and then by resolving that vertical stress normal to the plane of surface we are going to get this expression $\sigma' = (\gamma_t H_1 + \gamma_{sub} H_2) \cos^2 \beta$ is equal to $\sigma' = (\gamma_t H_1 + \gamma_{sat} H_2) \cos^2 \beta$ where H1 is here this is the depth of the water table and γ_t , I have taken the unit rate of this particular soil mass.

And, plus γ submerged into H_2 , H_2 is a H minus H_1 H_2 is this depth of the saturated soil. So, when I am calculating the normal stress, this weight we will be taking as γ saturated minus γ submerged γ_w , so that will be γ submerged. So, the weight of this portion will be in terms of γ submerged into H_2 .

Then, you will be able to calculate the vertical stress that will be into \cos of β and its component normal to the rupture plane another $\cos \beta$ will come into picture and this will be the expression for normal stress γ_t into H_1 plus γ submerged into H_2 and whole this multiplied by $\cos^2 \beta$ where H_2 is nothing but this depth of the saturated soil. And, by resolving the vertical stress along the rupture plane, we are going to get the shear stress.

The stress which is being applied at this rupture plane, τ will be equal to γ_t into H_1 that is the weight of the particular portion, but now please remember this instead of submerged now will be taking γ saturated. It will be total weight which will try to destabilize it and into H_2 and this whole multiplied by $\cos \beta$ this will be vertical stress and then multiplied by $\sin \beta$, so you are going to get the component which is parallel to the rupture plane.

Important point which you must remember here is in fact, that here we are taking the effective stress. In fact, this was the quantity γ_t into H_1 same quantity I had like this, γ saturated into H_2 . And then here, minus γ_w into H_2 , because that is the buoyancy of water, so the effective stress will be γ saturated minus γ_w .

So here in this case, the term which has to be remembered which you should make note of is γ submerged and in shear stress, it is γ saturated. This is very important point as far as the analysis is concerned.

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For stable slope

$$\tau = c_m' + \sigma' \tan \phi_m'$$
$$\Rightarrow c_m' + (\gamma_t H_1 + \gamma_{sub} H_2) \cos^2 \beta \tan \phi_m'$$
$$= (\gamma_t H_1 + \gamma_{sat} H_2) \cos \beta \sin \beta$$

$$\Rightarrow c_m' = (\gamma_t H_1 + \gamma_{sat} H_2) \cos \beta \sin \beta$$
$$- (\gamma_t H_1 + \gamma_{sub} H_2) \cos^2 \beta \tan \phi_m'$$

Putting $H_2 = H - H_1$

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So for a stable slope, the shear stress which is acting on the rupture plane should be equal to the mobilized shearing resistance. And let us say the parameters, the mobilized parameters are C_m , C_m is the cohesion and ϕ_m it is internal friction mobilized value of internal friction.

So, for stable slope τ will be equal to C_m dash plus σ dash tan of ϕ_m dash, this is the mobilized shearing resistance and it is the shearing stress. Now, put the value of σ dash here from the previous slide, so C_m dash plus γ_t into H_1 plus γ_{sub} into H_2 this into $\cos^2 \beta$, this value was σ dash into tan of ϕ_m dash. And here, it is the value of τ that $\gamma_t H_1$ plus γ_{sat} into H_2 into $\cos \beta \sin \beta$.

And, when I rearrange this expression you will be getting C_m dash is equal to $\gamma_t H_1$ plus, γ_{sat} into H_2 into $\cos \beta \sin \beta$ minus $\gamma_t H_1$ plus γ_{sub} into H_2 $\cos^2 \beta \tan$ of ϕ_m dash. Now, replace this H_2 by $H - H_1$ H_1 is the depth of the water table.

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$$\frac{c_m}{\gamma_{sat} H} = c \cos^2 \beta \left[\left(\frac{\gamma_t H_t}{\gamma_{sat} H} + \frac{H - H_t}{H} \right) \tan \beta - \left(\frac{\gamma_t H_t}{\gamma_{sat} H} + \frac{\gamma_{sub} (H - H_t)}{\gamma_{sat} H} \right) \tan \phi_m \right]$$

Put $c_m = c/F_c$; $\tan \phi_m = (\tan \phi)/F_\phi$; $F_\phi = 1$

$$N_s = \frac{c}{F_c \gamma_{sat} H} = c \cos^2 \beta \left[\left(1 - \frac{H_t}{H} \frac{\gamma_{sat} - \gamma_t}{\gamma_{sat}} \right) \tan \beta - \left(\frac{\gamma_{sub}}{\gamma_{sat}} + \frac{H_t}{H} \frac{\gamma_t - \gamma_{sub}}{\gamma_{sat}} \right) \tan \phi \right]$$

And, rearrange this expression in that old form which is very convenient, C_m upon γ into H this is a very convenient term which is used in analysis, it was called as stability number.

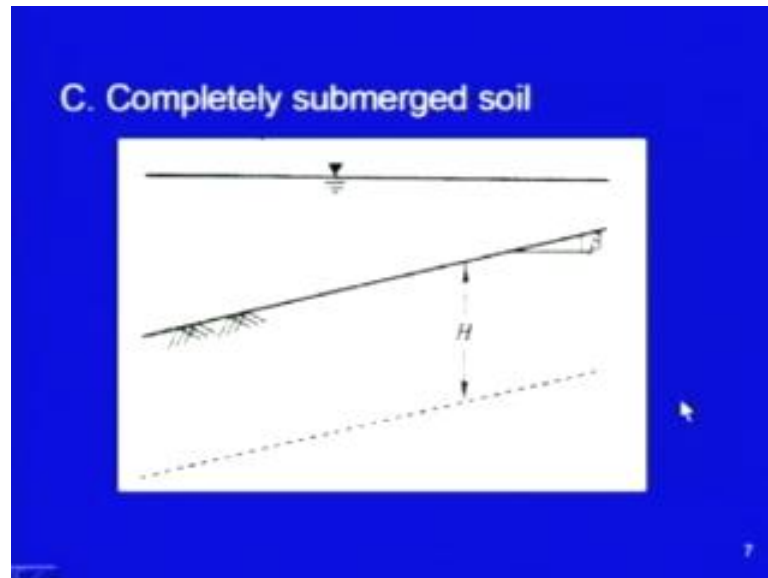
So, C_m dash upon γ saturated into H is obtained as \cos square β , inside bracket $\gamma_t H_t$ upon γ saturated into H , plus H minus H_t upon H \tan of β minus $\gamma_t H_t$ into H γ saturated into H , plus γ submerged times H minus H_t upon γ saturated H multiplied by \tan of ϕ_m . So, this is the general expression for this case and as that we have different factors.

You can define the different factors in terms of the mobilized cohesion and mobilized angle of friction. So, let us say replace C_m dash by C dash upon F_c , so F_c is the factor of safety against cohesion, C dash is the maximum value of the cohesion which it can mobilize and C_m dash is the value of the cohesion which is mobilized. So, C_m dash will be equal to C dash upon F_c and for a stable slope F_c will be more than one.

Similarly, replacing $\tan \phi_m$ equal to $\tan \phi$ dash upon F_ϕ and then I replace F_ϕ by 1. In fact, all our most of the times our computations in terms of stability charge in those computation we use F_ϕ equal to 1. So, here this stability number comes out to be equal to C dash upon $F_c \gamma$ saturated H into equal to \cos square β inside bracket 1 minus H_t upon H γ sat minus γ_t upon γ saturated into \tan

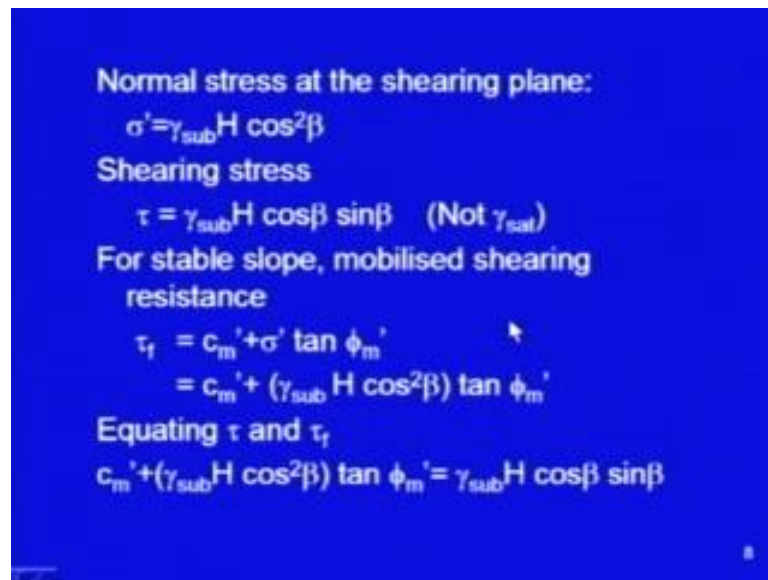
β minus inside bracket γ submerged upon γ saturated plus H 1 upon H
 γ t minus γ submerged upon γ saturated into \tan of ϕ dash.

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So, this was the case of partially submerged case, now I am taking third case here completely submerged soil. So, here the water table is not parallel to this, it is not seepage which is occurring here. In earlier cases, we were taking seepage to be taking place, whether it was through the entire mass or it was taking partially through the mass. Now, it is water table which is resting and the complete slope is now submerged, there will be difference now in the analysis.

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Normal stress at the shearing plane:
 $\sigma' = \gamma_{\text{sub}} H \cos^2 \beta$

Shearing stress
 $\tau = \gamma_{\text{sub}} H \cos \beta \sin \beta$ (Not γ_{sat})

For stable slope, mobilised shearing resistance
 $\tau_f = c_m' + \sigma' \tan \phi_m'$
 $= c_m' + (\gamma_{\text{sub}} H \cos^2 \beta) \tan \phi_m'$

Equating τ and τ_f
 $c_m' + (\gamma_{\text{sub}} H \cos^2 \beta) \tan \phi_m' = \gamma_{\text{sub}} H \cos \beta \sin \beta$

Let us see what is that difference, the basic philosophy is same. The normal stress at the shearing plane as usual σ' will be equal to $\gamma_{\text{sub}} H \cos^2 \beta$. I have been discussing, this is H . And, shearing stress on that shearing plane will be equal to $\gamma_{\text{sub}} H \cos \beta \sin \beta$ when you resolve the vertical stress, in fact the process is same as was discussed in previous cases. First we get the vertical stress from vertical stress we resolve it in tangential direction and in normal direction.

Now, the difference at this place is that when you calculate the shearing stress here for the shearing stress also γ_{sub} is used. And I have noted it here, I have placed a note here that it is not γ_{sat} , but it should be γ_{sub} , this is important point here.

So instead of here, in the last case we took γ_{sat} , because that was the weight of water itself was creating instability. It was seeping water, here it is submerged condition, under submerged condition the weight of the soil will be given by its effective weight. So, shearing stress also will be calculated in terms of γ_{sub} , so this is the main difference as far as analysis is concerned.

Rest the steps are same for stable slope, mobilized shearing resistance will be equal to $c_m' + \sigma' \tan \phi_m'$. And now, put these values σ' is equal to $\gamma_{\text{sub}} H \cos^2 \beta$ and $\tan \phi_m'$ and put this τ and

tau fequal to each other. So, we get this equation C_m dash plus gamma submerged H cos square beta into tan of phi m dash equal to gamma submerged into H cos beta sin beta.

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$$\frac{c'_m}{\gamma_{sub} H} = \cos^2 \beta [\tan \beta - \tan \phi'_m]$$

If $F_c = F_\phi = F_s$

$$\frac{c_m}{F_s \gamma_{sub} H} = \cos^2 \beta \left[\tan \beta - \frac{\tan \phi'_m}{F_s} \right]$$

If $F_c \neq F_\phi; F_\phi = 1$

$$N_s = \frac{c'}{F_c \gamma_{sub} H} = \cos^2 \beta [\tan \beta - \tan \phi']$$

When we rearrange these terms, we get a very simple expression this time, C_m dash divided by gamma submerged into H. This is the stability number c upon gamma H is always stability number, it is a known dimensional number which is very useful in analysing in doing that tedious cal calculations. So, this comes out to be in this case equal to cos square beta into tan of beta minus tan of phi m dash.

And, if I take F_c F_ϕ and F_s same factor of safety with respect to cohesion with respect to internal friction with respect to shearing strength. If I take the factors same, then the stability number will be given as C_m upon F_s gamma submerged into H equal to cos square beta tan beta minus tan phi m dash upon F_s .

If these two quantities are not same, then N_s will be equal to c dash upon $N F_c$ gamma submerged into H cos square beta into tan beta minus tan of phi dash.

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EXAMPLE-1

Given for an infinite slope:
 $c' = 30 \text{ kN/m}^2$; $\beta = 25^\circ$; $\phi' = 22^\circ$; $e = 0.67$;
 $G = 2.7$. To compute stability number and critical height of the slope for (i) dry condition; (ii) water seeping through the entire mass; (iii) water table 2 m below the ground surface; (iv) slope is completely submerged.

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Let me now, solve some examples, it is given for an infinite slope, c' is 30 kilo Newton per meter square. These are the shear strength parameters given for a particular problem, slope angle is 25 degree ϕ' is 22 degree and e is void ratio is given as 0.67 and G value is given as 2.7.

We have to compute stability number and critical height of the slope for number first dry condition, second water seeping through the entire mass and third water table when it is 2 meter below the ground surface and fourth case is slope is completely submerged.

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Solution

$e = 0.67$; $G = 2.7$

$$\gamma_d = \frac{G\gamma_w}{1+e} = \frac{2.7 \times 9.81}{1+0.67} = 15.86 \text{ kN/m}^3$$
$$\gamma_{sat} = \frac{\gamma_w(G+e)}{1+e} = \frac{9.81 \times (2.7 + 0.67)}{1+0.67} = 19.79 \text{ kN/m}^3$$
$$\gamma_{sub} = 19.79 - 9.81 = 9.98 \text{ kN/m}^3$$

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Let us, first find out the physical properties of the soil mass it is given e equal to e is the word ratio it is equal to 0.67 and specific gravity is 2.7.

Now, γ_d the dry unit weight is $G \gamma_w$ upon $1 + e$, so this becomes 2.7 into 9.81 upon $1 + 0.67$ that is equal 15.86. Above the water table in this particular case, we shall be taking γ equal to γ_d , if some other value is available you should use that value.

γ_{sat} is given as γ_w into $G + e$ upon $1 + e$ these are the standard expressions which are available from our principles and when I put these values here. So, γ_w is 9.81 kilo Newton per meter cube, G is 2.7, e is 0.67 and I shall be getting the γ_{sat} as 19.79 kilo Newton per meter cube and γ_{sub} will be this γ_{sat} minus γ_w 9.98.

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(i) Dry soil mass

$$N_s = \frac{c'}{F_c \gamma H} = \cos^2 \beta (\tan \beta - \tan \phi')$$

$$N_s = \cos^2 25 (\tan 25^\circ - \tan 22^\circ) = 0.051$$

$$0.051 = \frac{c'}{F_c \gamma H}$$

For critical height $F_c = 1$

$$H = \frac{c'}{0.051 \gamma} = 37.09\text{m}$$

Now, the first case was to solve the problem of dry soil mass and this was the equation of the stability number you can refer to the previous lecture N_s was C dash upon $F_c \gamma H \cos^2 \beta \tan \beta - \tan \phi$ dash. So, put these values, N_s will be equal to $\cos^2 25$ β is the slope angle, so \tan of 25 minus \tan of 22, N_s comes out to be 0.051. So, 0.051 is now known to us and this quantity C dash upon $F_c \gamma H$.

And, when we take the critical height critical height means maximum height for which the slope may remain will remain stable. So, for critical height F_c is taken as 1 and H will be the critical height, so H will be take it on the side C dash upon this is point this is N_s gamma into H and when you put these values answer is 37.09 meters, so this is the stable height.

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(ii) Seepage through entire soil mass

$$N_s = \frac{c}{F_c \gamma_{sat} H} = \cos^2 \beta \left[\tan \beta - \frac{\gamma_{sub}}{\gamma_{sat}} \tan \phi' \right]$$

$$N_s = \cos^2 25 \left[\tan 25 - \frac{9.98}{19.79} \tan \phi' \right] \text{ for } F_\phi = 1$$

$$\Rightarrow N_s = 0.216$$

$$\text{Critical height } H_c = \frac{30}{(19.79 \times 0.216)}$$

$$= 7.02 \text{ m}$$

If the seepage is occurring through the entire soil mass, then the expression for the stability number was N_s equal to C dash upon F_c gamma saturated into H equal to \cos square beta inside bracket \tan of beta, beta is slope angle, minus the ratio of gamma submerged and gamma saturated multiplied by \tan of phi dash.

Please remember that, in these expression we use F_ϕ equal to 1, so N_s will be equal to \cos square 25 \tan of 25 minus gamma submerged upon gamma saturated into \tan phi dash for F_ϕ equal to 1 and when you put these values N_s comes out to be 0.216 and now put this expression here, so H_c is equal to take N_s on this side 30 upon this value and you will be getting the critical height 7.02 meter.

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(iii) water table 2 m below the ground surface

$$N_s = \frac{c'}{F_c \gamma_{sat} H} = \cos^2 \beta \left[\left(1 - \frac{H_1}{H} \frac{\gamma_{sat} - \gamma_t}{\gamma_{sat}} \right) \tan \beta - \left(\frac{\gamma_{sub}}{\gamma_{sat}} + \frac{H_1}{H} \frac{\gamma_t - \gamma_{sub}}{\gamma_{sat}} \right) \tan \phi' \right]$$

For critical height $F_c = 1$. The term H is coming on both sides:
 \Rightarrow Trial are required; Start with some assumed value of H

Third case is, it is the case of partial submergence it is given that water table is 2 meter below the ground surface. So, H_1 is given to you and here it is the equation of the stability number, C' dash upon $F_c \gamma_{sat} H$ equal to $\cos^2 \beta$ inside bracket $1 - H_1$ upon H this ratio, H_1 is the depth of water table.

So, that ratio multiplied by this ratio $\gamma_{sat} - \gamma_t$ upon γ_{sat} into \tan of β , minus inside bracket γ_{sub} upon γ_{sat} plus H_1 upon H $\gamma_t - \gamma_{sub}$ upon γ_{sat} into \tan of ϕ' .

Now, it is in this case, you can see we cannot solve in one step, because when we are putting F_c equal to 1, the term H this is coming on this side as well as this is coming on this side. So, we have to use some trials and this is one of the way that you can start with some assumed some reasonable value of H put it on right hand side, you will be getting some value solve that get new H again put that new H into this expression and repeat the steps.

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Say H = 35
Putting the values

$$N_s = \frac{c}{\gamma_{sat} H_c} = \cos^2 25 \left[1 - \frac{2}{35} \frac{19.79 - 15.86}{19.79} \tan 25 - \left(\frac{9.98}{19.79} + \frac{2}{35} \frac{15.86 - 9.98}{19.79} \right) \tan 22 \right]$$

$N_s = 0.21 \Rightarrow H_c = 7.21$
Again put H = 7.21 ; get N_s and $H_c = 8.85$
After some trials solution converges at
 $H_c = 8.54$ m

So, let us say we start with H is equal to 35, so putting this value we will be getting this expression N_s is equal to $\frac{c}{\gamma_{sat} H_c} = \cos^2 25$.

And, now every term on this right hand side is known to us, we have assumed H, so everything is known it comes here and when you solve it this value comes out to be equal to 0.21. Once it is available now I can calculate this, so H_c comes out to be 7.21, this H_c is in fact is next trial value. Now, put next trial value as 7.21 and get N_s again from N_s again get H_c this time you will be getting 8.85.

So in fact, it needs only around 5 or 6 trials and every time, what you are going to do is, you are solving the right hand side value, then you are putting it here getting new value of H_c again put it here again get new value of H_c and so on. So, we then around 5 to 6 trials, the solution is going to converge and you will be getting the final value, so in this case this was the final answer.

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(iv) Slope is completely submerged

$$N_s = \frac{c'}{F_c \gamma_{sub} H} = \cos^2 \beta [\tan \beta - \tan \phi']$$
$$N_s = \frac{c}{F_c \gamma_{sub} H} = \cos^2 25 [\tan 25 - \tan 22]$$
$$\Rightarrow N_s = \frac{c'}{F_c \gamma_{sub} H} = 0.051$$

For critical height H_c ; $F_c = 1$

$$H_c = \frac{30}{0.051 \times 9.98} = 58.94 \text{ m}$$

The fourth part of the problem was that if the slope is completely submerged, so this was the expression for the completely submerged slope N_s was equal to C dash upon F_c gamma submerged H equal to \cos square beta \tan beta minus \tan of phi dash.

So, put this values here \cos square 25 \tan 25 minus \tan 22, this time it is gamma submerged and N is equal to 0.051 you solve it for critical height H_c , F_c will be equal to one. So, H_c comes out to be 58.94

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Discussion

For dry condition $H_c = 37.09 \text{ m}$
For seepage at GL, $H_c = 7.02 \text{ m}$
For seepage at 2m below GL, $H_c = 8.54 \text{ m}$
Completely submerged, $H_c = 58.94 \text{ m}$

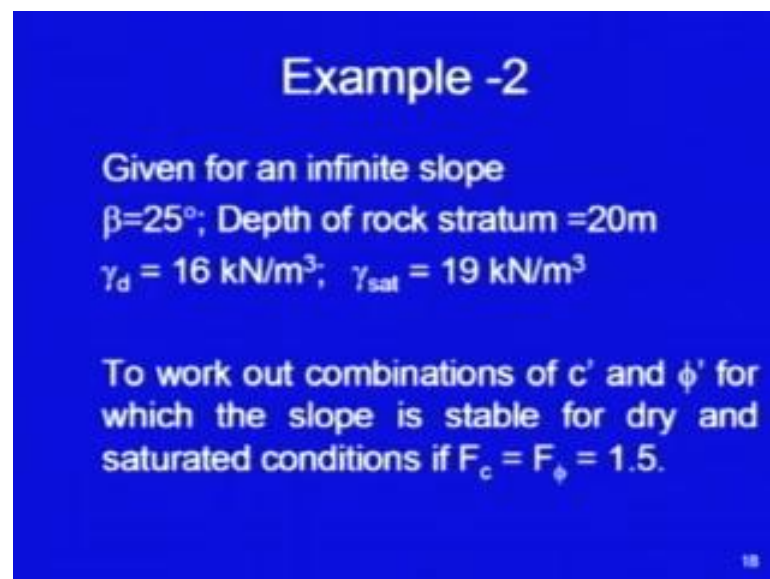
The effect of seepage conditions is substantial!

So, we have solved all the four cases and let us see it is interesting to see, what the results are? For the dry condition the maximum height for which the slope can remain stable was about 39 meter.

If it was a case of seepage the height reduced only to 7.02 meter, if the seepage was 2 meter below the ground level height was this much and if it was completely submerged height was 58.94. So, you can see it is varying minimum is around 7 and maximum goes around 58.

So you can see the drainage conditions, the effect of water table is very very substantial it is a very important factor which governs the stability of the slopes. And, you can appreciate it from here that because of these conditions the safe height is varying from almost 7 to 58 or 59 meter.

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Example -2

Given for an infinite slope
 $\beta=25^\circ$; Depth of rock stratum =20m
 $\gamma_d = 16 \text{ kN/m}^3$; $\gamma_{\text{sat}} = 19 \text{ kN/m}^3$

To work out combinations of c' and ϕ' for which the slope is stable for dry and saturated conditions if $F_c = F_\phi = 1.5$.

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Let us take another example, it is given in this problem that for an infinite slope, the slope angle is 25 degree and a hard stratum of rock is available at depth of 20 meter. The dry unit weight of the soil mass is 16 kilo Newton per meter cube and the saturated unit rate is 19 kilo Newton per meter cube. And, it is asked to work out the combinations of C and phi C dash and phi dash for which the slope is stable for dry as well as saturated conditions and F c and F phi can be taken same equal to 1.5.

In fact, this kind of the problem we use in back analysis, many times it is not possible to do the laboratory test and sometimes even if you do the laboratory test these analysis, these results from the back analysis they are more reliable. So, if we have some slope which is stable or which is just at the verge of failure or if the slope has just failed, we can analyse that slope and we can work out the shear strength parameters and those values can further be used

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(i) Dry soil mass

$$N_s = \frac{c'}{F_c \gamma H} = \cos^2 \beta \left(\tan \beta - \frac{\tan \phi'}{F_\phi} \right)$$

$$\frac{c'}{1.5 \times 16 \times 20} = \cos^2 25 \left(\tan 25 - \frac{\tan \phi'}{1.5} \right)$$

ϕ' (deg)	c' (kPa)
10	138.4
15	113.3
20	88.06
25	61.16
30	31.97

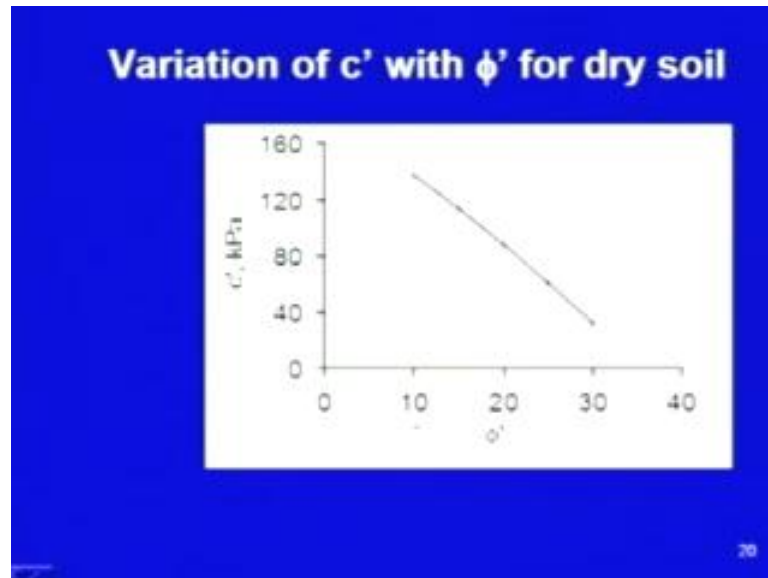
So, coming back to this example let us solve it for first case that is dry soil mass, the stability number is given as $C \text{ dash upon } F_c \gamma H$ equal to $\cos^2 \beta$ multiplied by $\tan \beta$ minus \tan of ϕ dash upon F_ϕ . Different values of F_ϕ and F_c could be taken as per the given case, so here they are same.

So, I put $C \text{ dash upon } 1.5$ into γ , this is γ dry and into 20 at height of the slope it is failing at that depth, this is the depth of the rupture plane. So, which we have taken at the hard stratum at the interface of the soil mass with the hard stratum and this will be equal to $\cos^2 25 \tan$ of 25 minus $\tan \phi$ dash upon F_ϕ .

Now, you can solve this equation for different combinations for example, we have taken here ϕ equal to 10 and when I take ϕ equal to 10 and solve this equations $C \text{ dash}$ comes out to be this much. Then I go on varying ten or I go on varying ϕ dash, so ϕ dash equal to 15, another value ϕ dash equal to 20 another value and so on.

So, as the value of ϕ' is increasing the required value of c' which is required to make the slope stable it is decreasing.

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You can see it is decreasing, here I have shown the variation of c' on y axis, we have plotted c' on x axis we have plotted ϕ' and you can see there is a variation of c' with ϕ' .

In fact, if you are able to predict, if you are able to assess one of them in the field let us say I am very confident about predicting ϕ' and I predict that ϕ' in the field is 20. So, I can correspondingly take this should be the value of c' and this curve gives me, how the variation is there, how c' varies with ϕ' for keeping this the slope stable.

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(ii) For saturated slope

$$\frac{c'}{F_c \gamma_{sat} H} = c \cos^2 \beta \left[\tan \beta - \frac{\gamma_{sub}}{\gamma_{sat}} \frac{\tan \phi'}{F_\phi} \right]$$

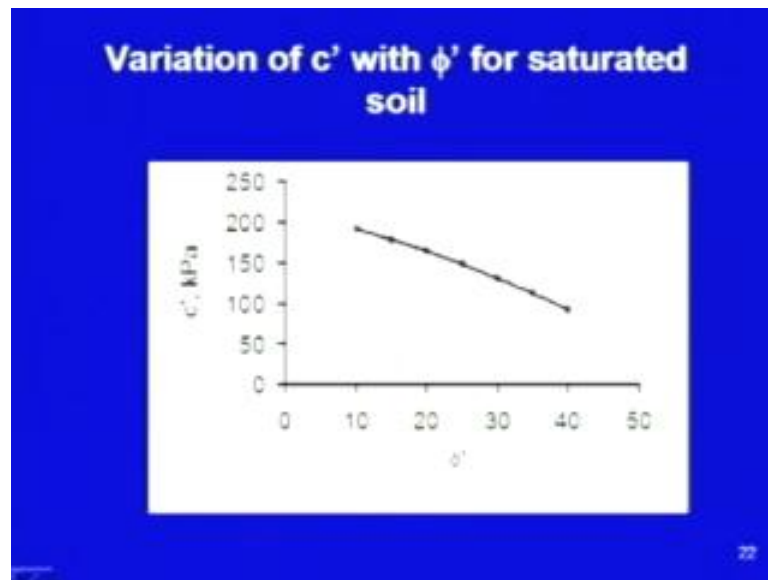
$$\frac{c'}{1.5 \times 19 \times 20} = c \cos^2 25 \left[\tan 25 - \frac{19 - 9.81 \tan \phi'}{19 \times 1.5} \right]$$

ϕ' (deg)	c' (kPa)
10	191.7
15	177.92
20	163.42
25	147.97
30	131.20
35	112.64

The second part of the problem is for saturated slope, so we have considered it to the fully saturated C dash upon F_c into γ_{sat} into H equal to $\cos^2 \beta \tan$ of β minus γ_{sub} upon γ_{sat} $\tan \phi'$ upon F_ϕ . Put the values unknowns are ϕ and C , so C dash upon these values.

So here, I am using now saturated value $\cos^2 25 \tan 25$ minus γ_{sub} is 19 minus γ_w divided by γ_{sat} into $\tan \phi'$ upon F_ϕ . Again we solve them, we solve this equation by taking several various combinations of ϕ , in fact, infinite number of combinations will be possible, so at ϕ equal 10 C is 191 at ϕ equal to 15 C dash is 177 and so on. Again the same thing is happening you can see as the ϕ dash value is increasing as it increases C dash goes on decreasing.

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And here, the variation is plotted, on x axis it is phi dash, on y axis it is C dash and C dash you can see it is decreasing with increasing phi dash.

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Discussion

ϕ' (deg)	Dry c' (kPa)	Saturated c' (kPa)
10	138.4	191.7
15	113.3	177.92
20	88.06	163.42
25	61.16	147.97
30	31.97	131.20

For the sake of comparison, we have put these values together. You can see for 10 degree, if phi is 10 degree, then required C dash is 138.4 in case of dry slope and if it is saturated you can see a very high value 191.7 is required.

Similarly, for 15 degree phi value, C dash required here for dry case is 113 where it is almost 178 for saturated case. So, you can see these this is you can compare these two

values and you will find that for saturated soil mass higher value of the shear strength parameters is needed. You can see for ϕ dash equal to 30 for dry you need only is 31.97 and this value is quite large.

So, for saturated conditions you can do some sort of parametric analysis, you can do some sort of analysis in which you can tell that if this much happens, if this thing happens for example, suppose rainfall occurs and because of the rainfall the water table increases.

Then, you can assess, how the factor of safety will be varying if and when the slope is likely to fail or whether it is going to remain stable in all the conditions or whether it is going to remain to be stable for some conditions and unstable for the other conditions.

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So friends we have discussed, the first part of the slopes the natural slopes these were infinite in extent.

In general, we took the rupture plane as a planar surface which was plane which was parallel to the ground surface and then we have taken several cases. We have taken the seepage, we have taken the dry cases, we have taken the partial seepage and also we have taken when the slope was completely submerged.

And now, let me move on to next important part slopes of finite height, those were slopes of infinite height now these are finite height slopes. Now, for finite height slopes,

failure is on these are the some important point which I will discuss first and then we will go on to the analysis.

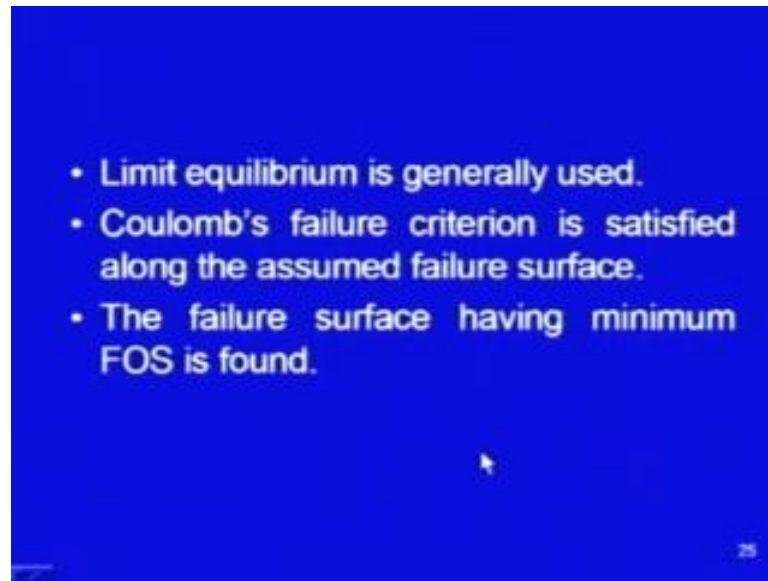
So, failure is considered on a curved surface, in general for the finite slopes means small with the extent is the small, generally they are manmade slopes, the infinite slopes are natural slopes. For these finite slopes failure is generally considered on curved surface, most of the times we assume a circular surface in fact, but in some cases planar surface it is also considered.

See, what happens, if the soil mass is completely isotropic we can assume a circular failure surface, but if there is some dominating discontinuity. For example, if it is a weathered rock and there is a bedding plane which is a dipping towards the slope to which is dipping towards the valley or something like that, in that case when it the discontinuity is there then planar surfaces can also be considered.

And in another example, is when the slope is very steep, then also it has been observed that the planar surface may be assume to be the ruptured surface may be assumed to be planar. Secondly, the failure surface may pass through the toe or it may pass above the toe or it may pass below the toe. So, this is the second point it can pass through any point through the toe above the toe or below the toe.

Now, this method Swedish method, this was the first method which was started which using with stability analysis was done It is quite popular and in this case the circular surfaces are considered.

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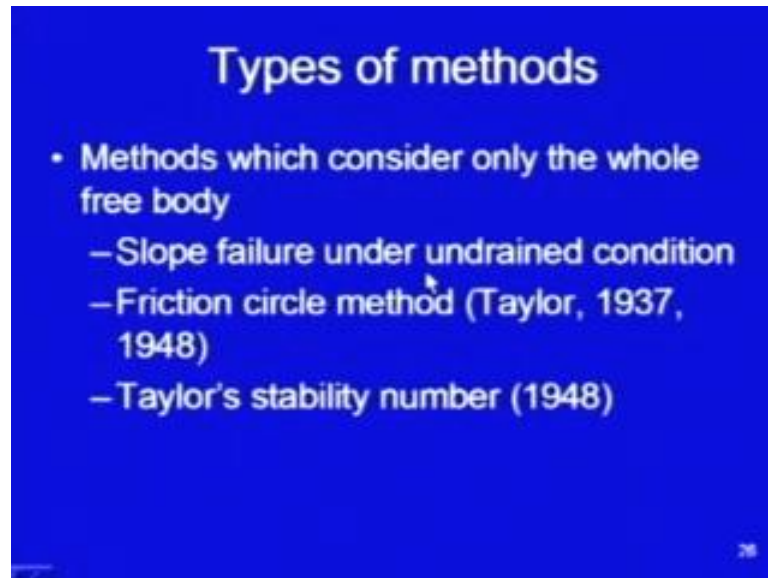


And again, limit equilibrium is generally considered as I told you limit equilibrium means we will be considering surfaces on those surfaces we will be finding out the shear strength. We will be comparing those shear strength with the applied shear stress and then we will know what is the factor of safety.

Next point is that, coulombs failure criterion is satisfied along the assumed failure surface. So, as usual in last time also we considered the coulombs failure criterion, it is a linear criterion and shear strength parameters c dash ϕ dash will be used or sometimes total strength parameters may also be used.

And, the failure surface having minimum factor of safety is found, so when you are considering a circular failure surface. There are infinite numbers of surfaces which may be possible and we will be finding out the factor of safety and that will correspond to the critical surface. Critical surface means that is having minimum factor of safety.

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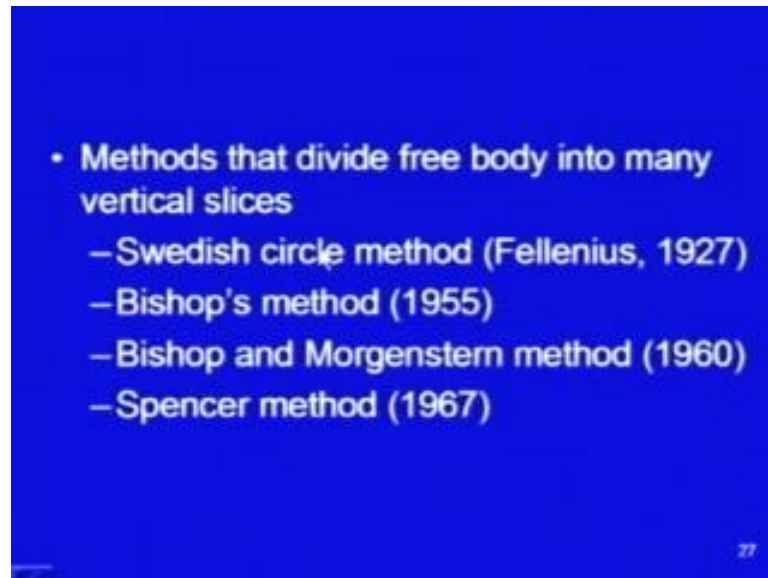


Coming to there are different types of methods which we are going to use, the first category of the methods is the methods which consider only the whole free body.

So, here entire body will be considered as a whole the subcategories the examples of this methods are slope failure under undrained condition. For undrained clay soils, the slope failure may occur under such under such condition, for the slope failure under undrained condition generally this type of the method is sometimes used.

Second is friction circle method, in this method is also we used the entire body and we treat the equilibrium of the entire body of the failing mass. And, Taylor's stability number also uses the same concept

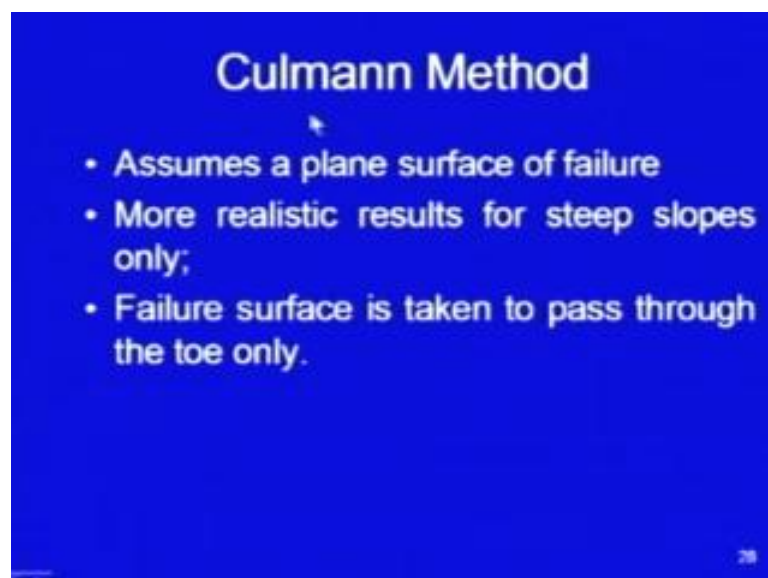
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There are other methods which do not consider the entire body as a whole, but it divides the body into many vertical slices.

So, these methods are Swedish circle methods, Bishop's method, Bishop and Bishop and Morgenstern method, Spencer method. So, what happens in these cases, is we do not consider the entire body as a whole, we divide that body into slices and then the slices are considered for the analysis.

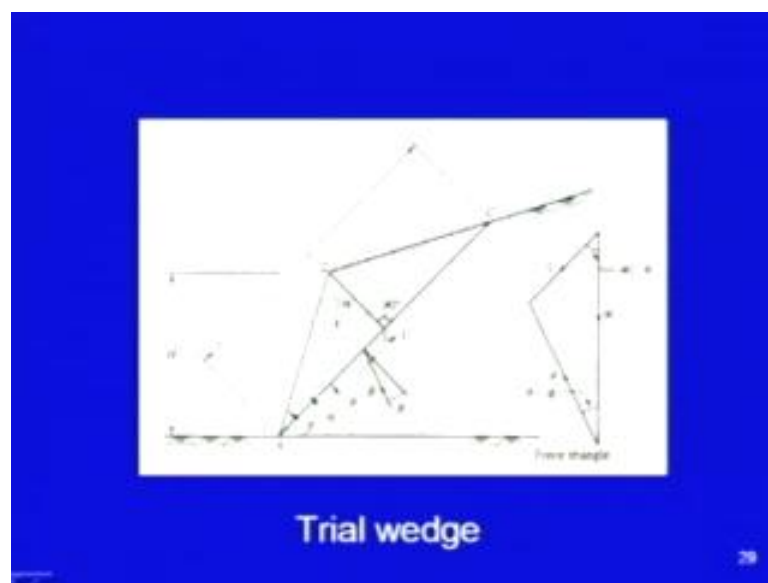
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Let me, come to one method Culmann's method, it belongs to the category one in which the entire body is considered. And it assumes a plane surface of failure it does not assume circular failure surface, it assumes a planar failure surface and it will give you more realistic results when the slopes are steep. If the slopes are not steep, it has been observed that the failure surface is curved most of the times it resembles with circle.

Third point is that failure surface is taken to pass through the toe only. So, it is the failure surface is assumed to pass through the toe, so this is the method.

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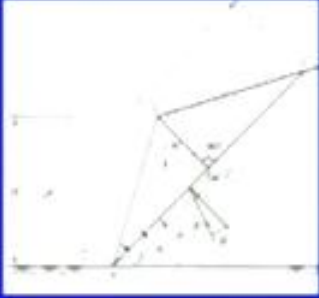
What we do is, let us say this is the slope here, this is the toe of the slope and this is the entire body, what we are going to do, we will be taking one trial wedge. So, we consider one trial wedge means one trial surface, so this is the trial surface we have considered.

Our assumption is that, this wedge is failing and sliding is occurring on this surface. So, rest of the analysis mechanistically is same as we did in previous case, what we are going to do, we are going to find out the weight of this particular wedge.

This weight will be acting in the downward direction, we will then resolve this weight if we will find out the components and when we will consider this stability analysis in the Culmann's method. In fact, it is basis is graphical approach, so this analysis is done little in little bit different manner.

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Taking a wedge ADC
Area = $\frac{1}{2} \times L \times DE$
AD = H cosec β
DE = AD sin ($\beta - \theta$)
Area = $\frac{1}{2} L H \text{ cosec } \beta \times \sin (\beta - \theta)$
W = $\frac{1}{2} \gamma L H \text{ cosec } \beta \times \sin (\beta - \theta) \times 1$
Cohesive component of shearing force
C = $c' \times L \times 1 = c' L$



So, taking a wedge, this is the wedge we have taken and one can find out the area of this wedge and if you know the area you can find out the weight. So, weight area of this wedge will be equal to half into this base length into this perpendicular distance, so half into L into D E and this dimension A D can be obtained as now this angle is beta.

The slope angle is beta and the rupture surface which we have assumed trial surface let us say it is inclined at an angle theta. So, this angle is theta, this angle is beta, so in between this angle will be beta minus theta and this dimension it is the height of the slope, it is a finite slope it is not an infinite slope now.

So, this height is it is vertical distances H, so once this H is available and this angle you can find out this. So, AD will be equal to H cosec of beta and once this A D is available now you can find out D E, D E is here, so D E will be equal to this dimension into sin of this angle. So, D E will be equal to A D sin of beta minus theta and A D is equal to H cosec beta, so once these two dimensions are available I can find out the area of the wedge.

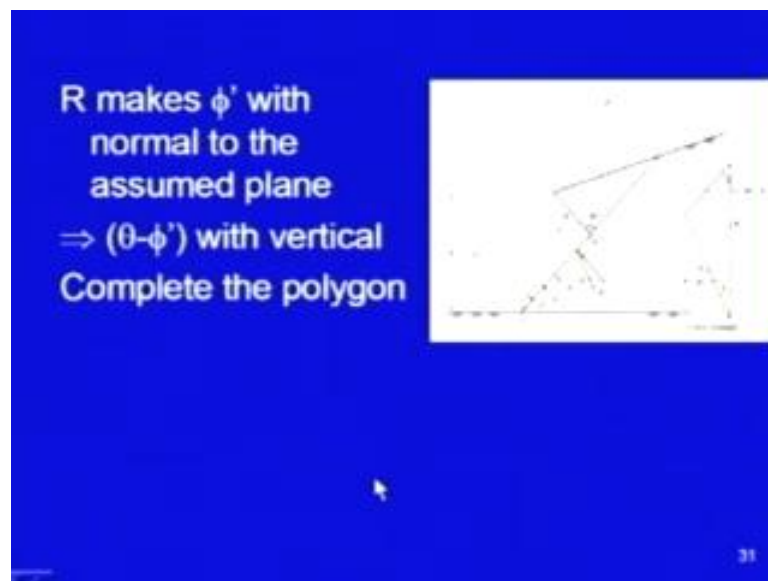
Area will be equal to half into L, L is this dimension, and this dimension also you can find out, because you have this angle available with you. And so, W is equal to area is equal to half L H cosec of beta into sin of beta minus theta this comes out with the area. Once you know the area our assumption is that perpendicular to this plane of paper, the dimension is 1

So, we are calculating the weight per unit length, so this is the weight per unit length perpendicular to the plane of paper. So, W will be equal to half, you have to multiply the area by 1 and multiplied by γ , so W is equal to half $\gamma L H \csc \beta \sin$ of β minus θ into 1.

Now, this is 1 meter you have to keep it in mind that we are taking it in meters, so it will be in meters and in some cases when people used to solve the problem in terms of feet then it use to be per feet. So, the values will be changing. Now, the second components, so this is first force it has considered.

Weight is the first force in this stability analysis, second component is cohesive component of the shearing force and it is taken as this length is available, length is L . So, L into 1 that gives the area, so L into 1 and C dash let us say it is the cohesion, so C dash into L that gives you the shearing the cohesive component of the shearing force.

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Now, we use a graphical method here, so in this case, this is the resultant R which has inclined at an angle ϕ dash with the normal. And so, you can find out the inclination of this resultant R , R is the resultant of the frictional component and the reaction.

So, its inclination with the vertical its inclination with normal to the assumed plane R , R is making ϕ dash with normal to the assumed plane and it will be making θ minus ϕ dash degree with the vertical.

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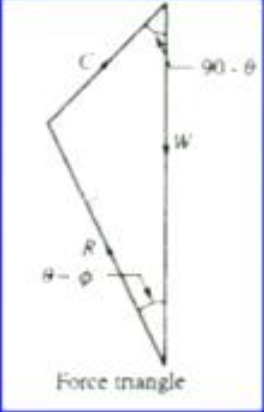
$$\frac{C}{\sin(\theta - \phi')} = \frac{W}{\sin(90 + \phi)}$$

$$\frac{C}{W} = \frac{\sin(\theta - \phi')}{\cos \phi'}$$

Putting values of C and W

$$\frac{c'}{\gamma H} = \frac{1}{2} \frac{\sin(\beta - \theta) \sin(\theta - \phi')}{\sin \beta \cos \phi'}$$

Several trial values of θ may be taken.



Force triangle

So, we now know the direction of this result resultant R and we know the direction of C, we know the direction of W. So, one can complete this force polygon, so this force polygon is completed.

So, once this force polygon is completed, now use the property of the triangle. This is cohesive component, so here the analysis in terms of C is the cohesive component, R is the reaction plus frictional component and W is the weight of the entire body. So, this is the difference in this method, we are using the entire body of the sloping of the failing mass as a whole.

So from this triangle, now from this force polygon you can it is a triangle, so from the property of the triangle. This factor C divided by sin of this angle, so C divided by sin of theta minus phi dash will be equal to W divided by this angle and you already know that R was making theta minus phi dash angle with the vertical and C also you can find out, because it is it is parallel to the rupture plane.

It is vertical its inclination with the vertical will be ninety minus theta where theta is the arbitrary angle which we have taken the rupture circle surface. So, you can find out this angle, it comes out to be 90 plus phi that is 180 minus this plus this angle, so you will be getting this angle. So, C upon sin of theta minus phi dash will be equal to W upon sin of 90 minus phi dash.

And from here, we will be getting this expression capital C upon W equal to sin of theta minus phi dash divided by cos of phi dash. And, when you put the values of C and W the equation converts into this form, C dash gamma H equal to half sin of beta minus theta sin of theta minus phi dash upon sin of beta into cos of phi dash. I had not shown this calculations how to calculate W you can find out in terms of the geometry of the body.

And then finally, this equation is obtained C dash upon gamma H equal to half sin of beta minus theta sin of theta minus phi dash upon sin beta cos of phi dash. Now, theoretically we have to take several trials for theta and then we have to take the critical one most critical one.

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For critical value of θ

$$\frac{\partial}{\partial \theta} \left[\frac{\sin(\beta - \theta) \sin(\theta - \phi')}{2 \sin \beta \cos \phi'} \right] = 0$$

$$\sin(\beta - \theta) \cos(\theta - \phi') = \sin(\theta - \phi') \cos(\beta - \theta)$$

Critical value of θ

$$\theta_c = (\beta + \phi')/2$$

Putting value of θ_c , the stability number for the critical plane may be obtained.

And, to do it we can do it mathematically, for critical value of theta, let us differentiate this term it should be equal to 0, so del by del theta of sin of beta minus theta, sin of theta minus phi dash upon 2 sin beta cos of phi dash will be equal to 0. Differentiate it and when you differentiate it first function differentiation of second minus second function differentiation of first and take it on this side it will be 0.

So, finally, you will left with this expression sin of beta minus theta into cos of theta minus phi dash will be equal to sin of theta minus phi dash cos of beta minus theta. And by solving this, then you will getting the critical value of theta that value of theta which gives you minimum factor of safety and that comes out to be theta c is equal to beta plus

phi dash upon 2. So, once the theta c is available now we can put this theta c and the stability number of critical plane may be obtained.

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Stability number for critical plane

$$\frac{c'}{\gamma H_c} = \frac{1 - \cos(\beta - \phi')}{4 \sin \beta \cos \phi'}$$

For any other stable height H, and mobilised shear strength parameters

$$c_m = \frac{c'}{F_c} \quad \text{and} \quad \tan \phi_m = \frac{\tan \phi'}{F_\phi}$$

$$\frac{c_m}{\gamma H} = \frac{1 - \cos(\beta - \phi_m)}{4 \sin \beta \cos \phi_m}$$

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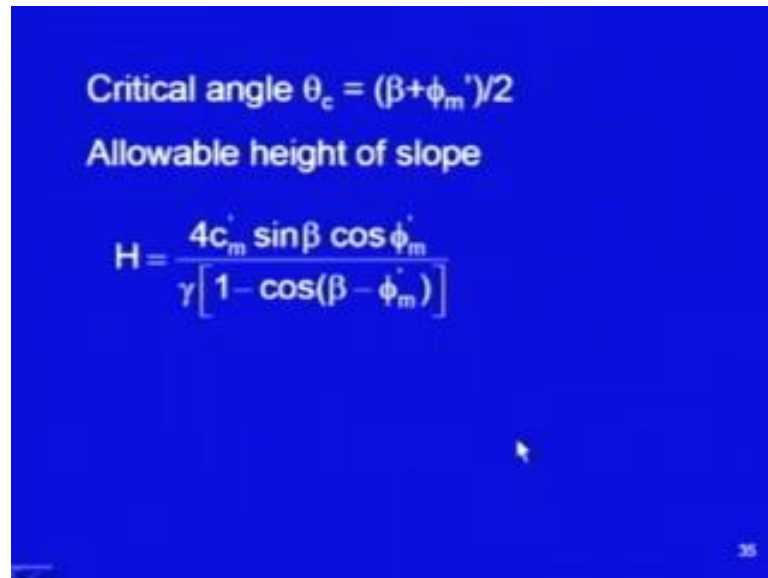
So, stability number for critical plane comes out to be equal to, when you put value of theta c, it comes out equal to C dash upon gamma H c equal to one minus cos of beta minus phi dash upon 4 sin beta cos of phi dash. So, this is the expression for the stability number for the critical plane.

You can change this expression for other planes also let us say for any other plane. So, for any other height also, this was for the critical plane.

So, for any other stable height H and mobilized shear strength parameters, let us say C m and phi m these are the mobilized shear strength parameters. You can change this equation, so instead of H c it will become H. And then, instead of C dash you have to replace it with C m dash and phi m phi dash you have to replace with phi m dash, so the equation for this stability number for a critical for any other plane.

For, any other slope stable slope with height H will be equal to this is the expression, C m dash upon gamma H equal to 1 minus cos of beta minus phi m dash divided by 4 sin beta cos of phi m dash.

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Critical angle $\theta_c = (\beta + \phi'_m)/2$
Allowable height of slope

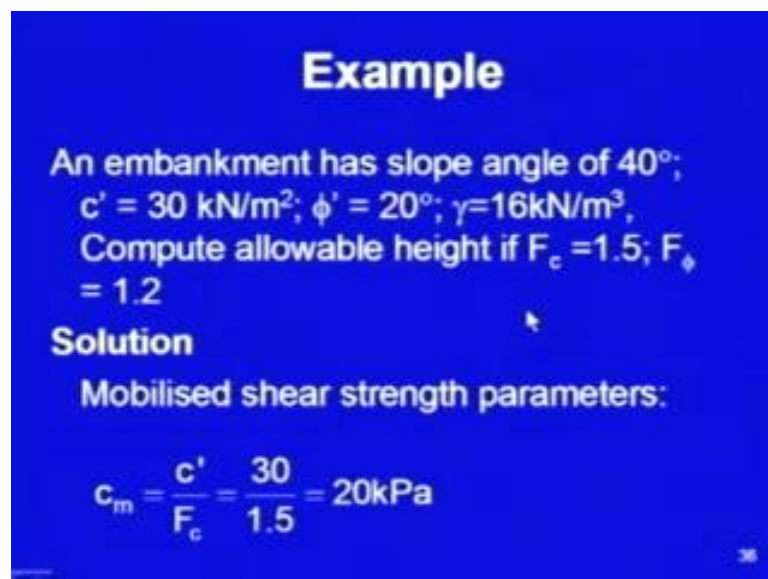
$$H = \frac{4c'_m \sin \beta \cos \phi'_m}{\gamma [1 - \cos(\beta - \phi'_m)]}$$

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And, critical angle expression is same that phi m dash is put here is, so the equation becomes theta c equal to beta plus phi m dash upon 2.

And, allowable height of the slope can be obtained using this expression from the previous expression. So, H is equal to four c m dash sin of beta cos of phi m dash divided by gamma one minus cos of beta minus phi m dash.

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Example

An embankment has slope angle of 40° ;
 $c' = 30 \text{ kN/m}^2$; $\phi' = 20^\circ$; $\gamma = 16 \text{ kN/m}^3$,
Compute allowable height if $F_c = 1.5$; $F_\phi = 1.2$

Solution

Mobilised shear strength parameters:

$$c_m = \frac{c'}{F_c} = \frac{30}{1.5} = 20 \text{ kPa}$$

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Lastly let me take an example, it is given there is an embankment it has slope angle of 40 degree, C dash is given as 30 kPa phi dash is 20 degree. The unit weight is 16 kilo

Newton per meter cube and we have to compute allowable height and it is given that factor of safety against cohesion is 1.5 and F phi is 1.2.

So, let us first calculate the mobilized shear strength parameters, so C m is equal to C dash upon F c that is 30 upon 1.5 it comes out to be twenty 20 kPa.

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$$\tan \phi'_m = \frac{\tan \phi'}{F_\phi} = \frac{\tan 20}{1.2} = 0.253$$

$$\Rightarrow \phi' = 14.18^\circ$$

$$H = \frac{4c'_m \sin \beta \cos \phi'_m}{\gamma [1 - \cos(\beta - \phi'_m)]}$$

$$H = \frac{4 \times 20 \times \sin 40 \cos 14.18}{16 [1 - \cos(40 - 14.18)]}$$

$$= 31.12 \text{ m}$$

And, tan of phi m dash is equal to tan phi dash upon F q, so tan 20 upon 1.2, so tan of phi m dash comes out to be 0.253. From there, you get phi dash is equal to 14.18.

Now, this was the expression which we had derived for the safe height, H is equal to 4 times C m dash that is 20 sin of beta cos of phi m dash divided by gamma bracket 1 minus cos beta minus phi m dash. And finally, you get 31.12, this is the height.

So friends today, we have we started with some cases of the infinite slopes, in infinite slopes we have considered several sub cases and then we started discussing about the finite slopes. And, in finite slopes today we have discussed Culmann's method and in the next term we shall be proceeding further with the finite slope.

Thank you.