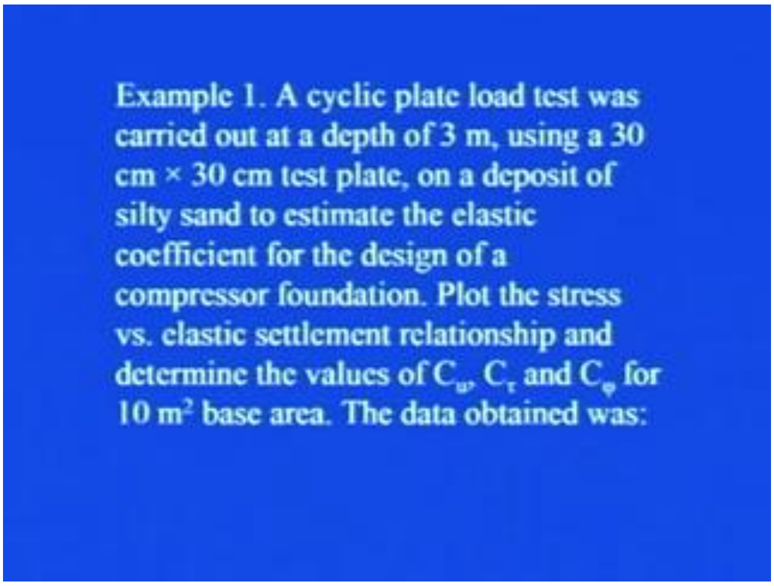
Foundation Engineering
Dr. Priti Maheshwari
Department Of Civil Engineering
Indian Institute Of Technology, Roorkee

Module - 02
Lecture - 15
Machine Foundations - 3

Hello viewers, In the last class we discussed, that how we can find out the elastic constants of equivalent soil spring using cyclic plate load test data and then block vibration test data. In block vibration test data, we saw two type of test, that is vertical vibration test and another one was horizontal vibration test. So today, let us try to first take, an example, based on cyclic plate load test data and then see, that how we can find out various constant, spring constant from this cyclic plate load test data.

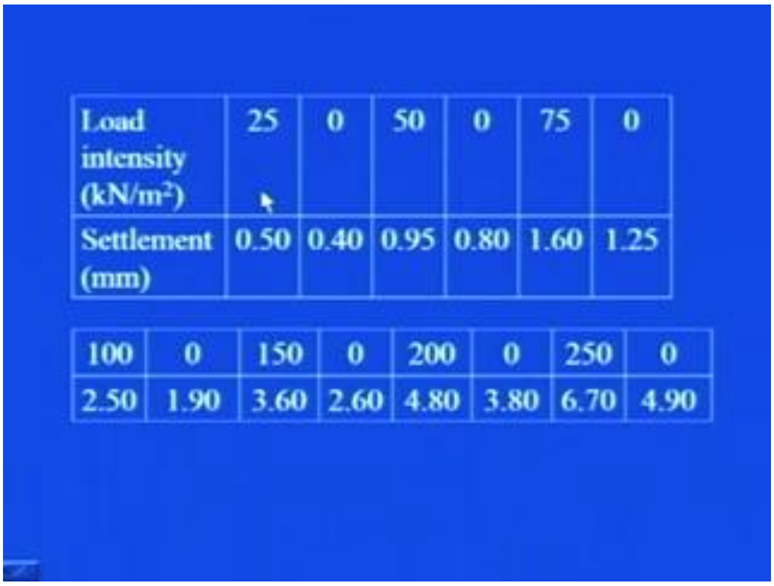
(Refer Slide Time: 01:07)



So, the statement of the example is that, a cyclic plate load test was carried out at a depth of 3 meter using a 30 centimeter by 30 centimeter test plate, on a deposit of silty sand to estimate the elastic coefficient for the design of compressor foundation. Then, in the last class we saw that, there were three types of machine and in that, when we were discussing the salient features of them, we saw that reciprocating machine is to be used for compressor foundation.

So, you see here, we have to design a compressor foundation, so that is, that machine will be reciprocating machine. So, further the statement of the example says that, plot the stress verses elastic settlement relationship and determine the values of C u, C tau and C phi for 10 meter square base area.

(Refer Slide Time: 02:04)



Load intensity (kN/m²)	25	0	50	0	75	0
Settlement (mm)	0.50	0.40	0.95	0.80	1.60	1.25

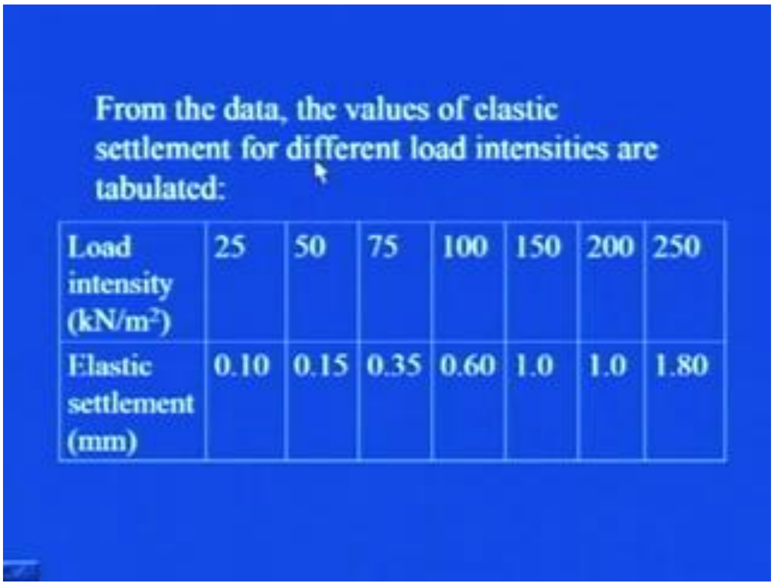
100	0	150	0	200	0	250	0
2.50	1.90	3.60	2.60	4.80	3.80	6.70	4.90

The data obtained from the test was since, it is cyclic plate load test, so loading and unloading and then reloading is there, so you see for the first loading step, when load intensity is 25 kilonewton per meter square, the corresponding settlement recorded was 0.50, when it was unloaded the settlement recorded was 0.40, then again in the second step, for 50 kilonewton per meter square, the corresponding settlement is 0.95 millimeter and the second unloading cycle the settlement is 0.80 mm.

Likewise, for 100 kilonewton per meter square load intensity, settlement observed was 2.5 millimeter and then, in this unloading cycle, when it was unloaded from 100 to 0, the corresponding settlement obtained was 1.90. Likewise, it has been given, for 150 kilonewton per meter square to 200 and 250 and then corresponding settlements are given. What we have to do is, first we have to plot the load intensity verses elastic settlement curve.

And then, we have to estimate the equivalent spring constants or coefficients, that is Cu, C tau and C phi. So, first let us try to find out, that how we can estimate the elastic settlement corresponding to each loading intensity.

(Refer Slide Time: 03:31)



Load intensity (kN/m²)	25	50	75	100	150	200	250
Elastic settlement (mm)	0.10	0.15	0.35	0.60	1.0	1.0	1.80

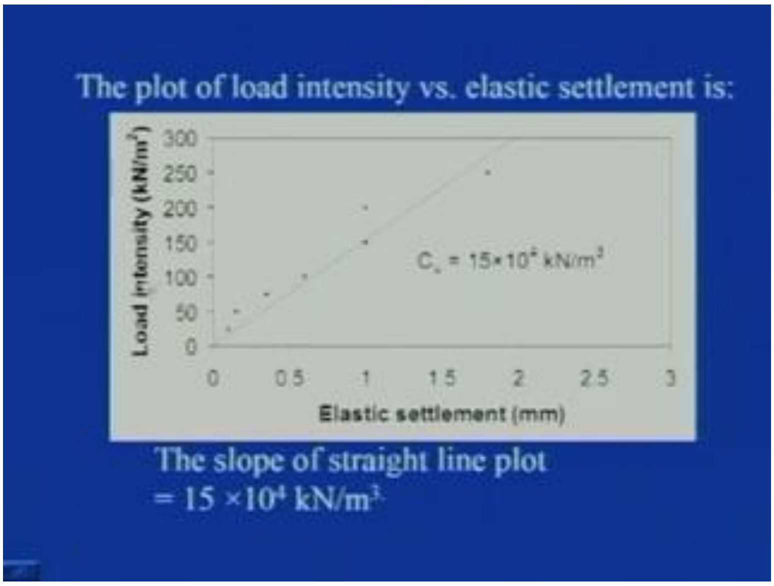
So, from the data the values of elastic settlements they can be calculated, you can see here, that when this maximum loading intensity of 25 was there, then the settlement obtained was of the was 0.5 millimeter and with, when this loading intensity was 0, then the settlement was 0.40. That means, that the recovery, that is 0.1, that is, the settlement corresponding to the maximum load intensity to this and the difference to, the settlement corresponding to the loading intensity equal to 0, will give us the value of elastic settlement, for that particular loading cycle.

So, in this case, corresponding to this load intensity 25, elastic settlement is 0.10, you can see here. ((Refer Time: 04:25)) That, this is 0.50 and 0.40, so the difference of these two is 0.1, which is the elastic settlement in case of this 25 kilonewton per meter square. However, in case of 50 kilonewton per meter square, you have to see that, corresponding to this 50 kilonewton per meter square, the settlement is 0.95 millimeter and when it is, when it has been brought down to 0, the corresponding settlement is 0.80.

So, here it is the recovery is 0.15, 0.95 minus 0.85 correspondingly, you can find for each loading cycle and tabulate them, in this manner. So, to correlate with the pervious table and this table, elastic settlement can be obtained by the difference, taking the difference of the settlement at the maximum loading intensity and when it has been brought down to 0. So, for 25 kilonewton per meter square, this elastic settlement of 0.10 mm was observed.

However, it was 0.15 in case of 50 kilonewton per meter square and so on, you can see here, that for 150 and 200, it is 1 mm and for 250 kilonewton per meter square, it is 1.8 mm. Now, we know this load intensity and corresponding elastic settlement also, let us try to see that, how we can plot it and can get the value of the elastic coefficients.

(Refer Slide Time: 05:57)



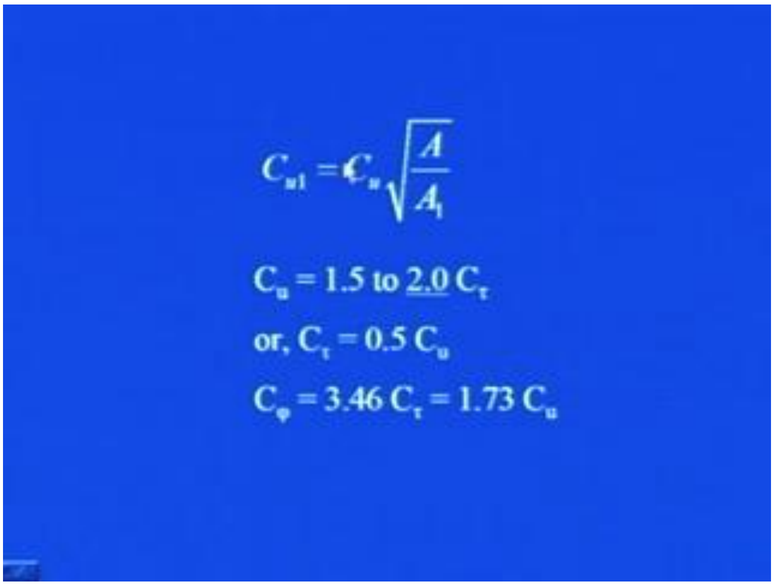
So, the plot of load intensity verses elastic settlement, on Y axis, we are plotting load intensity in kilonewton per meter square and on X axis, this elastic settlement in millimeter has been plotted. So, the, you know that, when the experimental data are there, they do not fall along line properly, so you have to take the best fit of that. So, these are the data points, which we are getting ((Refer Time: 06:28)) from this particular table and those data points have been plotted and then, a line has been fitted.

Such that, it is a slope, we can find out, so it is a slope will be giving us that C u for the test plate. Now, as we were discussing last time, that when you are find finding out this, these constants in the field and then, if you are applying them in case of foundation, you have to apply some correction measure, because their values, they were depending on the contact area of the block foundation and the soil. So, in case of test place, test plate, the test plate is of a definite size like 30 centimeter by 30 centimeter in this case.

However, the area of the foundation can be quite high and in the statement of the problem, it has been given that, we have to get the, these coefficients or constants for base area of 10 meter square. However, the base area of the test plate is 0.09 meter square, so we have to convert this value of C u which is for the plate, to corresponding value of the foundation. So, that we can use this data confidently as far as analyses of the foundation is concerned.

That, how we can do that, you know that we discussed last time these formulas.

(Refer Slide Time: 07:58)

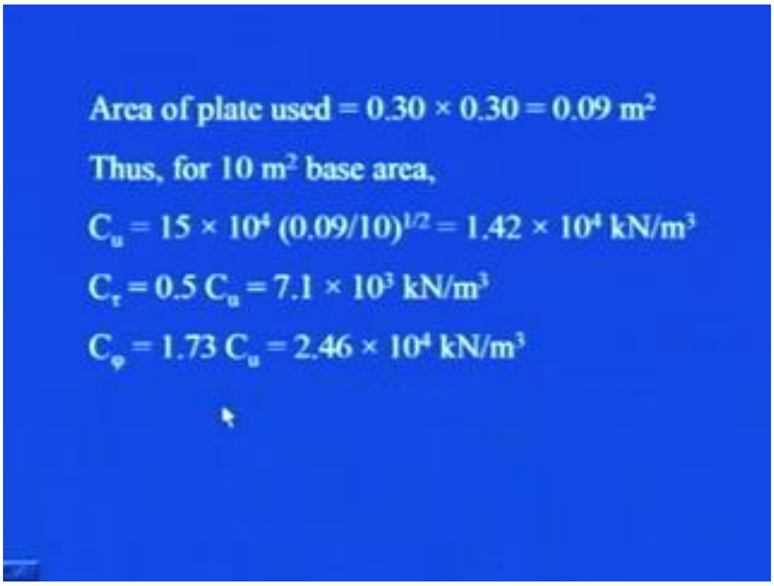


That, C u 1, that is the spring constant correspondingly for the foundation, C u is for plate, A is area of the plate and A 1 is area of the foundation. So, we know this C u, ((Refer Time: 08:11)) from this particular curve, we have the area of the plate, that is 0.09 meter square and then A 1, that is area of the foundation or base area, which is 10 meter square, that is given. Then, once we know this C u, then you know that, the, this is an empirical relation between C u and C tau, as that C u is equal to 1.5 to 2.0 times C tau.

Now, let us say that, for this particular example, I consider C u to be equal to 2 times C tau. So, correspondingly, C tau will be equal to half of C u, so knowing the value of C u for the test plate, I can get this value of C u for the foundation, once the C u value is known to us, we can substitute here in this particular expression and can get the value of C tau. Then, again we know, that C phi is equal to 3.46 C tau, all these things we have already discussed in the last class.

So, if this C tau is half of C u, so you substitute this particular expression, here in this expression, that will result in to 1.73 times C u. So, using these three relation that is, this one, this one and this one, we can get the value of C u for the foundation, from the value of C u which we have found out for the test plate and then, once we know this C u 1, that is C u for the foundation, using this expression we can get corresponding C tau and then using this particular expression, we can get corresponding C phi.

(Refer Slide Time: 09:52)



You can see here in this, that area of the plate which is used is 0.30 into 0.30, because the size of the plate is 30 centimeter by 30 centimeter, so in meter, it will be 0.30 and which will result into an area of 0.09 meter square. So, for the area of 10 meter of base, you can get, ((Refer Time: 10:16)) using this particular expression as, this C u value 15 into 10 to the power of 4 that we have observed or we have obtained, from the load intensity verses elastic settlement curve, for the test plate.

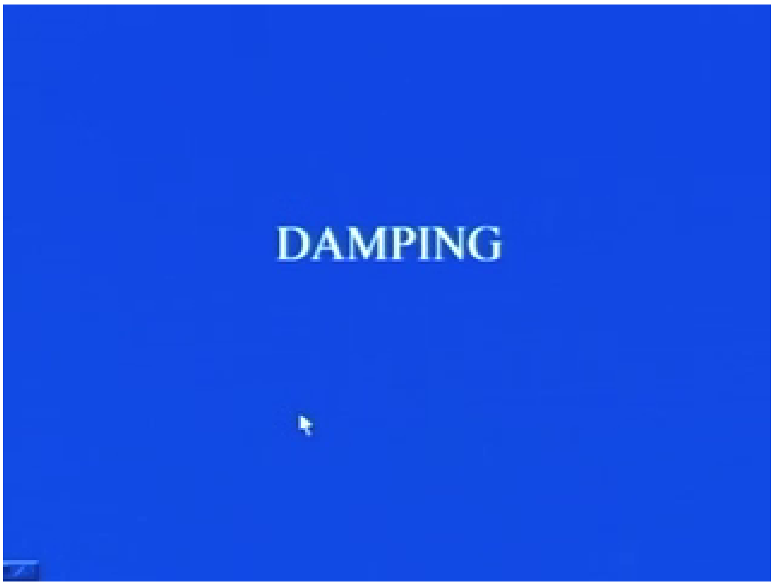
That has been converted it, this, here in this particular manner for the foundation, so this is, what is C u into, this area of the plate divided by the area of the foundation base, which is 10 in this case. Let us say, if it is more than 10, then you have to restrict it to 10, so this expression, I mean if you solve this, then you will be getting this value of C u to be equal to 1.42 into 10 to the power of 4 kilonewton per meter cube. Then, using this particular expression, that is C tau is equal to 0.5 times C u.

I can get here, that 0.5 times C u, which is C u is this value now, 1.42 into 10 to the power of 4, so then this will become 7.1 into 10 cube kilonewton per meter cube. Similarly, we have this expression for C phi, which is equal to 1.73 C u and C u we have got as, this 1.42 into 10 to the power of 4 kilonewton per meter cube and so, substituting this particular value here in this expression, you will be getting the value of 2.46 into 10 to the power of 4 kilonewton per meter cube as C phi.

So, you, I hope that, now you can appreciate more, that how we can get these equivalent soil spring constants or coefficient using cyclic plate load test data, because in cyclic plate load test, what you will be getting as raw data is that, loading intensity and corresponding settlement, corresponding to each loading cycle and each unloading cycle. So, from there, we can find out the elastic settlement and once we plot that elastic settlement with loading intensity, with corresponding loading intensity.

And then, if you fit a best line to that data, then we can, the slope of that line will give us the constant C u, which will be there for test plate, which can be converted for the foundation, depending on whatever is the area of the foundation, you can get the value of C u for foundation and then using the available empirical relations, further, you can get the other elastic constants like C tau and C phi.

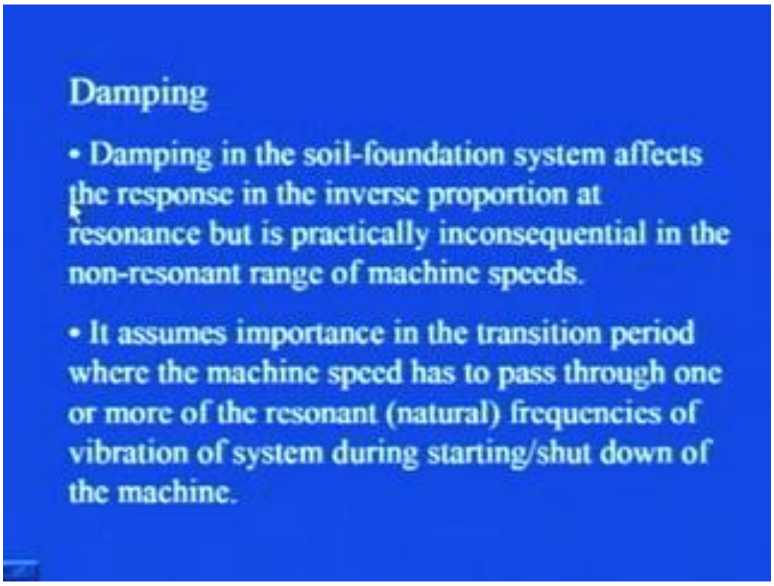
(Refer Slide Time: 12:48)



Now, as we were discussing, that in all the system existing in the nature, everywhere, some or other amount of damping is present. There is no system, in which the damping is not present, so that is why, damping becomes an important parameter, as far as machine foundation or dynamic analyses of any structure is concerned. Since, this machine foundation, they are subjected to dynamic loads, in addition to static loads, so there, damping plays an important role.

Let us try to see, some of the salient points related to this particular topic, that is damping.

(Refer Slide Time: 13:34)



Damping, in the soil foundation system affects the response, in the inverse proportion at resonance, but is practically inconsequential, in the non resonant range of machine speed. So, we have to really see, that during the resonance when the amplitude becomes almost infinite, so the damping effect is in inverse proportion. It assumes importance in the transition period where, the machine speed has to pass through one or more of the resonant or natural frequencies of vibration of system, during starting or shut down the machine.

You see, when you either start or shut down the machine, what happens is that, when the machine takes, picks up it, it is speed, it crosses it is some of it is natural frequencies. So, at that particular point of time, whenever it is crossing that it is natural frequency, then at that time the damping becomes quiet important.

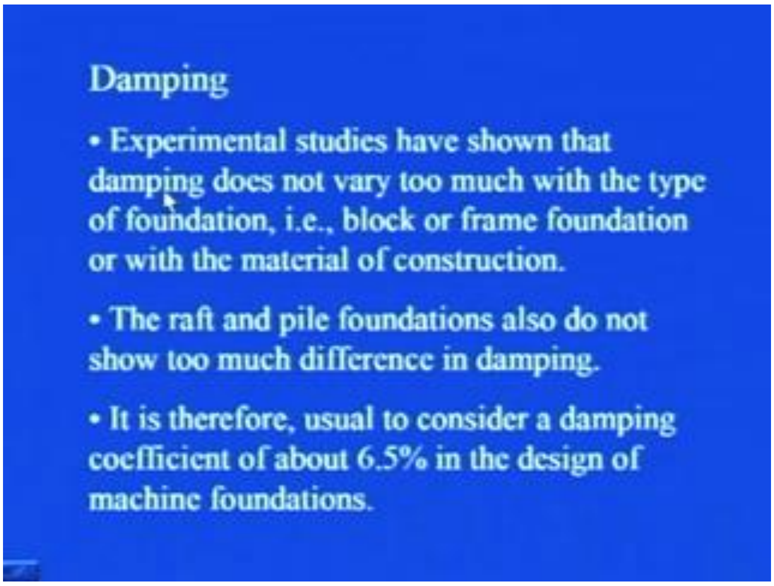
(Refer Slide Time: 14:38)

In the case of block foundations, damping is contributed almost entirely by the soil, so that, it is value depends much on embedment of the foundation. You have seen that, in case of block foundation, what has been done is, that a concrete block is casted and in top of that, that motor and the machine, they are mounted and it is mass is also very heavy. So, we, since we are discussing the case for block foundation, in block foundation the, this damping which is there in the system will be mostly contributed by the soil, on which the concrete block will be resting.

So, obviously, you will not be putting the block just on the ground surface, you take some depth of embedment. So, depending on that depth of embedment, that, what exactly is the depth of embedment, what exactly is the soil cover that, that is present above the concrete block, depending on that the damping will be there. So, damping in framed foundation also increases considerably due to participation of soil, we saw last time, the three types of foundation were there.

One was this block foundation, then box foundation and wall foundation, that was all frame foundation. So, in that case also, this damping is mostly contributed by the soil which lies beneath the concrete slab.

(Refer Slide Time: 16:12)



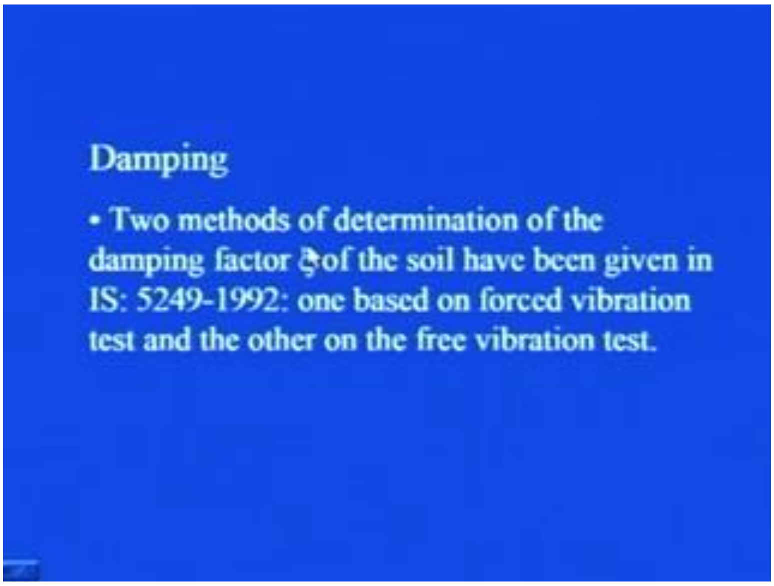
Then, experimental studies have shown that, damping does not vary too much with the type of foundation, that is, whether it is block foundation or frame foundation or with the material of construction. So, whatever is the material of construction, be it concrete or any other material or whatever is the type of foundation, be it block foundation, be it box foundation or be it wall foundation, the damping does not vary with the, with these type of different foundation and material of construction.

The raft and pile foundations also do not show much, too much difference in damping, so whether it is raft foot, raft footing that you are providing as foundation or you are providing piles as foundation, whatever is the damping, it does not make much of the difference. It is therefore, usual to consider a damping coefficient of about 6.5 percent, in the design of machine foundation. Since, the amount of damping is not dependent on type of foundation, it is type of construction material, whether it is raft foundation or pile foundation.

That is why, usually it has been seen that, if you consider a damping of 6.5 percent, that serve the purpose for the design of machine foundations. Now, we have to really, now we were, we were just talking in the qualitative terms this damping, we have to quantify it that, how we can find out, that what exactly is the damping in any particular system. So, to this damping is being represented quantitatively by this damping factor zeta.

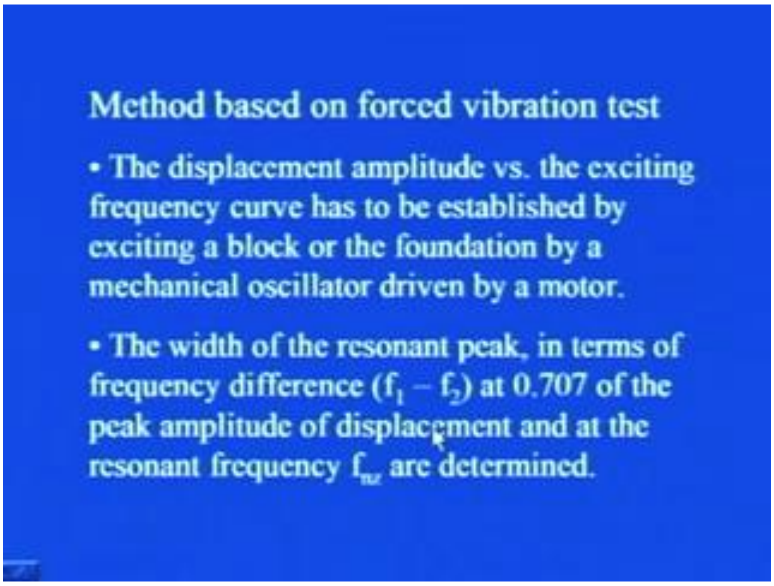
So, to, when you determine this damping factor zeta, that give you, gives you the measure of damping, that whatever is the percentage of the damping or amount of damping which is present in any particular system.

(Refer Slide Time: 18:15)



So, there are basically two methods for determining this damping factor zeta and they are given in the Indian standard code of practice, that is I S 5249 1992, one method is based on forced vibration test and the other on the free vibration test.

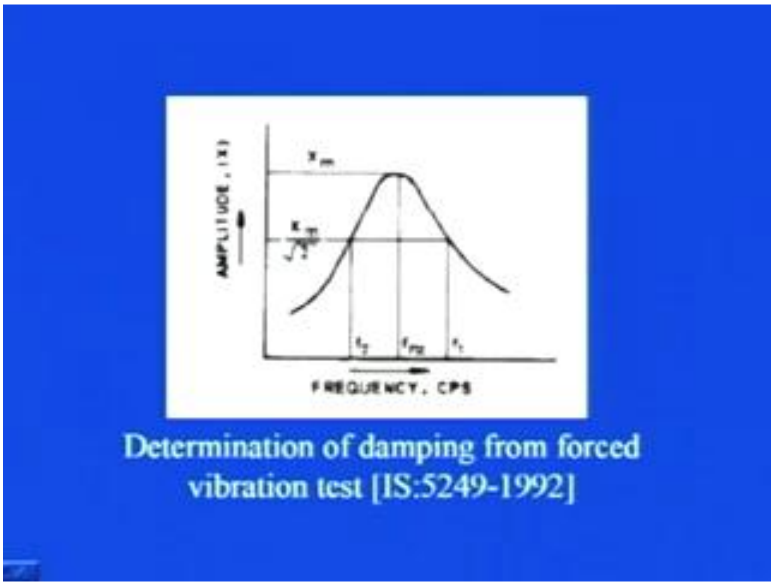
(Refer Slide Time: 18:34)



So, first let us see the, what exactly is the method, which is based on forced vibration taste test. The displacement amplitude verses the exciting frequency curve, has to be established by exciting a block or the foundation by a mechanical oscillator driven by a motor. In the last class, we discussed various features of this block vibration test, whether it was forced one or free one, in that one, we saw that ultimately what we plot, get is that, this amplitude verses the frequency curve.

And from there, for corresponding to the peak amplitude, what was the frequency that we were calling as, the natural frequency of that particular system. So, in this case, we get this particular curve, which we get by exciting the block or the foundation, on which this mechanical oscillator and the motor has been mounted. The width of the resonant peak, in terms of frequency difference, say f 1 minus f 2 at 0.707 of the peak amplitude of displacement and at the resonant frequency f n z are determined, how.

(Refer Slide Time: 19:51)

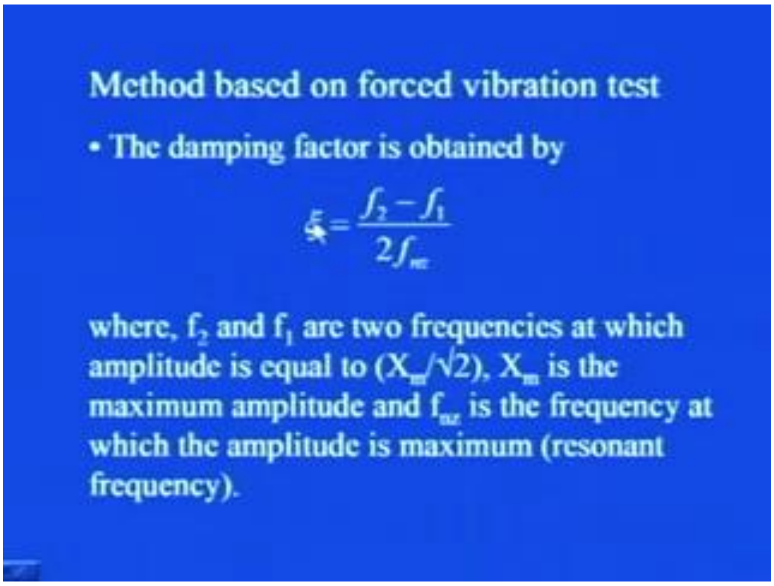


You, it is more clear in this particular figure, that you see that, this one is the curve that you will be getting, using that forced vibration test. Now, as ((Refer Time: 20:24)) this statement says that the frequency difference f 1 minus f 2 at 0.707 of peak amplitude of displacement, so if, X m, I represent as peak amplitude of the system, corresponding to that, whatever is the frequency that we are calling as f n z, that is the natural frequency of the system.

And then, if I divide it by square root 2 or multiply it by 0.707, it is one and the same thing, so corresponding to that, you will be getting two frequencies. You see here, this X m is here, that is, which is peak amplitude and you divide it by square root of 2 or multiply it by 0.707, it will get reduced to this particular level and if you draw a line, which is parallel to this particular frequency axis, then that line will be intersecting this particular curve at these two points, one point is this, another point is this.

So, corresponding to these two points, you have two frequencies, one is f 1 and other another is f 2, so the difference of this f 1 and f 2 is used as.

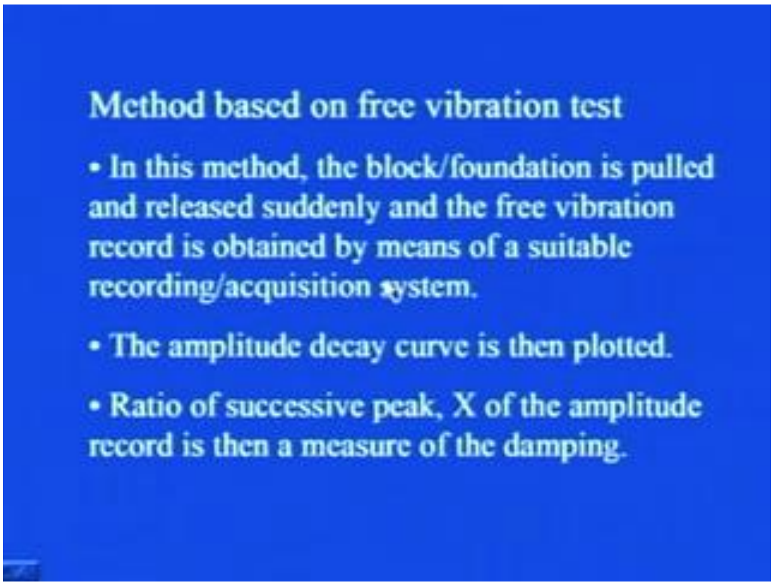
(Refer Slide Time: 21:14)



The, to determine that damping factor as, by this expression, that is zeta is equal to f 2 minus f 1 by 2 f n z, where f 2 and f 1 are two frequencies at which amplitude it equal to X m by square root of 2, where X m is the maximum amplitude and f n z is the frequency, at which the amplitude is maximum, that is resonant frequency. So, once we know, the amplitude verses frequency curve, we can find out, this f n z which is the frequency corresponding to the maximum amplitude.

This frequency is also known as resonant frequency and then, we know the peak amplitude, if we reduce that by 0.707, then we get correspondingly two frequencies, that is for one amplitude, there are two frequencies and the difference of those two frequencies divided by two times, this natural or this resonant frequency of the system will give us this damping factor, which is a measure of the damping.

(Refer Slide Time: 22:27)



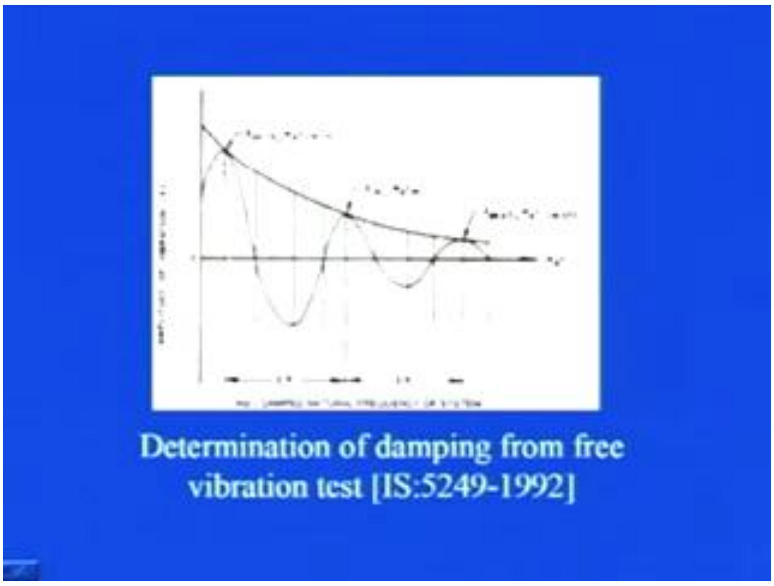
Then, methods which is based on free vibration test, in this method the block or foundation is pulled and released, suddenly and the free vibration recorded is obtained by means of a suitable recording and acquisition system. See, as the name suggest that, in forced vibration system, the oscillator was mounted to give the excitement to the system, that is, that was the forced vibration system. However, in this case, simply this block or foundation, whatever it is, is just pulled and then it allows to settle or deform.

And that condition, we know that, it is the free vibration condition, so when it is undergoing that free vibration, we provide some suitable measure to record these, this vibration. That, how it is vibrating, what exactly is the amplitude, what is the frequency etcetera, then the amplitude decay curve is then plotted. As you know that, some amount of damping is always there in any of the system, be it forced vibration system or free vibration system.

In free vibration system, we know that the system vibrates, under the influence of the forces which are inherent in it, there is no external exciting force, so whatever is the inherent force, once this block or foundation has been displaced, then it will be vibrating under the forces which are inherent in it, so under those forces, the amplitude of the, that particular system will go on decreasing with time. So, that amplitude decay curve can be plotted, because suitable measure has already been provided to record this amplitude.

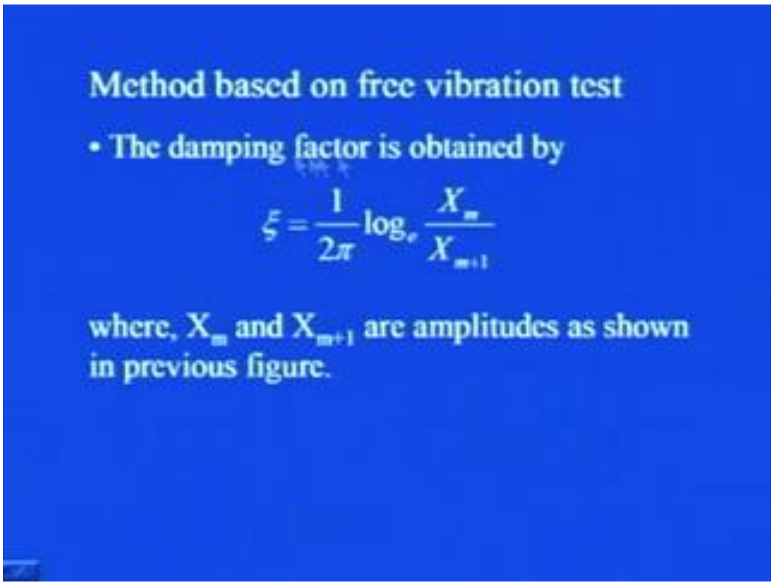
This, ratio of successive peak, that is X, of the amplitude record is then a measure of damping. So, you will get successive peaks and downs also, for that. So, the ratio of successive peaks of the amplitude, gives you the measure of damping. How it is, let us try to see with the help of this particular figure.

(Refer Slide Time: 24:48)



You see that, this is the amplitude decay curve, here is the, this is the frequency of the system, that is damped frequency and this is the amplitude of the vibration on Y axis. So, you see here, with time, this amplitude goes on decaying, so and then, it is the sinusoidal vibration which is taking place, so you see here, this is the one peak and this is another peak, so and this is another one. So, if I call this as X m plus 1, that is the amplitude corresponding to this particular peak is X m plus 1 and the previous one is X m and this s X m minus 1.

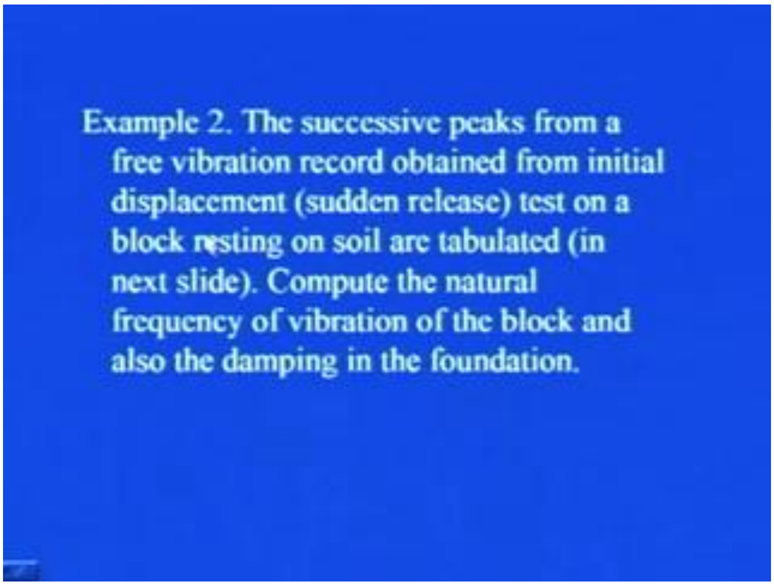
(Refer Slide Time: 25:31)



Then, we can find out the damping factor by using this particular expression as, 1 by 2 pi log of e X m upon X m plus 1 or let us say, if you are here, you had three peaks, that is X m minus 1, X m and X m plus 1. So, either you take these two or you take these two, X 1 and the same thing, where X m and X m plus 1 are the amplitudes like I have shown you here, this is X m plus 1, the amplitude, this much amplitude and this much amplitude, that is X m. So, using this expression, you can determine the damping factor which is a measure of damping.

So, you saw that, how we can quantify this damping using this forced vibration test data or free vibration test data. Now, let us try to take an example and try to develop some more understanding, as far as the measure of this damping is concerned. Here, I am taking one example, for this free vibration record.

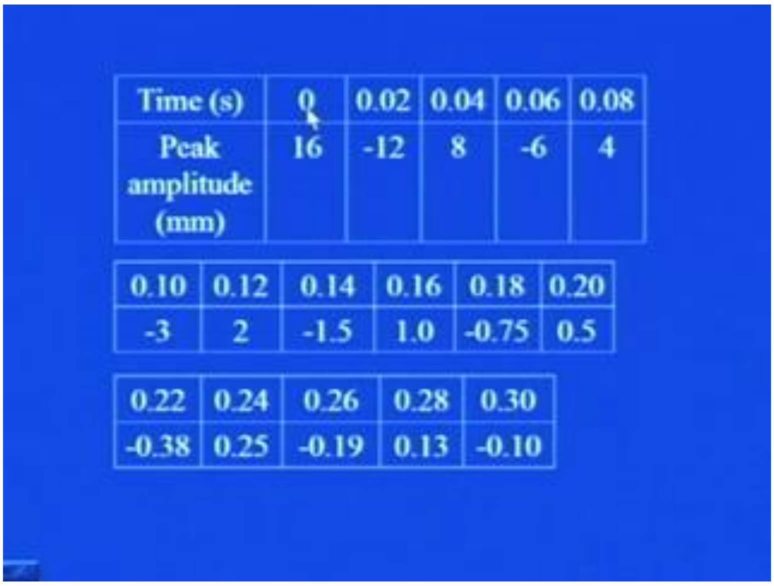
(Refer Slide Time: 26:39)



So, the successive peaks from a free vibration record obtained from initial displacement, that is sudden release, test on a block resting on soil are tabulated, I will be showing you in the next slide. Compute the natural frequency of vibration of block and also the damping in the foundation. So, you have to remember that, it is free vibration test, so in free vibration test, the two successive peaks of the amplitude, the difference gives you the, difference of these two gives you the magnitude of the damping factor.

So, how it is done, this is the data which has been supplied with the problem, that with time.

(Refer Slide Time: 27:19)



Time (s)	0	0.02	0.04	0.06	0.08
Peak amplitude (mm)	16	-12	8	-6	4

0.10	0.12	0.14	0.16	0.18	0.20
-3	2	-1.5	1.0	-0.75	0.5

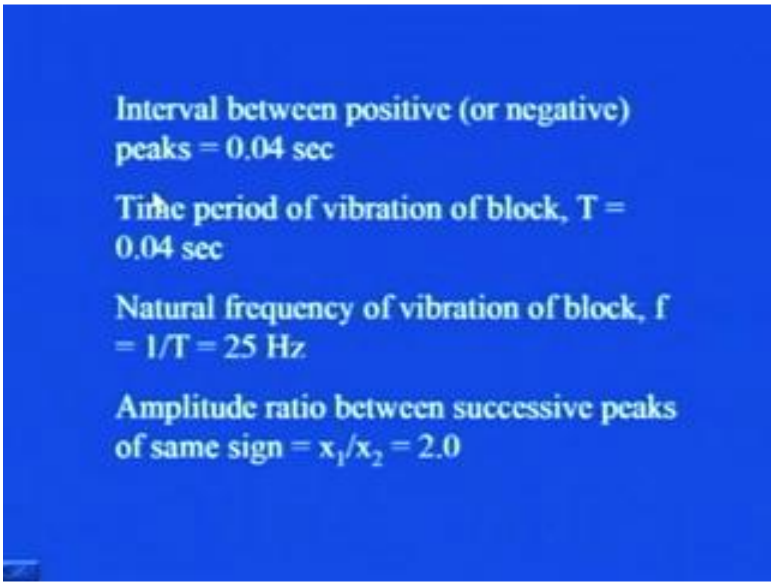
0.22	0.24	0.26	0.28	0.30
-0.38	0.25	-0.19	0.13	-0.10

You see, the time is increasing from 0 to 0.02 seconds, 0.04 seconds, 0.06, 0.08 then 0.10, 0.12 and then so on, till 0.30 and the corresponding peak, you see first, at time t is equal to 0, since it is free vibration test, so at time t is equal to 0, the foundation was just pulled and then it was released. So, at time t is equal to 0, the amplitude was maximum, you can see here, that it is the peak amplitude in millimeter, which is 16 at time t is equal to 0.

And then, it will, that, since it is sinusoidal, so next peak you will be getting in the negative direction. So, that is why, this negative sign has been shown here, that is, the next peak which is in negative direction is minus 12, the amplitude is 12, magnitude is 12. So, and then if, then again you will be getting a positive peak, which is 8 then, minus 6, 4, minus 3, 2, again minus 1.5. So, likewise, you are getting positive and negative peaks.

And you see that, as this time is increasing from 0 to 0.3, you see, here the peak was 16 mm, however, here at 0.30, that is, at time t is equal to 0.30 seconds, you have peak amplitude of the magnitude of 0.10 millimeter only. So, you see, how much is the difference, so why exactly is this difference is occurring, is due to the damping which is present in the system, how we can measure that.

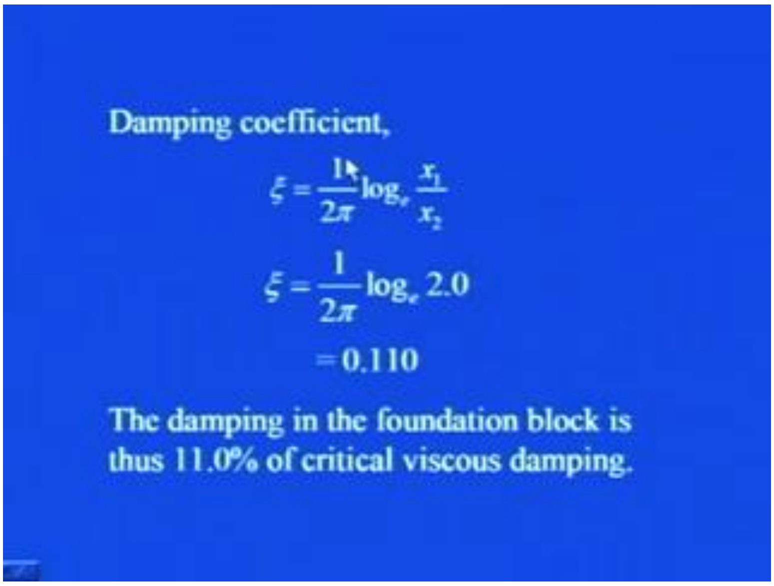
(Refer Slide Time: 29:03)



Let us try to see, that interval between positive or negative peaks, ((Refer Time: 29:06)) you can see here, that these are the two positive peaks, so the interval here between these two positive peaks, it is 0.04. Likewise, for negative peaks also, you can see here, that the time interval is 0.04 seconds, in all the cases. So, the time period of vibration of block, which are we represent usually by T, becomes 0.04 second, now, we can find out the natural frequency of vibration of block, by inverting this particular time period, which is 1 by t and that becomes 25 hertz.

So, amplitude ratio between successive peaks of same sign, that is x 1 by x 2 becomes 2.0, you can see here, the two successive peaks, if you take the ratio, that is 16 by 8 becomes 2, 12 by 6 is 2, again 8 by 4 is 2, you take this one, 2 and 1, the ratio is 2, then 1.5 and 1.75, the ratio is 2. So, the amplitude ratio between successive peaks of same sign, see you have to keep in mind, because, then only it is crossing it is equilibrium position, so x 1 by x 2 is working out to be 2.0, in this case.

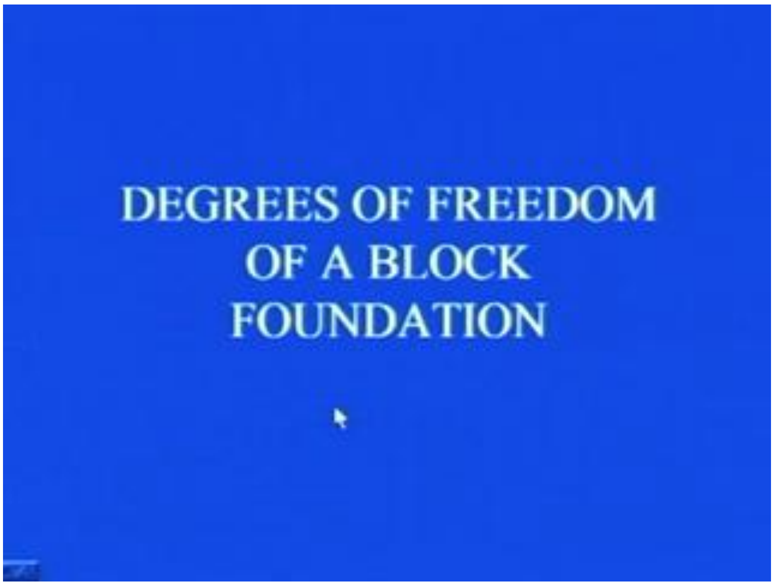
(Refer Slide Time: 30:20)



So, we can find out this damping coefficient or damping factor, using this particular expression, that is 1 by 2 pi log base e x 1 by x 2, this x 1 by x 2 from the data, we have got to be equal to 2, so this zeta will be working out as 0.110. So, if we talk in terms of percentage, so damping in the foundation block, is thus 11.0 percent of critical viscous damping.

So, I hope that, you got the idea, got the feel, that if, how the test result from this free vibration test can be utilized to get the magnitude of the damping which is present in the system, in the form of this damping coefficient or damping factor and then, it can be converted into percentage of critical viscous damping.

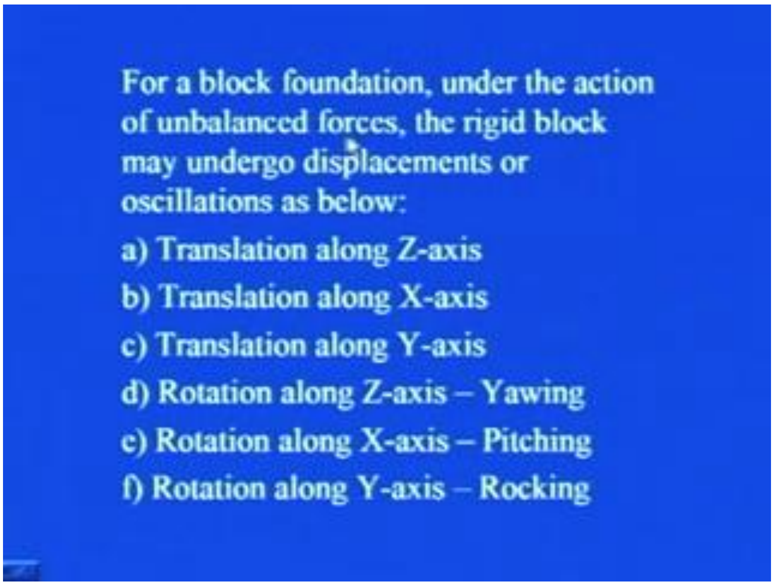
(Refer Slide Time: 31:17)



Now, let try to see, that what are the various degrees of freedom of a block foundation, as you know that, when we defined the degrees of freedom, what was the definition, was that how many number of independent coordinators that you require, to define the a particular location or the system of vibrating body. So, the number of coordinates which are required to define the state of any vibrating body is called as it is degree of freedom. So, now, here we are discussing the block foundation.

So, we will be discussing that, in what all are the modes of vibration, that is, that in what all manner or what all are the coordinates, which are required to define the vibration of this block foundation, they will be called as degrees of freedom.

(Refer Slide Time: 32:11)

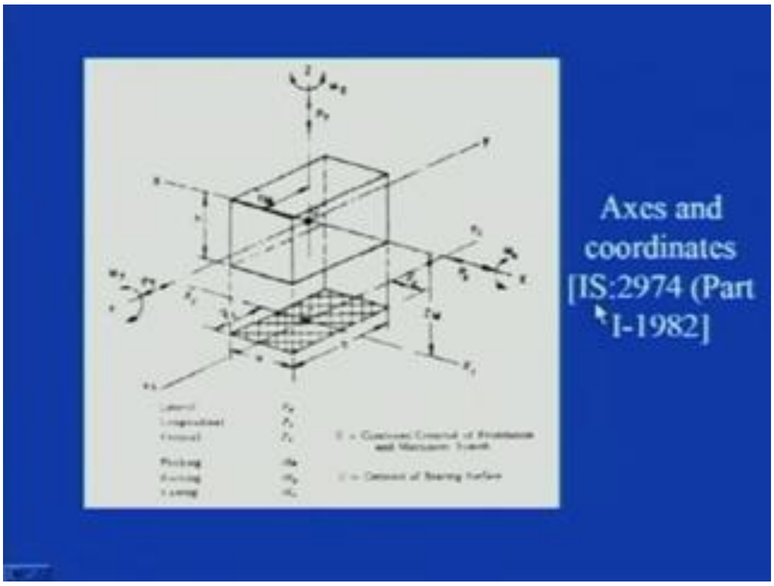


For a block foundation, under the action of unbalanced forces, the rigid block may undergo displacements or oscillation as will. As you know, that in case of any foundation it can translate or rotate, then, there are three axis, two axis in horizontal direction perpendicular to each other and one is vertical axis. So, there can be, the translation along this Z axis, which is the vertical axis, see, first I will be telling you about the vertical axis.

Because, usually we are more concerned about the settlement of the foundation, which is, which occurs usually there in the Z direction, then translation along X axis, translation along Y axis and similarly rotation about all these three axis. So, rotation along Z axis is, we call as yawing, rotation about along X axis, we call it as pitching and rotation along Y axis is called as rocking. So, wherever I use a term yawing, this picture should come into your mind, that there is a rotation, which is taking place along Z axis, that is which is vertical axis.

So, six degree of freedom, as far as the action of, under the action of this unbalanced force are concerned, they can occur, that six type of this deformation can take place, which is the translation along the three axis and the rotation along three axis. Out of which, the rotation about the vertical axis, that is Z axis is called as yawing, rotation along X axis is called as pitching and rotation along Y axis is rocking. Now, how it looks like in 3 D picture, let us try to see.

(Refer Slide Time: 34:05)



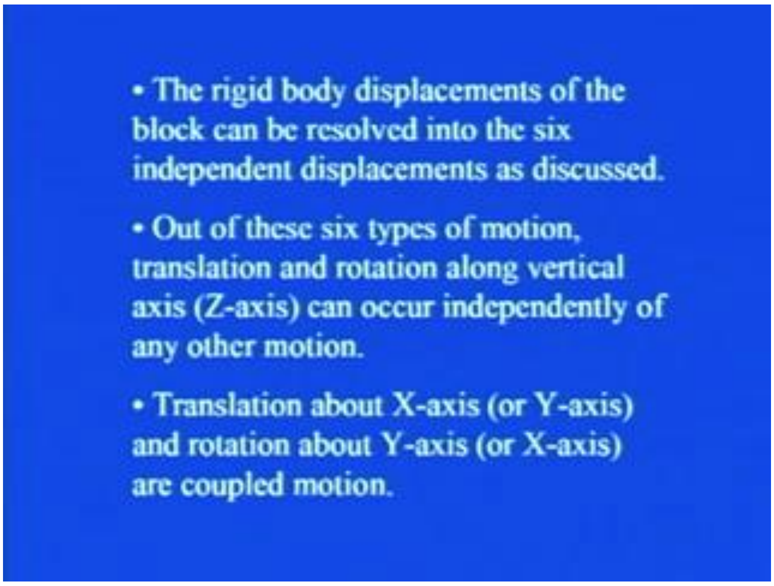
This is, what has been obtained from IS 2974 1982, which deals with the various aspects related to machine foundation. You can see that, here is the three dimensional plot, that is block and this direction is your X direction, that is in horizontal plain, perpendicular to this, this is Y direction and then vertical direction which is Z direction, is this. So, we have the translation along these three axis, so you can see, first if I take Z axis, which is P z that is, this is vertical one, that is P z, then lateral one is P x, which is in this particular direction.

You can see here, that, this is what is your X axis, this is Y axis and this is Z axis, this point O is the C G of this particular block, so this, any translation which takes place along this particular axis X is lateral transition and along Y axis, which is this P y, it is longitudinal one, then along vertical axis, it is P z. Then, rotation about this X axis, that you can see by this manner, from this armed arrow, that is m x, that is a moment which has been applied along X axis.

So, this will cause the pitching motion of the block foundation, then along Y, that is this, this will cause rocking motion of the foundation and along Z axis, if it is rotating along Z axis, that is in horizontal plain, then the motion is called as yawing. So, you have to keep in mind, that what, whenever we use these terminology, whether lateral, longitudinal , vertical, pitching, rocking and yawing, you should be able to make out that what exactly we are taking of.

Yawing is the rotation about Z axis, pitching is the rotation about X axis, however rocking is the rotation about Y axis.

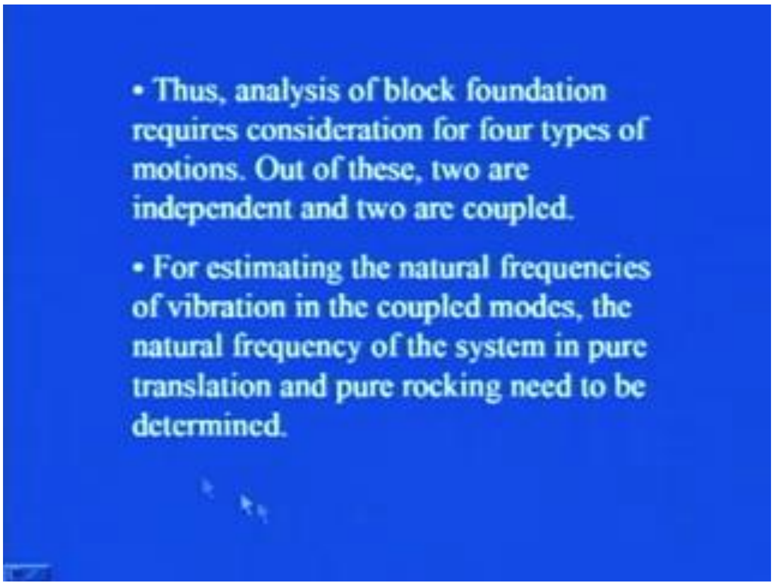
(Refer Slide Time: 36:30)



So, the rigid body displacements of the block can be resolved into these six independent displacement as we discussed, that is the translation and rotation about the three axis. Now, out of these six types of motion, translation and rotation along vertical axis, which is Z axis, can occur independently of any other motion. That is, the either the translation in X and Y direction or rotation in X and Y direction, so the translation and rotation along Z axis, they can occur independently of any other kind of motion.

Then, translation about X axis or Y axis and rotation about Y axis or X axis, so two combinations are there, translation about X axis and rotation about Y axis and another combination is, translation about Y axis and rotation about X axis, they are coupled motion.

(Refer Slide Time: 37:31)

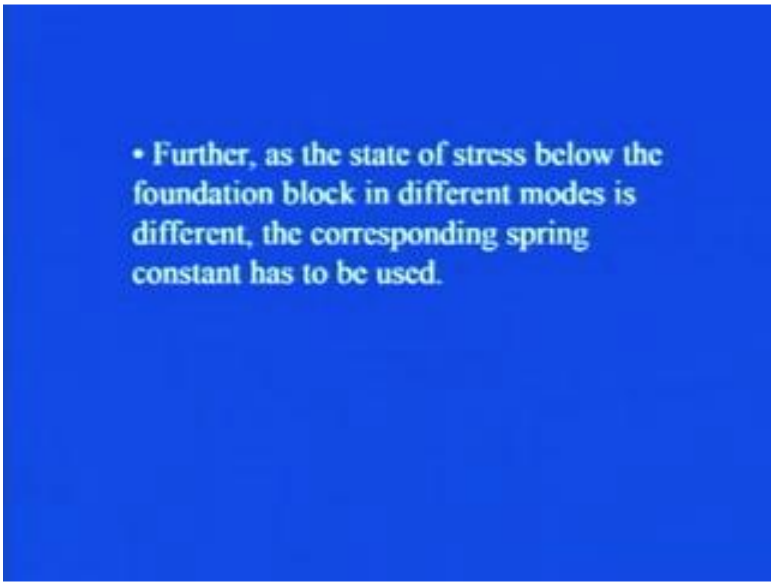


Thus, the analyses of block foundation requires, consideration for four types of motion, out of these, two are independent and two are coupled. As you see, you just now saw, that the translation along Z axis and the rotation along Z axis, they are independent and then, these two, that is translation about X axis ((Refer Time: 37:57)) and rotation about Y axis and translation about Y axis and rotation about X axis, they are the coupled one. So, you have two independent and two coupled, this motion, for the analyses of block foundation.

So, for estimating the natural frequencies of vibration in the coupled mode, the natural frequency of the system in pure translation and pure rocking, need to be determined, right. So, since the thing is that, when you are talking of that, rotation about X or Y direction, either pitching or rocking will come into picture. So, since they are coupled motion, first we need to know, that what exactly is the natural frequency, in only translation, that is pure translation and pure rocking condition.

How we do that, that we will see in subsequent slides.

(Refer Slide Time: 38:56)

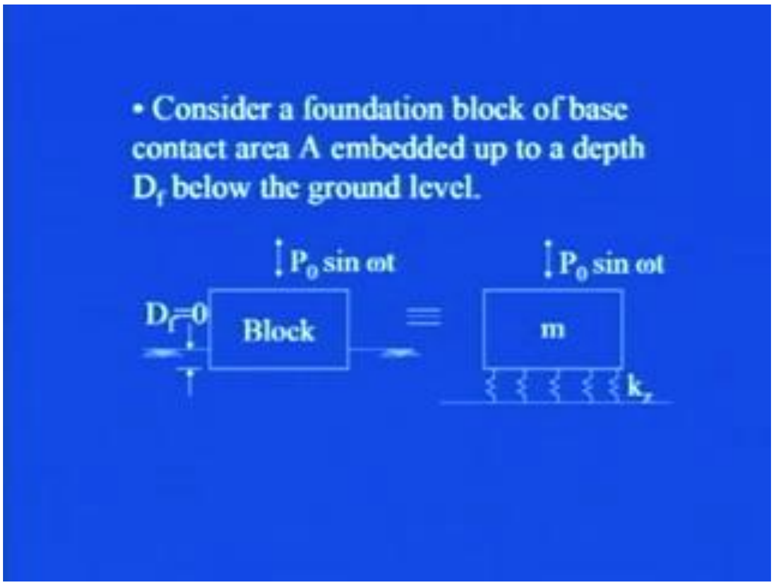


But further, as the state of a stress below the foundation block, in different modes is different, the corresponding spring constants to be used. You must be wondering, that why we are so much intend to find out the natural frequency in all different types of vibration or the mode of vibration. Because, you know that, depending on the mode of vibration or depending on the degree of freedom, you have to get the corresponding equivalent spring constants.

So, first we have to identify that, which type of motion is being there and then, what are the different combination that can be there. So, for all the combination, we have to get the corresponding equivalent spring constant, so that, we can find go ahead in the analyses procedure. Now, before finding this equivalent spring constant, for that, you need to know the natural frequency corresponding to each and every motion type, so that is what, we are attempting here.

So, first let us try to have a look, that how, what will be the natural frequency or how we can estimate natural frequency, as far as vertical vibration of a block foundation is concerned.

(Refer Slide Time: 40:19)



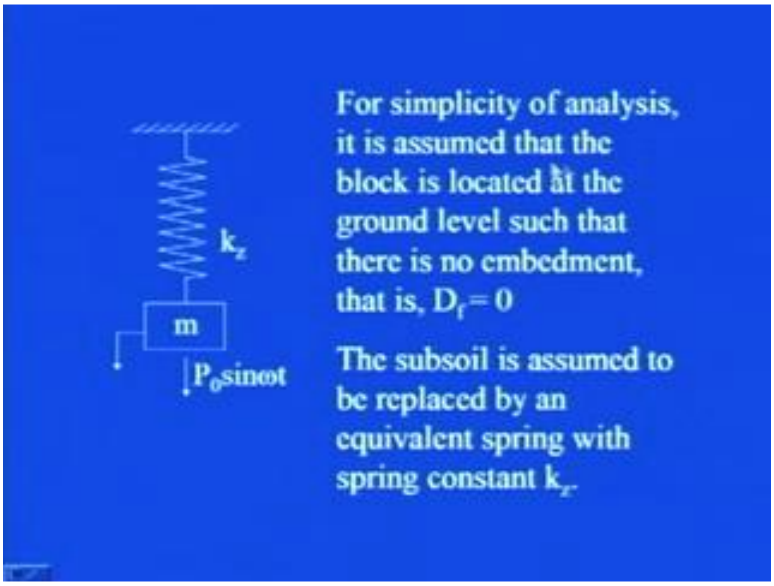
You can see here, that, you consider this block, of contact area which is A, embedded up to a depth D f, below the ground level, so you can see here, that this is the ground level and this block is embedded as D f. Now, why I have put here, as D f is equal to 0, because you know, when you analyze and you have to make some assumption to make the analyses simpler. So, to make the analyses simpler, this D f has been assumed to be 0, which I will be explaining you in the subsequent slides, when we will discuss that.

And then, this block is subjected to this force, that is P naught sin omega t, so this is what is the physical system, that, this is the soil which is lying here and with some embedment depth, the block is being casted, which is subjected to the external force, which is P naught sin omega t. Now, as I told you that this soil can be modeled as the springs, the series of springs.

So, how it is idealized, has been shown in this particular figure, that this soil, this particular soil, which is lying below the block has been replaced by equivalent springs of stiffness k z. See, we are talking of vertical vibration of a block foundation, so vertical direction is Z direction that we are considering. That is why, this subscript z is coming into picture here, when we are talking this stiffness of these springs. So, this k z is the stiffness of the spring, which are representing the soil, which is lying below this block

This block is having the mass m, so if you remember that, in one very first lecture, we discussed that linear weightless spring mass system. So, we have tried to idealize this whole physical system with the help of these spring and mass system and then, this mass which is representing this particular block, this mass m is subjected to this external exciting force of magnitude, this p naught sin omega t. Now, let us try to see, that how we can analyze this particular system and can get the natural frequency of this system.

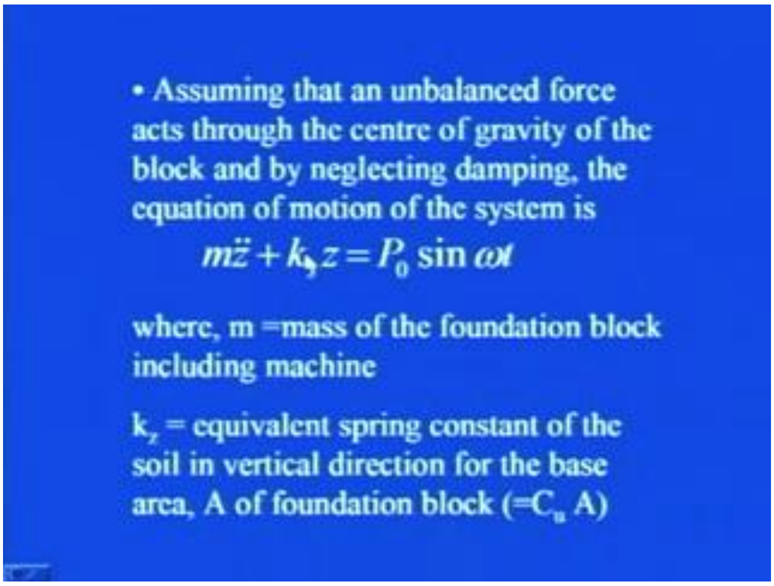
(Refer Slide Time: 42:48)



For simplicity of analyses, it is assumed that, block is located at the ground level, such that, there is no embedment, that is D f is equal to 0. So, as I was explaining you earlier, that just to make the analyses simple, this embedment depth has been assumed to be 0. However, for all practical purposes, this depth of embedment is not zero, but usually for theoretical point of view we have to assume, some simplification to get the analyses more simpler.

Now, you see that, the picture which was in inverted form, ((Refer Time: 43:25)) in this particular manner can be implied here like this. That, this is the rigid base and then, this is the soil, mass is there and which is subjected to this load p naught sin omega t. So, the subsoil is assumed to be replaced by an equivalent spring with spring constant k z, which you can see here, that, it is representing you the subsoil, m is the block or the foundation, that is with mass m and p naught sin omega t is the external force, which is acting on this particular block.

(Refer Slide Time: 44:04)

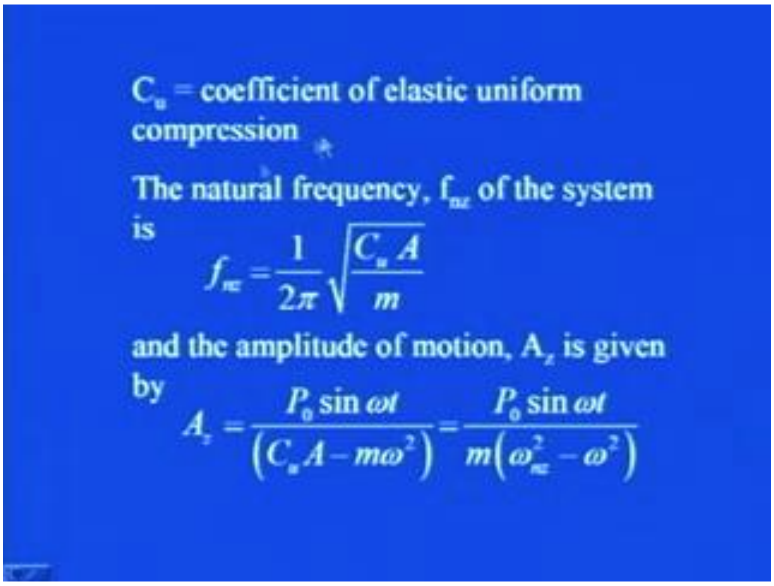


So, assuming that an unbalanced force, acts through the center of gravity of the block and by neglecting damping, the equation of motion of the system can be written as m z double dot plus k z into z equal to p naught sin omega t. If you remember that, in very first lecture, we discussed it how we can get this particular equation, that is summation of all the forces were equal to the mass into the acceleration.

So, you can see here, that this was the external force, this is the force, which will be there in the system, due to the spring which is representing the subsoil with this stiffness k z. So, and this m into z double dot, z double dot is the acceleration, which is the differentiation of z with respect to time, that is twice if you differentiate z with respect to time, you will get this z double dot. So, this m z double dot plus k z into z is equal to p naught sin omega t becomes the equation of motion, in the absence of damping.

Where, m is mass of the foundation block including the machine, k z is your equivalent spring constant of the soil in vertical direction for the base area, A and you now, that co, that the different coefficient that we found out was C u, C tau, C psi and C phi. So, from the definition of those, we can get the value of this equivalent spring constant as C u into A.

(Refer Slide Time: 45:40)

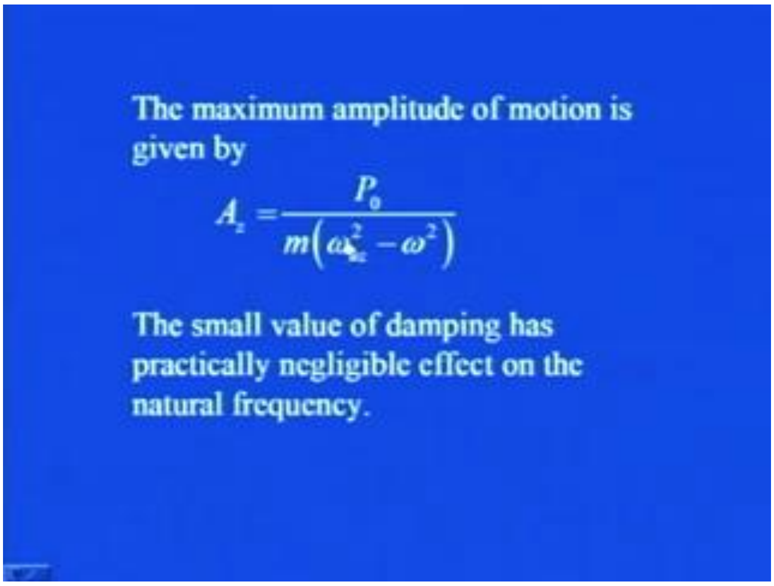


Where, C u is coefficient of elastic uniform compression, so in this case, ((Refer Time: 45:47)) if you try to solve this particular equation as you, we have already done in first few lectures. Then, you will be getting the natural frequency of the system, which is f n z as, 1 by 2 pi square root of C u into A by m, where this C u into A, see, it is as similar to the forced system in the absence of damping. So, that would have, it was there, we saw earlier that, it was 1 by 2 pi square root of k by m and here, k is equal to k z.

And that is equal to C u into A, so I am simply replacing that particular expression and the natural frequency in that case will become, 1 by 2 pi square root of C u A by m. Similarly, the amplitude of the motion, that is A z will be given by this particular expression, that is p naught sin omega t divided by C u A minus m omega square. Now, if I take this m to be common, so this will become, that C u A by m and that C u A by m, you can see here, that this C u A by m becomes 4 pi square f n z square and 2 pi into f n z is nothing but omega n z.

So, that is how, you are getting, once you take this m common, out of this particular bracket, you will be getting m omega n z square minus omega square. So, your A z, the expression for A z will become, p naught sin omega t divided by m into omega n z square minus omega square. So, this is how, you can find out the natural frequency and the corresponding amplitude for vertical vibration of the block foundation.

(Refer Slide Time: 47:40)

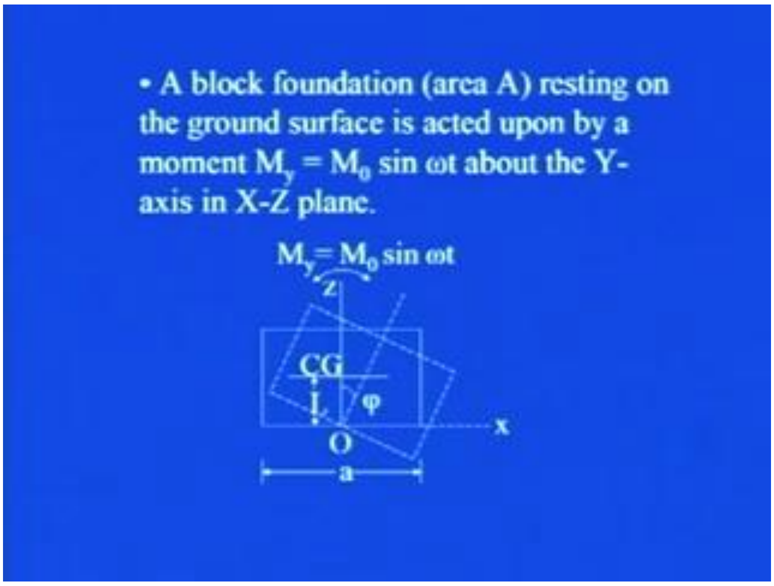


Then, in case, we want to find out the maximum amplitude, so maximum amplitude will occur when the sin omega t is equal to 1, so that will become p naught divided by m omega n z square minus omega square. So, a small value of damping has practically negligible effect on the natural frequency, that we have already seen, when we were discussing that forced system and free system. There we saw that, damping very is, if the damping value is quite small, it has negligible effect on the system. Now, this was what about, that vertical vibration of the block foundation.

(Refer Slide Time: 48:25)

Now, if we see that rocking vibration of a block foundation.

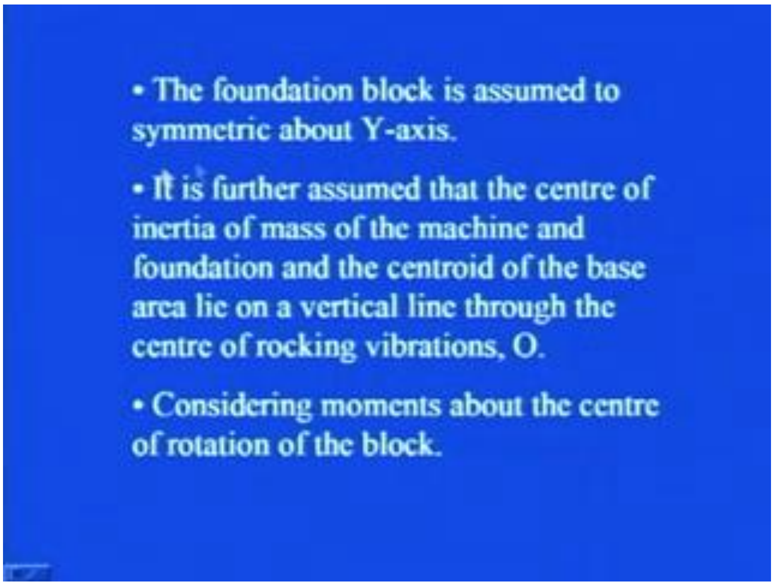
(Refer Slide Time: 48:28)



So, rocking vibration means, that, you are applying the rotation along Y axis, so you can see here, in this particular figure, that if it is a block, this is your Z direction, this is X direction and in this particular direction, you have this y direction. So, a block foundation with area A, resting on the ground surface is acted upon by a moment M y, which is equal to M naught sin omega t, about the Y axis in X Z plane. So, you can see here, that this is what, is your XZ plane.

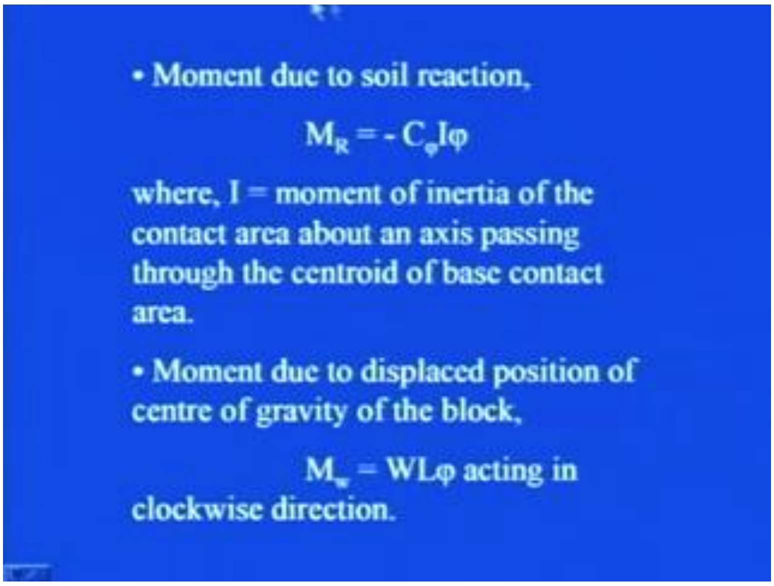
And in perpendicular to that, if you apply the moment, the rotation of the block will be taking place, in like this manner, which has been shown by the dotted ones and then, if this angle is equal to phi, that is the rotation angle is phi. So, how we can analyze this particular situation, because you have to get the corresponding value of soil spring constant. And for that, you have to first get the natural frequency, when this particular type of vibration is occurring.

(Refer Slide Time: 49:46)



So, the foundation block is assumed to be symmetric about Y axis, it is further assumed, that the center of inertia of mass of the machine and foundation and the centroid of the base area, lie on a vertical line, through the center of rocking vibrations, O. ((Refer Time: 50:03)) You can see here, that, this is what is the center of the vibration, that is O, then considering moments about the centre of rotation of the block.

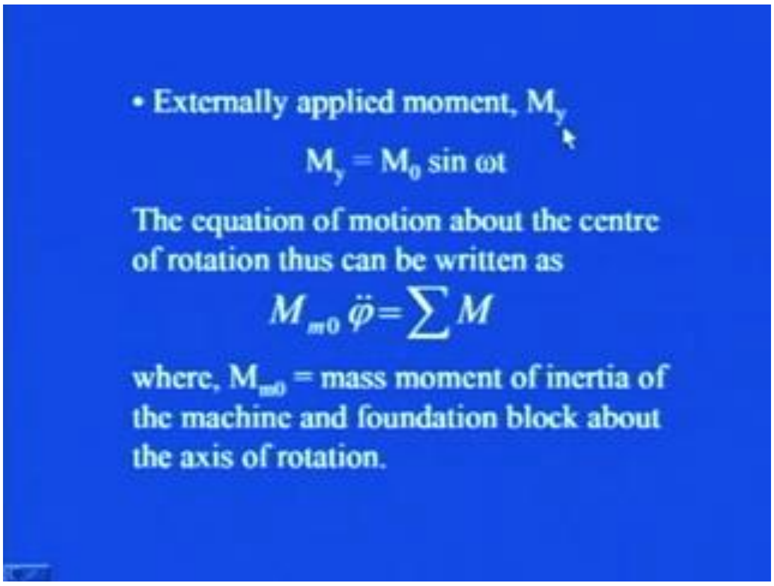
(Refer Slide Time: 50:15)



We get, the moment due to the soil reaction to be equal to, minus C phi into I into phi, where I is the moment of inertia of contact area about an axis, which is passing through the centroid of base contact area. Then, moment due to displaced position of center of gravity of the block is M w, let us say, which is equal to W into L into phi, which is acting in clockwise direction. That is why, we are putting it with minus, with positive sign here.

However, here we put that negative sign, that means that, it was acting in the anti clock wise direction, as you saw, that in case of the translation, we were taking the equilibrium of forces. However, here since we are subjecting this particular system to the rotation along Y axis, so that is why, we will be taking the moment equilibrium and that is how, we are trying to find out, that what exactly are the different moments, which will be there, when this is subjected to the rotation about the Y axis.

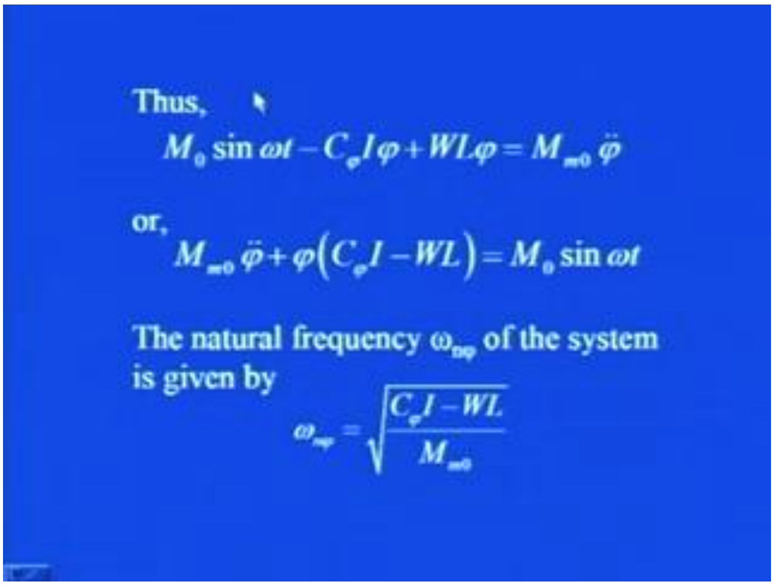
(Refer Slide Time: 51:32)



Then, externally applied moment, you know, it is M y, which is M naught sin omega t, so the equation of motion about the center of rotation can be written as, M m o phi double dot is equal to summation of M, summation of M gives you the summation of all the moments, where phi double dot gives you. Since here, it is, you must be wondering that, why it is not Z double dot here. See, in earlier case, it was translation, so that Z was coming into picture.

However, in this case, the deformation is rotational mode, so it is the angle which will be coming into picture and it was getting displaced by an angle phi. So, you have to differentiate phi with respect to time, to get this quantity that phi double dot, where M m o is mass moment of inertia of the machine and foundation block about the axis of rotation.

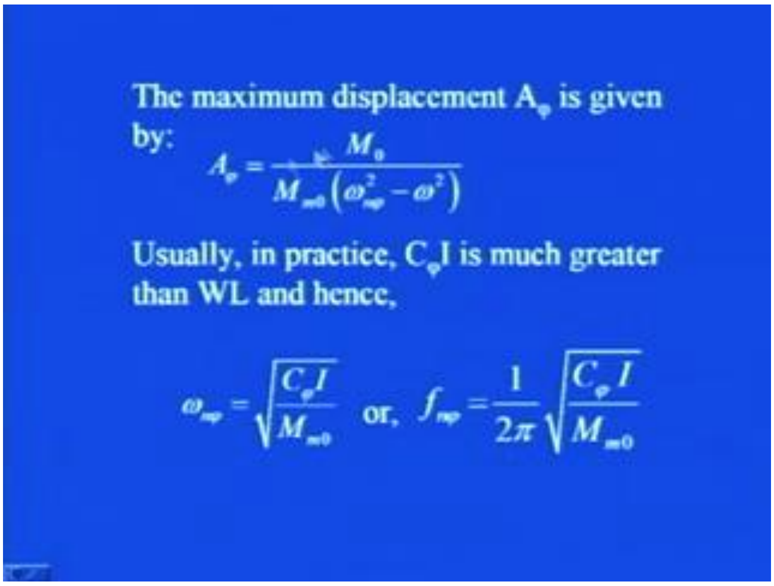
(Refer Slide Time: 52:37)



So, by putting all the moments in this particular equation, we will be getting this particular expression. This is the external moment, then we had the moment, as I told you earlier that C phi I into phi, which was in anticlockwise direction and plus W L phi, which was in clockwise direction. And that will become equal to the internal one, that is the moment and whatever is the acceleration because of that, that is M m o into phi double dot.

And this equation can be rewritten in this particular form and if you solve it, in the usual manner as you have already done in case of vertical vibration of the block foundation, then, you will be able to obtain the natural frequency of the system, which is represented by omega n phi, to be equal to, a square root of this C phi I minus W L divided by M m o. So, this is how, that you can get the natural frequency of the system.

(Refer Slide Time: 53:38)



So, the maximum displacement A phi is given by, that is A phi is equal to Mo divide by M m o omega n phi square minus omega square, now, usually in practice, it has been seen, that C phi I is quite greater than W L. So, you can neglect this particular term, that is C phi I, it is quiet high, so you can neglect this W L term, ((Refer Time: 54:08)) in this particular expression.

So, if you do that, the resulting expression you will get here as, C phi I by M m o or that natural frequency, that you can get as 1 by 2 pi of omega n phi and that will be this particular expression. So, in this class we saw, that how we can get the measure of damping, using the damping factor in case of free vibration test and in case of forced vibration test and after that, we saw, that how we can go for the analyses of block foundation.

And in that, we saw that, there are basically six degrees of freedom, that is translation and rotation about the three coordinate axis X, Y and Z. However, out of them, we saw that there are four independent vibration modes, out of which, two were in independent to each other and then, two were the coupled motion. So, it became necessary for us, that what exactly is the natural frequency, corresponding to all of these vibration mode, to evaluate the corresponding equivalent spring constant.

So, we saw, in that sequence, we went ahead and then we saw that, how we can get the natural frequency and the maximum amplitude for vertical translation of block foundation and for this moment about the Y axis, that is the rocking motion, so and the subsequent one, that is the remaining ones, we will see in the next class.

Thank you.