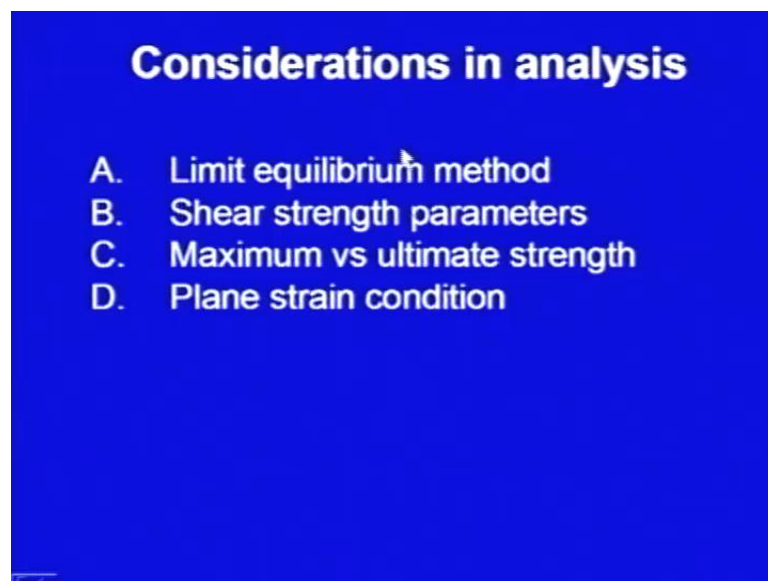


Foundation Engineering
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Module - 03
Lecture - 07
Stability of Slopes

Hello viewers, welcome back to the course to the lecture on the Stability of Slopes, we had started this particular topic on the last term.

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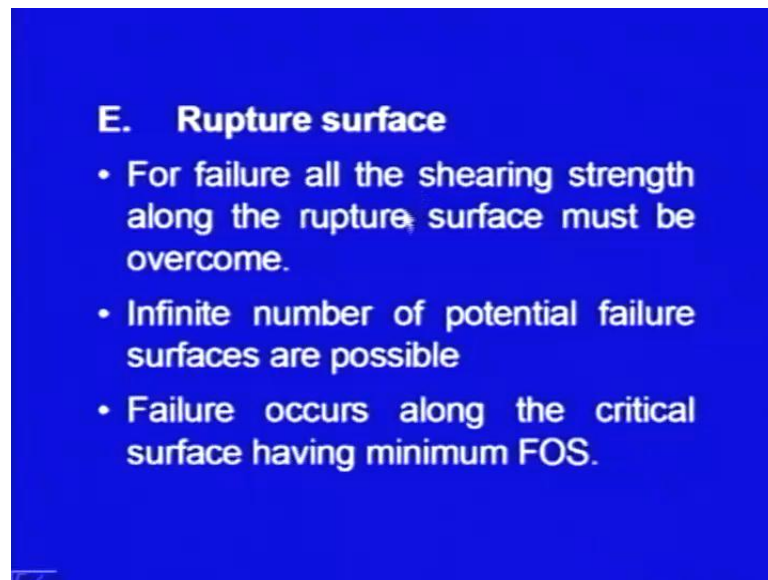


And we had discussed about the types of the slopes, then we discussed the different considerations, what is mechanistically we mean by stability of slope. We, discussed about the limit equilibrium method, limit equilibrium method means we consider some failure plane a potential failure plane is considered. And on that potential failure plane we find out what are the shearing stresses, what are the mobilized shearing stresses and what are the available shearing strength.

The ratio of the shearing strength and the mobilized shearing resistance that gives the factor of safety. Then, we discussed about the shear strength parameters, the Mohr-Coulomb parameters C and ϕ in general will be used, we also discussed about the use of maximum versus ultimate strength. In case of the clays the shearing strain has an influence on the shear strength parameters for high strains it has been found that the strength becomes slow.

So, it was suggested that especially for the over consolidated clays, not to use the maximum value of the shear strength parameters, rather to use the ultimate strength. Then, we also discussed about the plane strain condition, plane strain condition means we had considered the slope to be in two dimension and perpendicular to the plane of paper, unit length is taken. And it is considered that there is no deformation, which is occurring in the third dimension and this particular assumption gives somewhat conservative results that we should keep in mind.

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E. Rupture surface

- For failure all the shearing strength along the rupture surface must be overcome.
- Infinite number of potential failure surfaces are possible
- Failure occurs along the critical surface having minimum FOS.

Let us, now discuss the next consideration in the rupture surface as I told that in the limit equilibrium analysis, we assume failure plane potential failure plane. And on that failure plane for failure all the shearing strength along the rupture surface must be overcome. So, this is the assumption that along that failure plane, the shearing strength should be exceeded, if it is exceeded then and only then the failure will be taking place and secondly, there are infinite number of potential failure planes which are possible.

So, there can be infinite number of potential failure surfaces, we have to search for all of them and then failure occurs along the critical surface having minimum factor of safety. As per this procedure, our intention is to find out that particular surface, along which minimum factor of safety is obtained and that particular surface is considered critical.

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Types of safety factors:
Maximum shear strength:
 $\tau_f = c' + \sigma' \tan \phi'$
 τ = Average value of mobilised shearing resistance
 F_s = Factor of safety with respect to shearing strength

$$F_s = \frac{\tau_f}{\tau} = \frac{c' + \sigma' \tan \phi'}{\tau}$$

Now, let me go to different types of the safety factors, safety factors means it gives you an idea about the margin of safety. Let us, consider that there is a plane and shear strength parameters are c' and ϕ' , then the maximum shear strength which will be available is τ_f is equal to c' , c' plus $\sigma' \tan \phi'$, please note we are doing effective stress analysis. So, this is the maximum shear strength from the Mohr Coulomb criterion, and let us say τ is the average value of the mobilized shearing resistance, means if you resolve the forces and find out what is the acting shear stress that is τ .

Then, factor of safety is defined as factor of safety with respect to shear strength first of all I am taking this particular factor of safety, I am designating it as F_s this will be equal to the ratio of maximum available shear strength, this is the shear strength which is the maximum available value of this strength and τ , τ is the average value of mobilized shearing resistance or you can say what is being acted upon there, so this is the shear stress which is acting, this is the shear stress which is available.

So, if I put these values $c' + \sigma' \tan \phi'$ upon τ , this gives me a margin of safety, this tells us this gives the idea how safe this particular plane is, so this is defined as the factor of safety with respect to shearing strength.

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The mobilised shearing resistance is given as:

$$\tau = \frac{c'}{F_s} + \sigma' \frac{\tan \phi'}{F_s}$$

Defining mobilised cohesion and angle of internal friction as:

$$c_m = \frac{c'}{F_s} \quad \text{and} \quad \tan \phi_m = \frac{\tan \phi'}{F_s}$$
$$\tau = c_m + \sigma' \tan \phi_m$$

Yes the acting shear stress is tau, it is not equal to tau F it is less than tau F for a stable plane, then it can be written as from this equation, if I take tau on this side and F is on this side. So, I can rearrange this equation and the equation becomes tau equal to c dash upon F s, F s is the factor of safety plus sigma dash tan of phi dash divided by F s. This equation now I can write in little bit different manner, I now define mobilized cohesion and mobilized friction angle.

I defined them like this, c m or sometimes I can write c m dash also c m is equal to c dash divided by F s and tan phi m I am writing tan phi dash divided by F s. In fact, these parameters c m and phi m they are mobilized parameters, means when the slope is stable the mobilized value of cohesion and mobilized value of angle of shearing resistance or you can call it angle of internal friction will be this much tan this will be phi m and this will be c m.

So, I defined them like that then this equation tau equal to c dash upon F s I replace it by c m dash here and here sigma dash tan of phi m, so this is the new equation I am getting in terms of mobilized shear strength parameters.

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If the factors of safety with respect to cohesion and friction are different, mobilised shearing resistance :

$$\tau = \frac{c'}{F_c} + \sigma' \frac{\tan \phi'}{F_\phi}$$

Where: F_c and F_ϕ are factors of safety with respect to cohesion and friction respectively.

Now, if the factors of safety with respect to cohesion and friction are different, then mobilized shearing resistance will be this much. See here, in the previous equation instead of F_c here it was F_s we had taken, we as per our definition ((Refer Time: 08:00)) we have defined c_m is equal to c' / F_s and $\tan \phi_m$ equal to $\tan \phi' / F_s$, this value we had taken same, but in the field you can take them different.

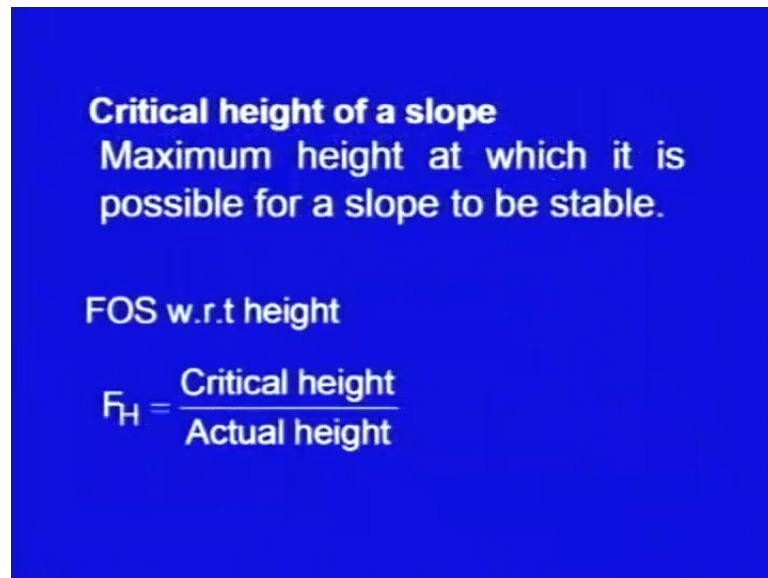
See what happens in the field is factor of safety is basically used to have more confidence in the work. For example, if we have calculated we have measured c' value, we have obtained ϕ' value also, so we if our confidence in c' , c' is small then we should use higher factor of safety. If our confidence is very if large, if I am sure that ϕ' value or c' value or these parameter whatever I am getting is likely to be the same it is I am having more confidence, then I will be using lesser factor of safety.

So, depending on the confidence in these parameters on c' and ϕ' , I can use we can use different factor of safety. In fact, on ϕ' you can use a smaller amount factor of safety because, ϕ' value we can obtain more confidently. So, if I use different values of the factor of safety for cohesion and friction, then τ will be equal to c' / F_c plus $\sigma' \tan \phi' / F_\phi$, so this will become $\tau = c_m + \sigma' \tan \phi_m$.

So, remember c_m will be equal to c' / F_c and $\tan \phi_m$ will be equal to $\tan \phi' / F_\phi$, where F_c F_ϕ are the factors of safety with respect to cohesion and

friction respectively. So, we have defined the factor of safety with respect to strength, factor of safety with respect to cohesion, factor of safety with respect to angle of internal friction F_{ϕ} .

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Critical height of a slope
Maximum height at which it is possible for a slope to be stable.

FOS w.r.t height

$$F_H = \frac{\text{Critical height}}{\text{Actual height}}$$

Now, there is another factor of safety which is sometimes used is factor of safety with respect to height, you can understand it this way that there is a critical height of a slope, this is the maximum height at which it is possible to keep a slope stable. You cannot have a slope more than that height and it will remain stable, if the height becomes more than critical height, it will become unstable.

So, there is always a maximum value of the height, for which it will remain stable and factor of safety with respect to height is defined as, that critical height means that maximum value of the height, that maximum value which the height can attain divided by actual height, which is present there. So, if it is a stable slope in case of a stable slope what is going to happen is, critical height will be more actual height will be less, so F_H factor of safety with respect to height will be more than one. In case critical height is less and actual height is more; that means, factor of safety with respect to height is less than one and it is going to fail it cannot remain stable.

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Example

Given: The shear strength parameters of a soil are:
 $c' = 30\text{kPa}$; $\phi' = 15^\circ$; $c_m' = 18\text{kPa}$;
 $\phi_m' = 12^\circ$.
Average $\sigma' = 100\text{ kPa}$.

Compute:
FOS with respect to shear strength, cohesion and friction: Extreme values of F_c and F_ϕ

Let me try to solve some examples to give you the idea, which we have discussed about the factors of safety, this is a problem given to us, shear strength parameters of a soil are c' equal to 30 kPa. So, we are doing the active stress analysis all the time ϕ' is equal to 15 degree and it is given c_m' is also given the mobilized cohesion, it is 18 kPa, kPa means kilo Newton per meter square and ϕ_m' is also available 12.

So, you can see here ϕ' was here 15, ϕ_m' is 12, c_m' is 30 and c' is 18. So, mobilized values of the shear strength parameters are less than their actual values, so means the slope is stable and it is given to us that the average value of the effective normal stress σ' is 100 kPa and we are required to find out factor of safety with respect to shear strength that is number one, with respect to cohesion and friction and also we have to work out the extreme values of F_c and F_ϕ for which this particular slope will remain stable.

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Solution

Average shearing strength along the failure surface

$$\tau_f = c' + \sigma' \tan \phi'$$
$$\tau_f = 30 + 100 \tan 15^\circ = 56.79 \text{ kPa}$$

Average value of mobilised shearing resistance:

$$\tau = c'_m + \sigma' \tan \phi'_m$$
$$= 18 + 100 \tan 12^\circ = 39.25 \text{ kPa}$$

This is the solution, let us first calculate the average shearing strength, shearing strength means what is maximum available shear strength, what is the maximum amount of which can be made available. So, τ_f will be equal to $c' + \sigma' \tan \phi'$, c' is 30, σ' is 100, ϕ' is 15 and when I put these values τ_f comes out to be 56.79 kPa, so this is the average shearing strength maximum strength.

Now, average value of mobilized shearing resistance, see what is being mobilized c_m and ϕ_m give us what is the shear strength that is being mobilized. So, it is being mobilized τ equal to $c'_m + \sigma' \tan \phi'_m$ instead of c' puts c'_m instead of ϕ' here put ϕ'_m or you can call it ϕ'_m dash also. So, c'_m plus σ' tan ϕ'_m c'_m is 18, σ' is 100, ϕ'_m is 12 and you get the mobilized shearing resistance 39.25.

Now, please note this is the mobilized, mobilized means we are applying or the slope is being acted upon this much by this much stress. Whereas, available is this much, so available strength is higher than the mobilized shearing resistance, so that means, the slope is stable.

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FOS w.r.t. shearing strength:

$$F_s = \frac{\tau_f}{\tau} = \frac{56.79}{39.25} = 1.45$$
$$F_c = \frac{c'}{c_m} = \frac{30}{18} = 1.67$$
$$F_\phi = \frac{\tan \phi'}{\tan \phi_m} = 1.26$$

And how to calculate the factor of safety it is very simple, factor of safety with respect to strength is it is available strength upon mobilized strength, mobilized will be equal to applied stress. So, 56.79 divided by 39.25 and the factor of safety is 1.45, in fact in case of the slopes it should be around at least around 1.5, so you can say it is just stable slope. And if you calculate the factor of safety with respect to cohesion, this is the ratio of c' dash, c' dash means maximum value of the cohesion that can be made available divided by the mobilized value of cohesion, which we are mobilizing or which the structure is mobilizing at present and this will be equal to 30 upon 18 it comes out to be 1.67.

Same way factor of safety against angle of internal friction, it is equal to $\tan \phi'$ dash it is the measure of the maximum available frictional resistance and $\tan \phi_m$, this is the mobilized frictional resistance and their ratio, so it is 1.26, 1.67 and 1.45, so the slope is stable.

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Extreme values:
To get extreme value of F_c put F_ϕ equal to one in expression for mobilised shearing resistance.

$$\tau = \frac{c'}{F_c} + \sigma' \frac{\tan \phi'}{F_\phi}$$
$$\Rightarrow 39.25 = \frac{c'}{1} + \frac{100}{F_\phi} \tan 15^\circ$$
$$\Rightarrow F_\phi = 2.89$$

Now, in the second part of the question it is required that we should get the extreme values of F_c and F_ϕ for which the slope is stable. In fact, you can work out large or I should say infinite number of combinations of F_c and F_ϕ for which the slope will remain stable, I again use the same equation τ is equal to this is c mobilized plus σ dash $\tan \phi$ mobilized. So, to get extreme value of F_c , the minimum value of F_c and F_ϕ you can say let me explain that minimum values of F_c and F_ϕ for a stable slopes should be 1.

So, if it is less than 1 then we will assume that it is going to fail, if it is less than 1 then also it is going to fail in friction. So, to get extreme value of F_c put F_ϕ equal to 1. So, put F_ϕ equal to 1, similarly to get maximum value F_ϕ put F_c equal to 1, so here mobilized shear strength was 39.25 and c dash I am putting F_c equal to 1 and putting F_ϕ as unknown and when I put these values σ dash is 100 I get F_ϕ equal to 2.89. So, this these are the extreme combination, this is one extreme combination; that means, F_c will be equal to 1 and F_ϕ will be equal to 2.89.

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Similarly to get extreme value of F_ϕ put F_c equal to one in expression for mobilised shearing resistance.

$$\tau = \frac{c'}{F_c} + \sigma' \frac{\tan \phi'}{F_\phi}$$

$$\Rightarrow 39.25 = \frac{c'}{F_c} + \frac{100}{1} \tan 15^\circ$$

$$\Rightarrow F_\phi = 2.40$$

Similarly any other combination can be worked out.

The another extreme combination is obtained by putting you can see in this equation it is again the same equation $c' / F_c + \sigma' \tan \phi' / F_\phi = \tau$ equal to mobilized shearing resistance. So, 39.25 is equal to c' / F_c now I am putting F_c as unknown and I am putting lowest value of F_ϕ , so 1 here, so when I put if one the extreme value of F_c sorry it should be F_c here that comes out to be 2.40. So, similarly you can work out all the combinations different combinations of F_c and F_ϕ can be worked out.

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INFINITE SLOPES IN SAND

Vertical stress on AB

$$\sigma_v = \frac{(\gamma z) b \cos \beta \times 1}{b \times 1}$$

$$\sigma_v = \gamma z \cos \beta$$

$$\sigma' = \gamma z \cos^2 \beta$$

Shear stress $\tau = \gamma z \cos \beta \sin \beta$

Now, let me go to the next part of this particular chapter, the infinite slopes and first I am taking infinite slopes in sand, sand means it is a granular material its cohesion is 0, so c is equal to 0. It is an infinite slope, so this is the ground surface it is extending up to very large distance, so for theoretical purpose we will be taking it as infinite, it is just like a hill slope. And here I have taken a plane, plane of rupture you can have it, in fact in case of the hills etcetera, there is a hard stratum available at certain depth, so we can use that stratum also.

So, this is the surface at which we are considering the stability, to consider the stability I am taking this a slide A, B, C, D its weight is acting in downward direction and here at the base it is trying to slide down. So, the frictional force is acting in upward direction and reaction, because of weight there will be reaction, reaction is acting in this direction and normal stress will be acting in this direction. This will be acted upon forces from downstream side as well as from upstream side, this P_1 and P_2 these two forces, but their values will be same and their line of action will also be same.

So, as far as this analysis is concerned we are not considering them, so let us take a point at this depth. The unit rate of this material is γ and the depth of this rupture surface is let us say Z , then I can find out the vertical stress which is acting at the base of this slice, this will be equal to γZ , γ is the unit rate, Z is the height. So, Z into $b \cos \beta$, b is this dimension please note we have taken the dimension b as the inclined dimension, so the thickness the horizontal dimension will be equal to $b \cos \beta$.

So, $b \cos \beta$ is this horizontal dimension, Z is vertical dimension, so $b \cos \beta$ into Z that gives me the volume and when I multiplied with γ , this gives me weight. So, weight of this particular body A, B, C, D will be $\gamma Z b \cos \beta$, this is the area sorry $b \cos \beta$ into Z gives me the area and I am taking one unit perpendicular to the plane of paper. So, volume will be $Z b \cos \beta$ and multiplied by unit weight that gives you the weight, so total weight is equal to $\gamma Z b \cos \beta$.

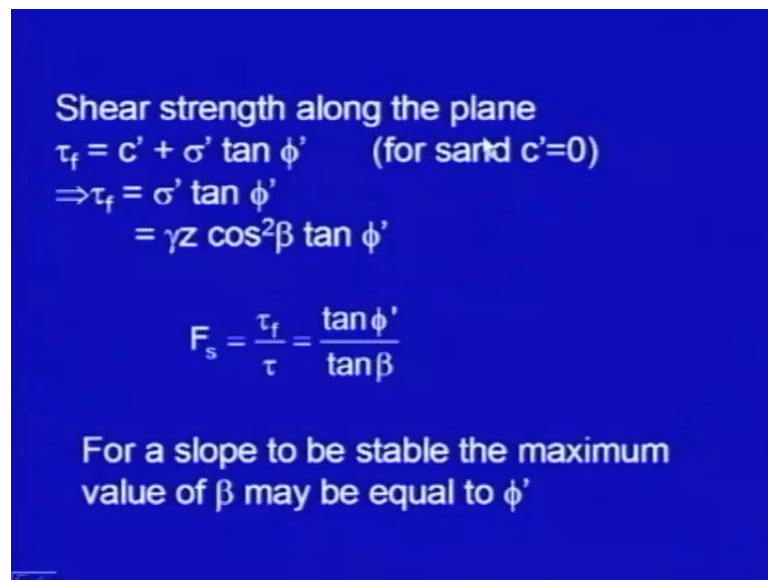
And the area over which this weight is acting is b in this direction and one perpendicular to the direction of the perpendicular to the plane of paper. So, the vertical stress will be $\gamma Z b \cos \beta$ upon b , so this b will cancel and vertical stress at a depth Z will be equal to $\gamma Z \cos \beta$. So, this vertical stress will be acting in vertical direction I can resolve this vertical stress in two components, one component will be normal

component, which will be acting normal to the plane of the rupture and second component will be parallel to the plane of rupture.

And if you remember, when I was discussing the basic mechanism of the slope failure this is what we did. The component of weight that is trying to create instability will be that component, which is along this tangential direction and the component of weight which is perpendicular to the plane of rupture is going to give you stabilizing force. So, here normal component will be equal to $\gamma Z \cos \beta$ into $\cos \beta$, you can see here this line is having this line is inclined at inclination β .

So, thus normal to the rupture and vertical they will be having β angle in between them. So, the normal component will be equal to $\sigma' = \gamma Z \cos^2 \beta$, this will be the normal component of the vertical stress, whereas the tangential component, the component that is trying to create instability that will be along the rupture plane and it will be equal to this value $\gamma Z \cos \beta$ and into \sin of β this \sin of β . So, shearing stress will be equal to $\gamma Z \cos \beta \sin \beta$, so now we have got this shear stress on this plane, we have got the normal stress on this plane.

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Shear strength along the plane
 $\tau_f = c' + \sigma' \tan \phi'$ (for sand $c'=0$)
 $\Rightarrow \tau_f = \sigma' \tan \phi'$
 $= \gamma z \cos^2 \beta \tan \phi'$

$$F_s = \frac{\tau_f}{\tau} = \frac{\tan \phi'}{\tan \beta}$$

For a slope to be stable the maximum value of β may be equal to ϕ'

Let us get the shear strength along this plane, as I told you shear strength is the maximum strength which will be available. So, τ_f will be equal to c' plus $\sigma' \tan \phi'$ and we are taking first example as the sand, so c' is equal to 0 in case of the sand cohesion is 0. So, τ_f will become $\sigma' \tan \phi'$ and

when I put the value of sigma dash from the previous slide you can see sigma dash was gamma Z cos square beta.

So, tau f will be equal to gamma Z cos square beta into tan of phi dash, so this is the shear strength, this is shear strength available. And the shear stress which is applied is this much or you can say this is the mobilized value of the shearing resistance. So, the factor of safety with respect to shearing strength F s is defined as the ratio between the shear strength and the shear stress tau f upon tau. And when you divide this value by the tau value, if this equation reduces to a very simple form F s is equal to tan of phi dash upon tan of beta.

So, it is a very simple equation which we finally, obtained and you can see if I put phi dash, if phi dash is less than beta this F s value will be less than 1, if phi dash is more than beta this F s will be more than one. In other words, if the angle of shearing resistance is higher than this it will be stable, so for a slope to be stable, the maximum value of its inclination, the maximum value of its inclination with the horizontal may be equal to phi dash.

So, this is the thumb rule which we can remember, that if it is cohesion less soil and it is a and the ground is making beta angle, then beta should always be less than phi dash for the slope to be stable.

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INFINITE SLOPE: c-φ SOIL

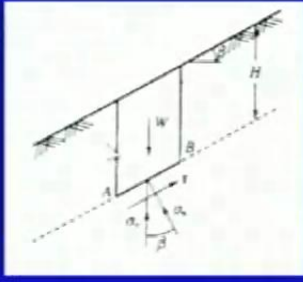
$$\sigma_v = \gamma H \cos \beta$$

$$\sigma' = \gamma H \cos^2 \beta$$

$$\tau = \gamma H \cos \beta \sin \beta$$

$$\tau_f = c' + \sigma' \tan \phi'$$

$$\Rightarrow \tau_f = c' + \gamma H \cos^2 \beta \tan \phi'$$

$$F_s = \frac{\tau_f}{\tau} = \frac{c' + \gamma H \cos^2 \beta \tan \phi'}{\gamma H \cos \beta \sin \beta}$$


Let me now consider the next case, which is more general it is an infinite slope, last time we had considered only the sandy soil c was 0. But, now it is a general case c phi soil

and here it is again the same thing, an infinite slope which is making angle beta and our analysis will assume and it has been observed in the field also, that plane of rupture is generally parallel to the ground surface or if there is hard stratum available here, we can take that hard stratum also at that particular stratum, we can take this plane of rupture and then do the analysis.

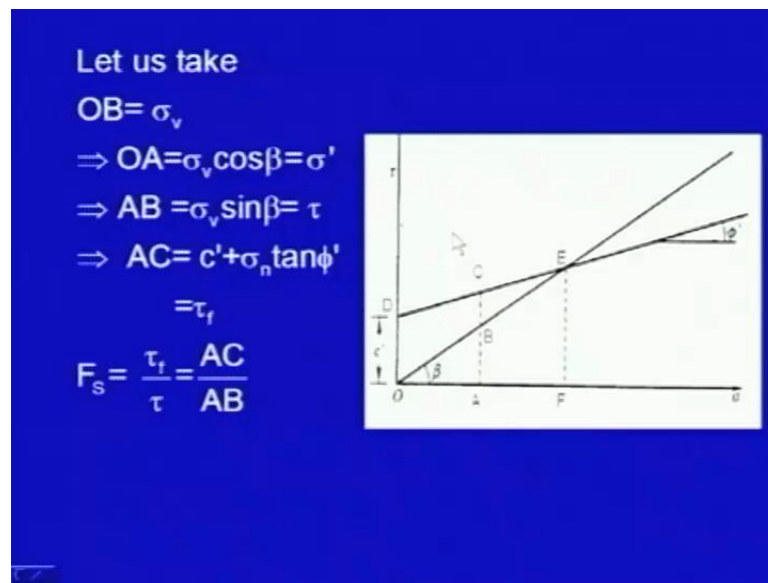
Similar to previous case, I am taking this body the one slice it is having weight w and weight w is acting over here I can find out the vertical stress and this vertical stress is again resolved into tangential component, as well as normal component. Tangential component is going to create instability, the normal component is going to give you higher frictional resistance and then we are going to get factor of safety.

Let us say the depth of the rupture plane here is H , so the vertical stress at that rupture plane is given as $\gamma H \cos \beta$, H is the height of this slice and $\cos \beta$ is the normal component. So, $\gamma H \cos \beta$ as in previous case, this is σ_v and it is normal component, normal means normal to the plane of rupture. So, normal component will be $\gamma H \cos \beta$ in σ_v into $\cos \beta$, so that becomes $\gamma H \cos^2 \beta$, it is tangential component, the shearing stress which will act which will try to create instability τ will be equal to this value into $\sin \beta$.

So, $\gamma H \cos \beta \sin \beta$, similar to the previous case to get the factor of safety we need to know, what is the maximum available shearing strength at this rupture plane. So, τ_f will be equal to $c + \sigma \tan \phi$, so τ_f will be $c + \gamma H \cos^2 \beta \tan \phi$ and when I put this in this equation factor of safety is shearing strength divided by available the mobilized shearing resistance or applied shearing stresses.

So, here it will be $c + \gamma H \cos^2 \beta \tan \phi$ divided by τ , so when I put them $c + \gamma H \cos^2 \beta \tan \phi$ upon $\gamma H \cos \beta \sin \beta$, this is the value of the factor of safety for $c \phi$ soil.

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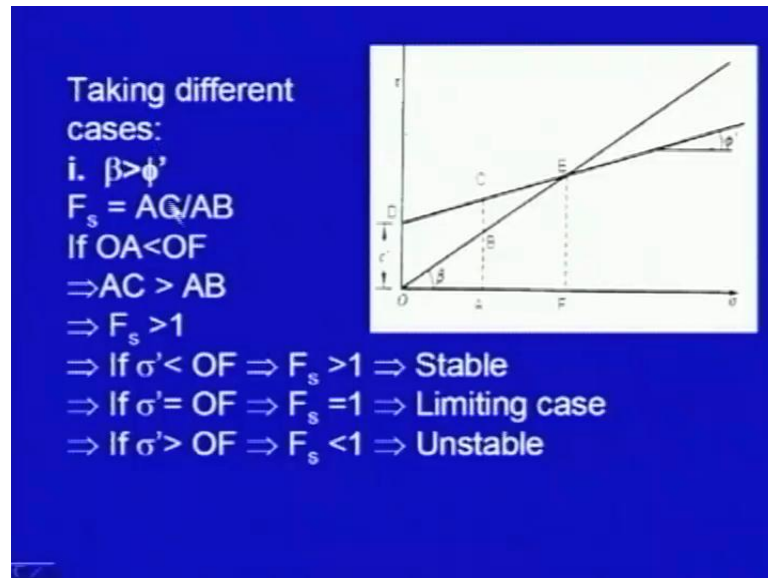
Now, this is an interesting case here let me plot here, this is x axis and I am plotting sigma and y axis I am plotting the shear stress and from origin. Let us plot a line at an angle beta, beta is the inclination of this slope. And here, this is the failure envelope which has given by Mohr Coulomb criterion on y axis, it is having an intercept c dash and it is having an inclination phi dash. Now, if I take one in this range let us say I take a line A B and I represent this line A B by the vertical stress sigma v, then O A this is the horizontal component, this is sigma v into cos of beta O A is from this figure it is equal to sigma v into cos of beta.

And if you remember in the previous figure, sigma v into cos of beta was nothing but, sigma dash normal stress. So, here O A is equal to sigma v cos of beta, so this intercept represents sigma dash the normal stress, which is acting over the rupture plane. Similarly, I can show that A B, A B will be equal to sigma v into sin of beta and from previous figure again, this should represent the shearing stress, so A B represents A B capital A, capital B this represents the shearing stresses.

So, here in this diagram if this is vertical stress, then O A is going to give you the normal stress and A B is going to give you the shearing stress. And also what does A C give, A C is nothing but, it is tau f it is equal to c dash plus sigma dash this is sigma dash into tan of phi. So, in fact this line the any point on this line gives you tau f, so A C represents the tau f, now the factor safety as I told you was tau f upon tau. So, in this diagram I can

write the factor of safety will be equal to AC upon AB , this is an interesting case and it will give us very important conclusions.

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Let me take different cases, the first case I am taking when this beta is more than phi dash, this is the general case in fact, to be which I started. So, as I told you in the previous slide that F_s is equal to AC upon AB , so this ratio AC upon AB , this ratio gives you the factor of safety. Now, if OA is within this range between O and F , if OA is less than OF in that case AC will be more than AB , so far up to this point E this AC is more than AB ; that means, the shear strength available is more than shear stress being applied.

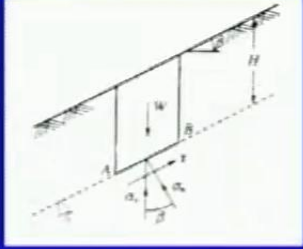
In other words, F_s , F_s is the ratio of these two straight lines these two values AC and AB , F_s will be equal to F_s will be more than 1; that means, it is a stable slope. So, if σ dash, σ dash represents the normal stress it is within this range O to F , then F_s will be equal to it will be less than 1, it will be more than 1 and the slope will be stable, when A point reaches at F . So, here AC and AB both will be equal and it will be limiting case, so factor of safety is exactly equal to 1, it is just at the verge of failure.

And if σ dash if this point OA comes somewhere here, in other words if σ dash is this much σ dash is higher than the factor of safety will be less than 1. Because, now you will be taking ratio of this and this, so it will unstable because, factor of safety will be less than 1. So, this is an interesting case here for certain reason in this region,

when the normal stress up to this value the shear strength is more whereas, in this region beyond A shear stress is more, so it will be unstable.

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Conclusions:
If $H < \text{Critical depth}$
 \Rightarrow Stable
If $H > \text{critical depth}$
 \Rightarrow Unstable
Where $H = \text{depth of hard stratum}$
If the depth of soil layer is more than critical depth it will be unstable.
i.e. slope is steeper than ϕ' , it will be stable only upto critical depth.

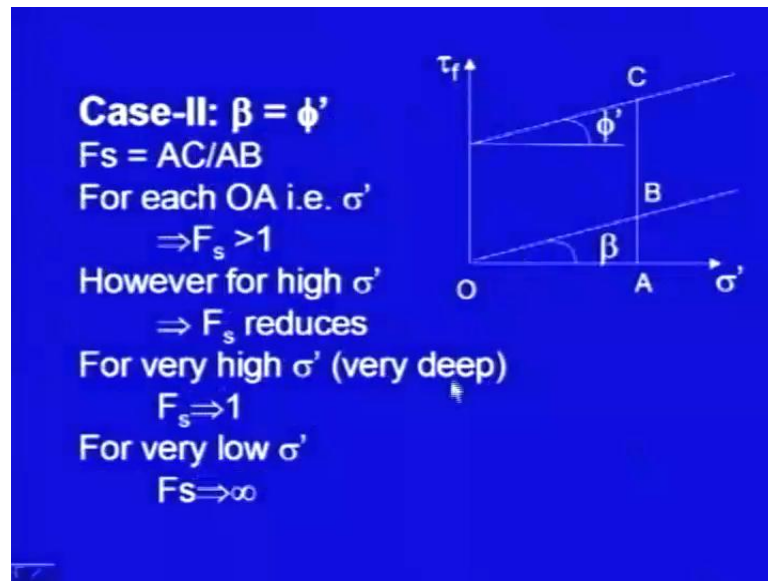


So, what we infer from this what are the conclusions, conclusion is that if H is less than certain critical depth, ((Refer Time: 37:19)) let me go back here, here the normal stress as you go on increasing the normal stress what does it represent, it represents that the weight is increasing or in other words z is increasing, the depth is increasing. So, at higher depth if the value of depth is large, then factor of safety is less than 1, so if this depth of the slope this H , this H value is less than certain critical value it is going to remain stable because, σ dash will be less than that particular critical value.

If H is more than particular value of the depth that we are calling here as critical depth, then this will represent an unstable slope, where H is the depth of the hard stratum. So, the conclusion is, if the depth of the soil layer is more than critical depth it will be unstable. So, what it gives us is that suppose there is a hard stratum here and if the stratum is at relatively shallow depth, this H will be small, if H is a small σ dash will be small. That means, there are chances that the slope will be stable, but if the hard stratum is available at a large depth, then it may become unstable.

So, finally it is concluded if the slope is steeper than angle ϕ dash, remember we had taken a case where the slope is steeper than ϕ dash, then it will be stable only up to certain critical depth beyond that it will not be stable.

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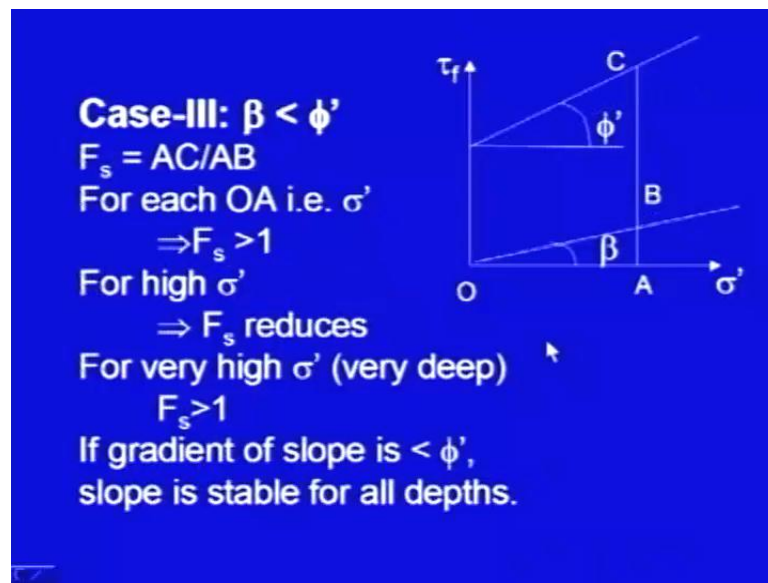


Let me now go to the case number 2, the relationship between beta angle and phi angle and here again this beta angle shows the inclination of the slope angle and this is the phi dash, we can again get this expression that factor of safety is equal to A C, this point is A, this point is C and upon A B, the factor of safety with respect to shear strength will be equal to this much ratio. Now, let us see what happens, if I go on increasing if I keep on changing point A from 0 and let us say up to theoretically let us say up to infinity.

So, for each O A now you can see, A C upon AB I am trying to find out for different values of sigma dash and here these two lines are parallel to each other because, we have taken beta equal to phi dash. So, it is a slope we are analyzing a slope which is having slope angle exactly equal to phi dash, so what is going to happen at each value of O A means at each value of sigma dash F s will always be more than 1. See, whether you take the point here, whether you take the point here or you take the point here, this ratio is always more than one.

And when this value becomes when sigma dash becomes smaller and smaller, if you take a point somewhere here, then this ratio is very large. And theoretically, if I take sigma dash is equal to 0, then this factor of safety in fact, becomes infinite, but as you go on this side as you go on increasing sigma dash, this F s value will go on reducing from infinite to some definite values and finally, at a very large value of sigma dash this F s will approach towards unity. So, far very high depth very deeper slopes F s will be approaching towards unity and for shallow depth it will be very high factor of safety.

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Let me go to now case number third, first we had considered when the slope was steeper than phi dash, second case we had considered when slope is exactly equal to phi dash. And in case in first case when the slope beta was steeper than phi dash, we had concluded that only up to certain depth this slope is going to remain stable, means if the hard stratum is available at a shallow depth only slope will remain stable. Whereas, in the second case when beta was equal to phi dash at all depths it was stable.

42:26 And now, let us see what happens in third case, now the slope is very flat beta angle is very small, this is phi dash phi dash is larger. And now you check it again F_s will be equal to AC upon AB , again this is the shear strength A to C and A to B this is the shear stress, there ratio represents the factor of safety. And again the same thing happens when sigma dash is very small, this value tends towards infinity factor of safety if I take theoretically at point O , where sigma dash is 0 you have AC equal to c dash and AB equal to 0 .

So, factor of safety at this point will be almost infinity, but as you go on increasing it goes on reducing, but because the ratio they are not parallel to each other. So, we cannot say what happens at very large value, but one thing is sure that whatever may value you may take for sigma dash, it may be a very large value, but this factor of safety is always going to remain more than one. So, the conclusion is if the gradient of the slope is less than phi dash, then slope is stable for all depths, so for all depths it will remain stable.

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Critical depth
Considering $\beta > \phi'$ and limiting case
(depth = critical depth, H_c):
 $\tau = \tau_f$
 $\Rightarrow \gamma H_c \cos \beta \sin \beta = c' + \gamma H_c \cos^2 \beta \tan \phi'$
 $\Rightarrow c' = \gamma H_c \cos \beta [\sin \beta - \cos \beta \tan \phi']$
 $\Rightarrow H_c = \frac{c'}{\gamma \cos^2 \beta [\tan \beta - \tan \phi']}$

Now, we can calculate some important parameters for example, critical depth, critical depth is dead depth for which the slope will remain stable. So, considering the first case which I took when the slope was steeper, beta was more than phi dash and considering the limiting case when I say critical depth means, the maximum depth for which the slope will remain stable or if it is the depth at which the hard stratum should be available, otherwise the slope will not remain stable.

So, considering the limiting case at limiting case means, when tau F is equal to tau, what do you mean by this, the entire shearing strength which is available it is being mobilized. So, you will be getting the maximum depth, so this is the critical depth H_c , so we can now use these values we have already derived, tau will be equal to gamma into H cos beta sin beta in place of H, I am putting now H_c . And for the shearing strength it will be C dash plus gamma $H_c \cos^2 \beta \tan \phi'$, this is the same equation which we use in the case number one.

So, from here c dash will be equal to gamma $H_c \cos \beta$ is taken common and inside bracket you will be getting sin beta minus cos beta into tan of phi dash. And finally, you will be getting this expression H_c is equal to c dash divided by gamma take cos beta common. So, gamma cos square beta tan beta minus tan of phi dash, so using this expression you can find out the critical depth, the depth which is make which gives you maximum value of the depth for which the slope will remain stable.

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Stability Number
Again considering $\beta > \phi'$
For stable slope
Mobilised shearing resistance $\tau = \tau_f$
 $\Rightarrow \gamma H \cos \beta \sin \beta = c_m + \gamma H \cos^2 \beta \tan \phi_m$
 $\Rightarrow c_m = \gamma H \cos \beta [\sin \beta - \cos \beta \tan \phi_m]$
 $\Rightarrow \frac{c_m}{\gamma H} = \cos^2 \beta (\tan \beta - \tan \phi_m) = N_s$
 $N_s =$ stability number (dimensionless)

There is another important quantity, which we call as stability number, again if I consider the same case when beta is more than phi dash and slope is stable. Then, the mobilized shearing resistance tau is equal to tau is equal to tau f, so here gamma H cos beta sin beta, this is equal to mobilized shearing resistance. So, now we are talking in terms of the mobilized shearing resistance, this is the applied shear stress and this is the mobilized shearing resistance.

And from here you will be getting see c m is equal to gamma H cos of beta and sin beta minus cos beta tan of phi m. And finally, you will be getting this expression c m upon gamma H is equal to cos square beta tan beta minus tan of phi m, this particular quantity cos square beta tan beta minus tan phi m or on left hand side it is c m upon gamma into H, this is a non-dimensional quantity, non-dimensional number, you can see c units of c will be the units of pressure and gamma is let us say kilo Newton per meter cube into H. So, it is also be ((Refer Time: 47:33)) kilo Newton per meter square, c m is also kilo Newton per meter square.

So, this is the dimension less quantity and if this quantity is known as stability number, this number is very important in analysis in the old during old days when the computers were not available, the computations were tedious and this stability number was very helpful in doing the complicated the computations.

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Stability number

- \propto required cohesion
- $\propto 1/\alpha$ allowable height

putting

$$c_m = \frac{c'}{F_c} \quad \text{and} \quad \tan \phi'_m = \frac{\tan \phi'}{F_\phi}$$

If F_ϕ is one

$$N_s = \frac{c'}{F_c \gamma H} = \text{Cos}^2 \beta (\tan \beta - \tan \phi')$$

This stability number as you can see is directly proportional to required cohesion and universally proportional to allowable height. ((Refer Time: 48:19)) And in this equation N_s equal to c_m upon $\gamma H \cos^2 \beta \tan \beta - \tan \phi'_m$, now if I replace c_m and ϕ'_m , c_m is the mobilized cohesion, mobilized cohesion is equal to c' dash upon F_c , this is the definition of the factor of safety with respect to cohesion we had given, F_c was equal to c' dash upon c_m same to way F_ϕ was equal to $\tan \phi'$ dash upon $\tan \phi'_m$.

So, now in this equation I am replacing c_m and $\tan \phi'_m$ and in general what we do is to define this stability number, normally we take F_ϕ equal to 1. So, the stability number will be equal to c' dash upon F_c into γH , ((Refer Time: 49:20)) so here it was c_m upon γH . So, put the value of c_m this becomes c' dash upon F_c into γH is equal to $\cos^2 \beta \tan \beta - \tan \phi'$ dash. Remember, in this equation whenever you use this equation for some stability analysis, it is inherent it is already assumed that F_ϕ equal to 1. So, this is the stability number, this is the definition of the stability number which is very useful in the analysis of slopes.

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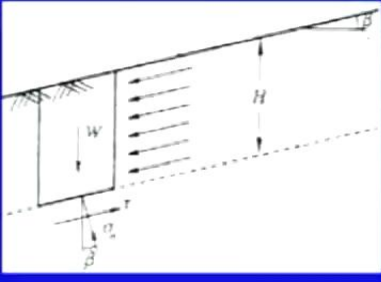
INFINITE SLOPE WITH SEEPAGE

A. Seepage through entire mass

Normal stress at the shearing plane:

$$\sigma' = (\gamma_{\text{sat}} - \gamma_w) H \cos^2 \beta$$
$$= \gamma_{\text{sub}} H \cos^2 \beta$$

Shear stress

$$\tau = \gamma_{\text{sat}} H \cos \beta \sin \beta$$


Now, let me take the another case of the infinite slope with seepage, here I be considering several sub cases. And first case I am taking seepage through entire mass, again this is an infinite slope, inclination is beta, depth is H and here is the plane of rupture on which we are considering the equilibrium and the seepage is taking place in this direction, we are considering this lies again it is weight will be acting in downward direction.

We can find out this stress acting on this plane and then we can resolve that stress into normal component, as well as the shearing component as usual and then we will find out the factor of safety. So, the normal stress at the shearing plane will be equal to sigma v into cos of beta and finally, this is the expression which we have been getting, the different here is that instead of gamma, now you will be using gamma saturated minus gamma w; that means, submerged unit weight because, this will be the stress which will really give the strength.

So, the normal component will be equal to gamma submerged into H into cos square beta and the shear stress, please note down here to compute the shear stress you should not use gamma submerged, but here you have to use gamma saturated. The total weight will be acting and total saturated weight will be creating the instability, so shearing stress will be equal to gamma saturated into H cos beta into sin of beta.

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For stable slope, mobilised shearing resistance

$$\tau_f = c_m' + \sigma' \tan \phi_m'$$
$$= c_m' + (\gamma_{\text{sub}} H \cos^2 \beta) \tan \phi_m'$$

Equating τ and τ_f

$$c_m' + (\gamma_{\text{sub}} H \cos^2 \beta) \tan \phi_m' = \gamma_{\text{sat}} H \cos \beta \sin \beta$$
$$\frac{c_m'}{\gamma_{\text{sat}} H} = \cos \beta \sin \beta - \frac{\gamma_{\text{sub}}}{\gamma_{\text{sat}}} \cos^2 \beta \tan \phi_m'$$
$$= \cos^2 \beta \left[\tan \beta - \frac{\gamma_{\text{sub}}}{\gamma_{\text{sat}}} \tan \phi_m' \right]$$

Now, we are considering the slope to be stable and let us talk in terms of the mobilized shear strength parameters. So, mobilized shearing resistance will be equal to c_m' plus σ' dash tan of ϕ_m' put the value of σ' dash here, so it becomes σ_m' plus $\gamma_{\text{sub}} H \cos^2 \beta$, which we have taken from previous slide. So, σ_m' is equal to $\gamma_{\text{sub}} H \cos^2 \beta$ and by putting this value we get τ_f is equal to this much.

Now, equate τ and τ_f the shearing stress which is acting and the shearing strength, then we will be getting this expression c_m' plus $\gamma_{\text{sub}} H \cos^2 \beta \tan \phi_m'$ will be equal to $\gamma_{\text{sat}} H \cos \beta \sin \beta$, this is the acting shearing stress. So, finally, I take the term c_m' on the left hand side and $\gamma_{\text{sat}} H$ on the left hand side, and we convert this equation into the form of this stability number c upon γ into H .

And this is the equation you are to get c_m' upon $\gamma_{\text{sat}} H$ equal to $\cos \beta \sin \beta$ minus $\frac{\gamma_{\text{sub}}}{\gamma_{\text{sat}}} \cos^2 \beta \tan \phi_m'$ and finally, you take $\cos^2 \beta$ common. So, c_m' upon $\gamma_{\text{sat}} H$ will be equal to $\cos^2 \beta \left[\tan \beta - \frac{\gamma_{\text{sub}}}{\gamma_{\text{sat}}} \tan \phi_m' \right]$.

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$$\text{If } F_c = F_\phi = F_s$$

$$\text{Using } c_m = \frac{c'}{F_s} \quad \tan \phi_m^i = \frac{\tan \phi'}{F_s}$$

$$\frac{c'}{F_s \gamma_{\text{sat}} H} = \cos^2 \beta \left[\tan \beta - \frac{\gamma_{\text{sub}}}{\gamma_{\text{sat}}} \tan \phi' \right]$$

$$\text{If } F_c \neq F_\phi$$

$$\frac{c'}{F_c \gamma_{\text{sat}} H} = \cos^2 \beta \left[\tan \beta - \frac{\gamma_{\text{sub}}}{\gamma_{\text{sat}}} \frac{\tan \phi'}{F_\phi} \right]$$

Now, if we use same factor of safety for cohesion for angle of internal friction, for shearing strength. Then the equation we will be replacing the parameters by this c_m will be c dash upon F_s and $\tan \phi_m$ dash will be $\tan \phi$ dash upon F_s and you will be getting this equation c dash upon F with respect to shearing strength γ of saturated into H will be equal to \cos square β \tan β minus γ submerged upon γ saturated \tan of ϕ dash or if you use if the values for F_c and F_ϕ which are not equal.

Then, this equation can be written as c dash upon $F_c \gamma$ saturated H \cos square β \tan β minus γ submerged upon γ saturated and here you will be having $\tan \phi$ dash upon F_ϕ .

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If factor of safety with respect to friction is one, the stability number with respect to cohesion is given as:

$$N_s = \frac{c'}{F_c \gamma_{sat} H} = \cos^2 \beta \left[\tan \beta - \frac{\gamma_{sub}}{\gamma_{sat}} \tan \phi' \right]$$

In the stability number normally we take the value of F_ϕ is 1, so if the factor of safety with respect to friction is one, the stability number with respect to cohesion is given as N_s is equal to c' upon $F_c \gamma_{sat} H$ equal to $\cos^2 \beta \tan \beta$ minus γ_{sub} upon γ_{sat} into $\tan \phi'$. So, this is the stability number we get for this particular case, so in today's lecture class we have discussed we started with the different types of the factor of safety's.

Then, we started discussing the infinite sloped, we have also given some idea about the stability number and we have given the how to analyze, how to get the factor of safety or how to get the expressions which is correlate factor of safety and other parameters and through the stability number for infinite slopes, so in the next class we shall continue with more cases on the infinite slopes.