Foundation Engineering
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Module - 2
Lecture - 13
Machine Foundations -1

Hello viewers good afternoon, today we will be starting a new chapter, named Machine Foundation. So, first let us, try to see, that what exactly, do this kind of foundation mean, why they are necessary, what are the different types of machine foundation, what are their salient features, we will be discussing, all these aspects detail in this particular class. So, let us, first try to see, that why, exactly machine foundations are necessary, what type of the forces, which come on this kind of foundation.

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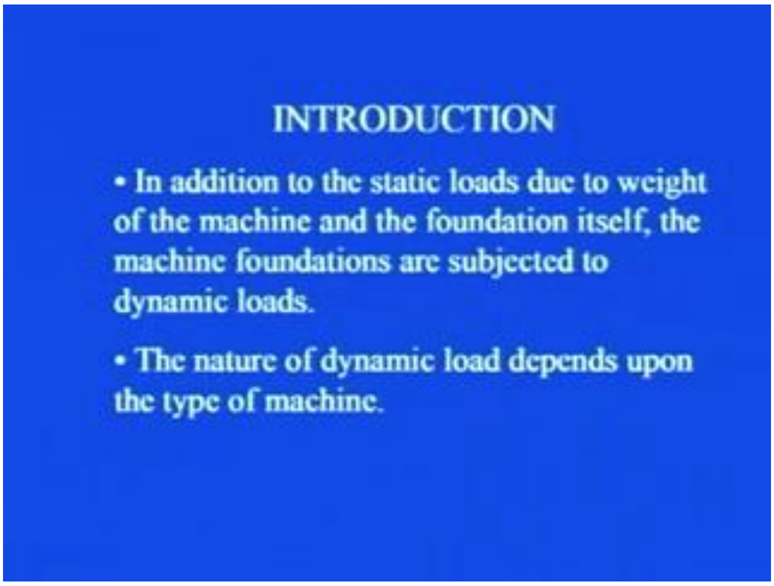

So, coming to the introduction part of the, same that in the addition of static loads, due to the weight of machine, and the foundation itself, the machine foundations, are subjected to dynamic loads. Let us say, that there this, some machine or motor kind of thing, which vibrates while operating, so in that case, that particular foundation will be subjected to dynamic loads, so in that case, you have to provide this machine foundation.

The nature of the dynamic load, depends upon the type of machine, so there are various type of machine, as far as their functioning are concerned, and corresponding to that you have to choose the appropriate type of foundation.

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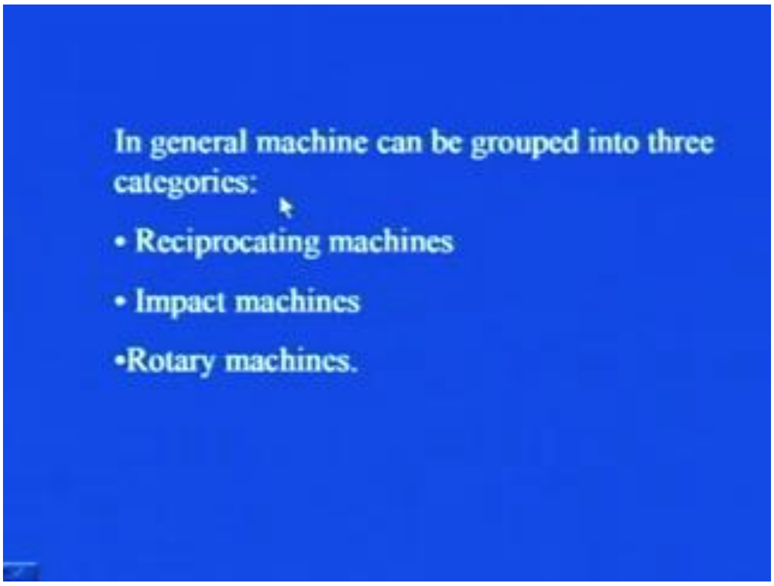

In general, machine can be grouped into three categories they are reciprocating machines, impact machines, and rotary machines. Now, let us try to see, one by one, the salient features of these three machines.

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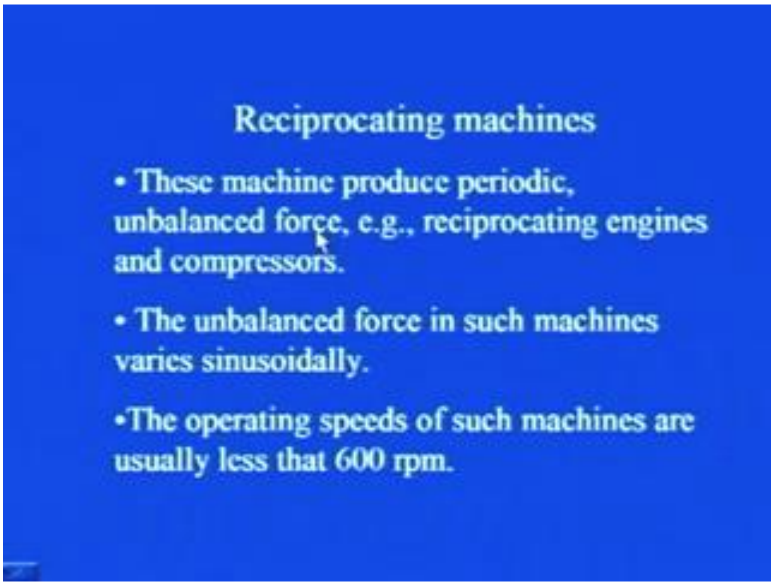

First reciprocating machines, these machines, produce periodic unbalanced force, for example, reciprocating engines and compressors. So, the machines, like reciprocating engines and compressors, they fall under the category of reciprocating machines, the unbalanced force, in such machines, vary sinusoid ally, that is it follows the sin curve, the operating speed of such machines, are usually less than 600 rpm.

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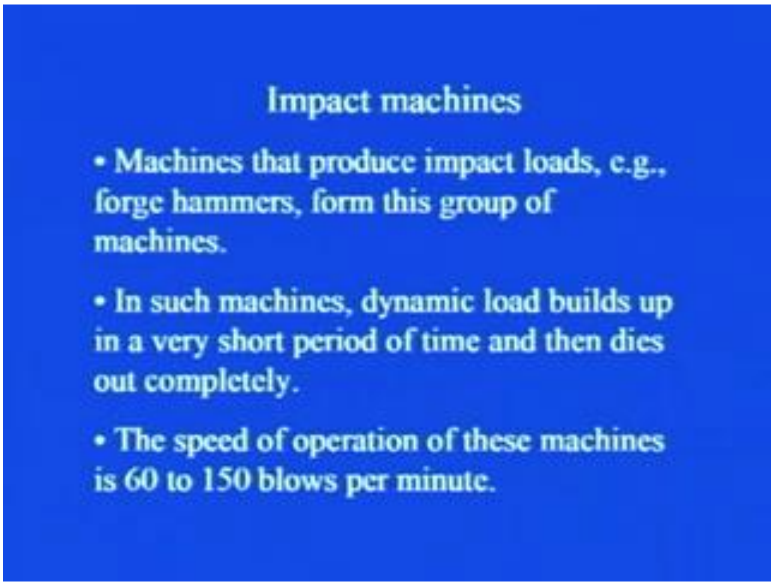

Now, impact machines, as the name suggest, these produce impact loads, for example, forged hammers, form this group of machines. In such machines, dynamic loads, they build up for very small duration of time, and after that they, completely die up. Then the speed of operation of these, machine is 60 to 150 blows per minute.

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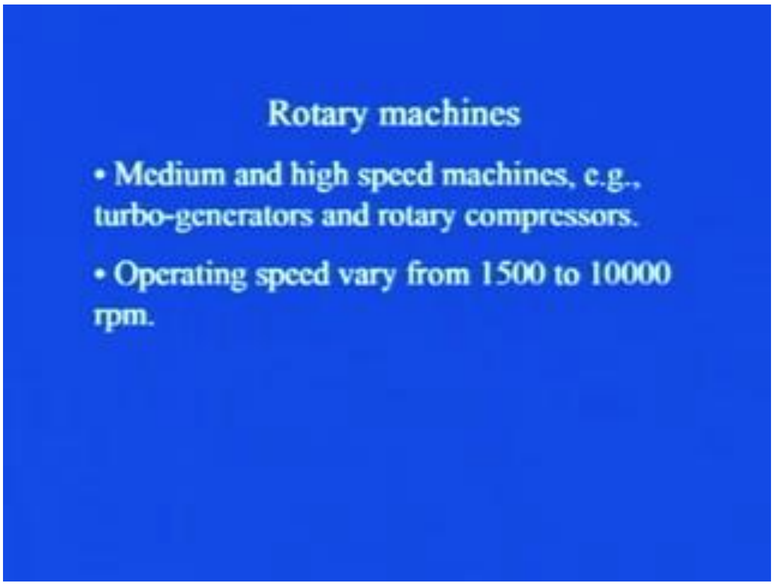

Then, coming to the rotary machines, medium and high speed machines, for example, turbo generators, and rotary compressors, and their operating speed, vary from 1500 to 10000 rpm. So, you see, that what exactly is the basic difference between these, three type of machine, first thing is that their operating speed, and the another one case, is that how what exactly are the different type of loads, which are going to come on these three type of machines.

As, you saw that, in case of reciprocating machine, the loading was sinusoid ally, in case of impact machine, it was the load, was that it dies out completely. However, in case of rotary machines, they are medium and high speed machines, and the operating speed, varies between 1500 to 10000 rpm.

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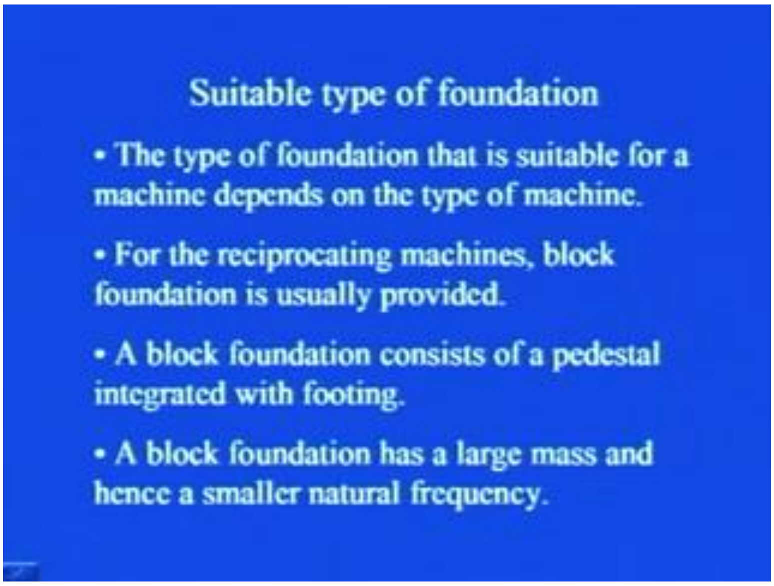

Now, depending on, which type of machine, you have to install, for that you have to choose suitable type of foundation. For, that some of the guide lines, have been given, which are as follows, that the type of foundation, that is suitable for, any type of machine, will depend on the type of machine. So, whatever is the type of machine, you have to choose, the type of foundation accordingly.

For the reciprocating machines, blocked foundation is usually provided, now we are introducing another term, as block foundation, what exactly is this let us, try to have a look. A block foundation consists of a pedestal integrated with footing, then a, block foundation has a large mass, and hence a smaller nature frequency.

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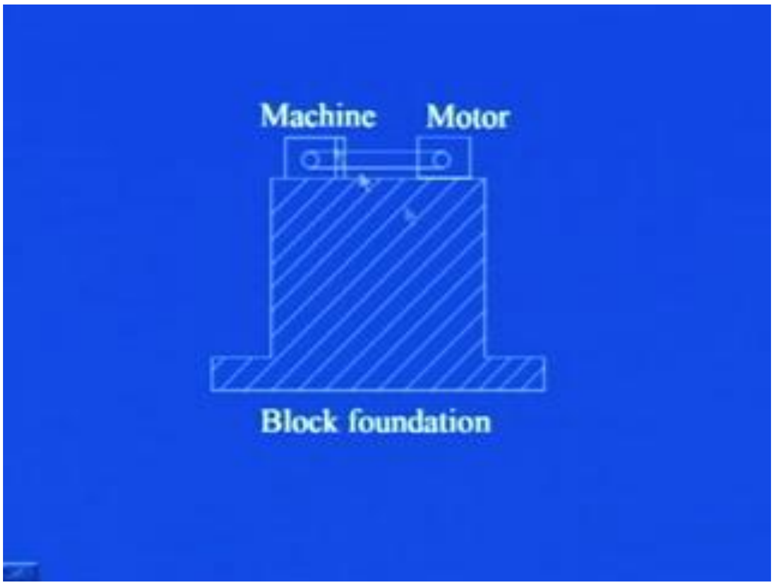

So, you see, you can see here, this is the pictorial view of block foundation, you see it forms kind of block, and on this one this machine as well as the motor, they are mounted, it is mass is quiet high, and so it is natural frequency is less.

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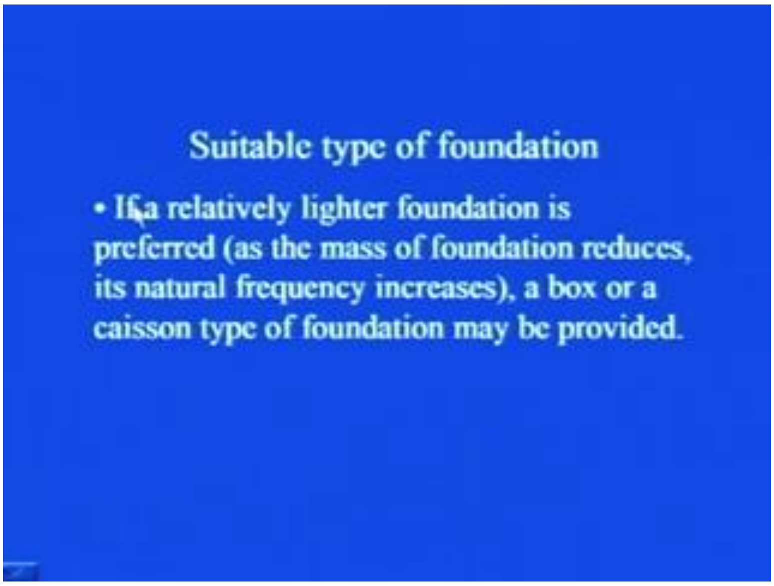

Now, if the relatively lighter foundation is preferred, because when the mass of the foundation reduces, it natural frequency increases. So, in case, you require such type of foundation, that you want more natural frequency, so in that case, you have to reduce the mass of the foundation. So, in that case, a box or a caisson type of foundation may be provided.

How is that ((Refer Time: 05:36)), you see here, this is a kind of box, there is no material in this particular area, so that reduces significantly, the mass of the foundation. That is, what is the difference between, the block foundation and the box foundation it is, only its, mass.

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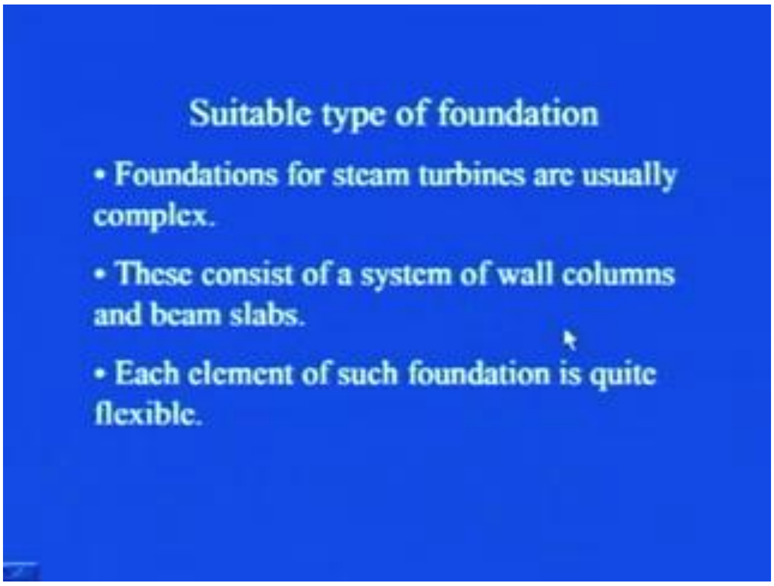

Then, foundation for steam turbines are usually complex, these consist of a system of wall columns and beam slabs. Each element of such foundation is quite flexible. So, you have seen in, that case of block foundation, the mass is very heavy, and if we have to go for lighter foundation, then you have to go for box foundation. However, both the foundations are quiet rigid in nature, and if, the type of machine, is such that, that you cannot go for such rigid foundation, in that case, you have to go for flexible kind of foundation.

And, in that case, the foundation consists of a particular type of system, in which wall columns are there, and beam slabs are there, how it looks like, let us have a look.

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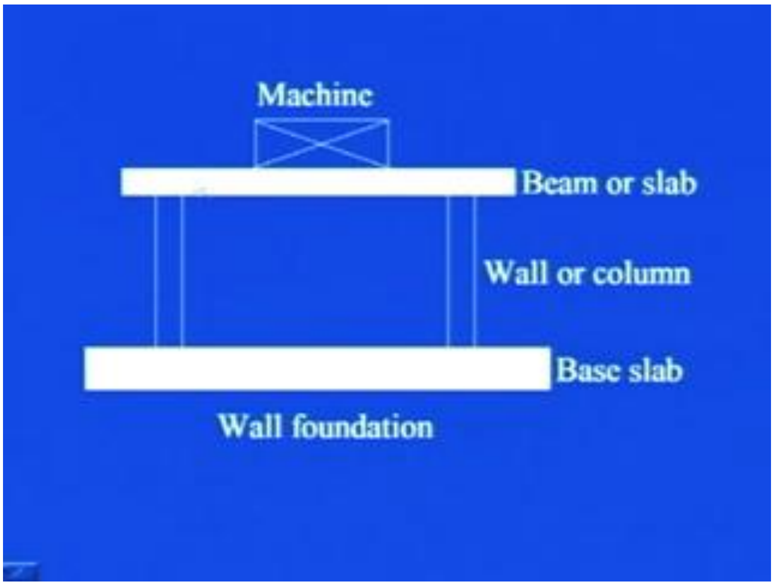

You see here, this is beam or a slab, this is wall or column, and this is base slab, which will be resting on the soil. So, that comprise of, you can see here, that each and every, part of this particular system, they are, although they are integrated together, to form this wall foundation. But, if you have a look, on the individual columns they are quiet, on the individual component of this particular foundation, they are quiet flexible, and on top of that this machine is mounted.

So, this is, ((Refer Time: 07:22)) suitable for steam turbines, which are usually complex, as you can see that, to work this all together, one unit, this construction should be monolithic, and to have, that kind of thing, it will be really, a complex thing.

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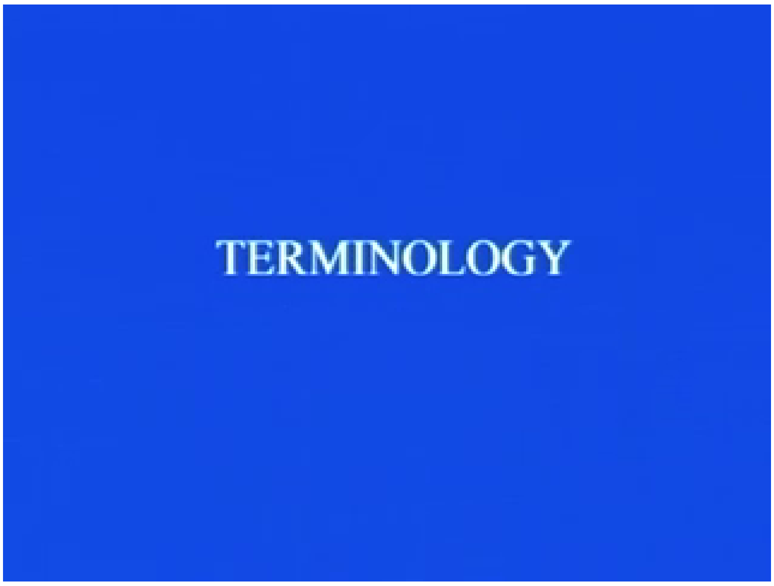

So, first let us, try to have, the look on this terminology, and what exactly do we mean, by the particular terms, that what exactly, are the definition of those terms. Let us, try to have a look one by one.

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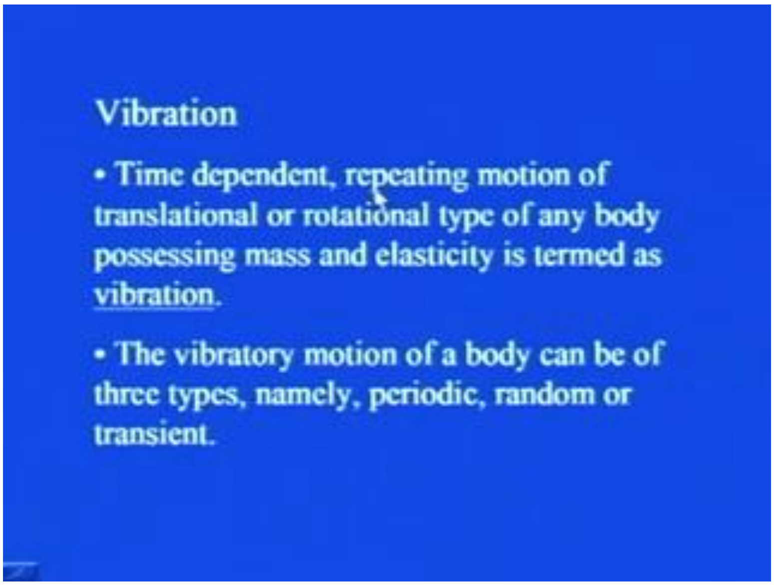

First is vibration, so time dependent, repeating motion of translation or rotational type of anybody, possessing mass and elasticity is termed as, vibration. So, whenever I will be using, this term vibration, you must have, this picture in your mind that time dependent behavior, repeating motion, of may be of translation, or rotational type of any type of body, which is possessing, some mass and the elasticity, so whenever I say, vibration this picture should come into, your mind immediately.

The vibratory motion of a body, can be of three types, namely periodic, random or transient, we will be seeing, that what exactly do we mean by, these particular terms, what exactly is the definition of these three terms, in subsequent slides.

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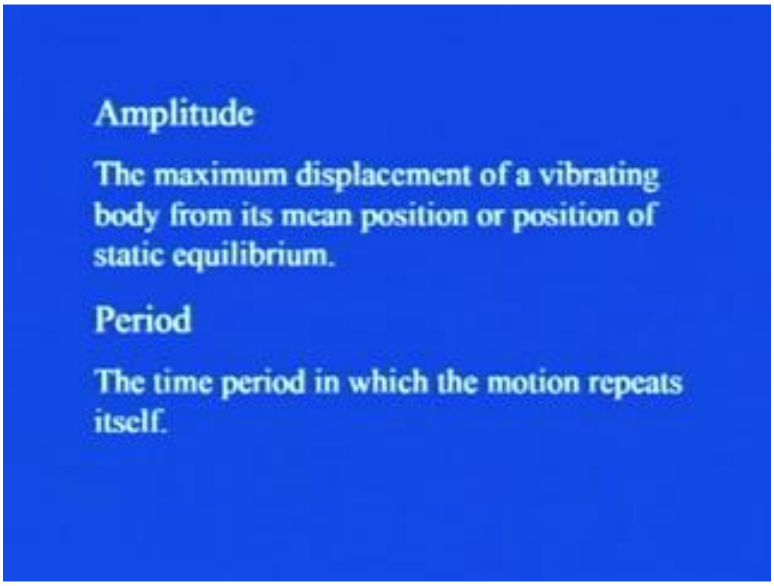

Then, we will be talking of amplitude, what exactly, does it mean, the maximum displacement, of a vibrating body, from it is mean position or position of static equilibrium. You, if you think of, a pendulum of a wall clock, then you see that it is, static position is the vertical one, and in case since it vibrates, along it is both side of it is, that particular position.

So, the maximum displacement, that it attains is known as, amplitude, so the maximum displacement of any vibrating body, from it is mean position, that is known as amplitude, period, the time period, in which the motion repeats itself.

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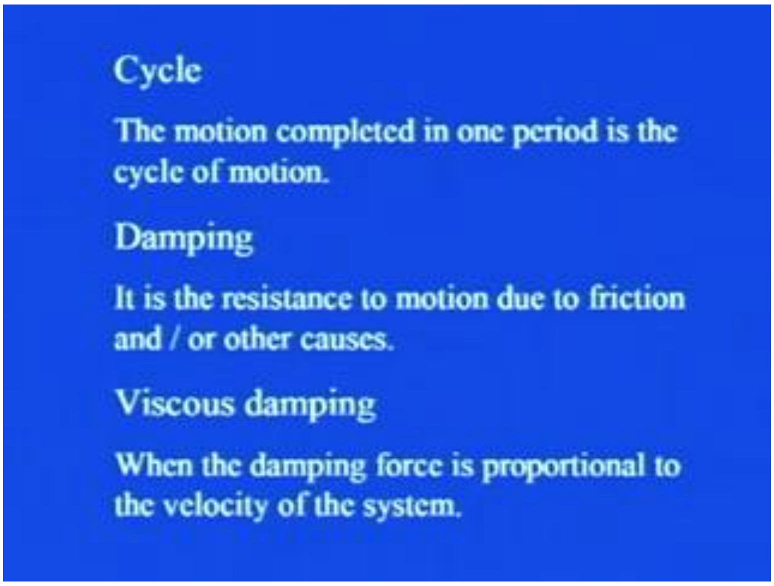

Cycle, the motion completed in one period is the cycle of motion, so period is, that when the motion is repeating itself, and the cycle is motion completed in, one particular period, is your cycle of motion. Damping, it is the resistance to motion, due to friction, on and or, any other causes, so you see that, in case, if there is no damping generally in nature, in all the system, there is damping, otherwise any vibrating body, would be vibrating for infinite time.

Viscous damping, when the damping force is proportional to the velocity of the system, in that case, we call that damping as viscous damping.

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Then degree of freedom, that number of independent coordinates, required to define a vibratory system, you see, that this degree of freedom, which we will be using here, is with respect to vibratory system. However, you, when you study anything in coordinate mathematics, then you in two dimensional, you require two coordinates to, mention or to locate any of the particular point, so that is, what is known as, it is degree of freedom.

However, in this case here, we talk in terms of, the independent coordinators, coordinates, which are required to define any vibratory system that is what is called your degrees of freedom. ((Refer Time: 11:35)) Free vibration, vibration of a system, when it is displaced, from it is equilibrium position, and it is left to vibrate, so you see, you simply, deflect or displace, that particular body, from it is equilibrium position, and simply you leave that, and then when it will be vibrating, that vibration is called free vibration.

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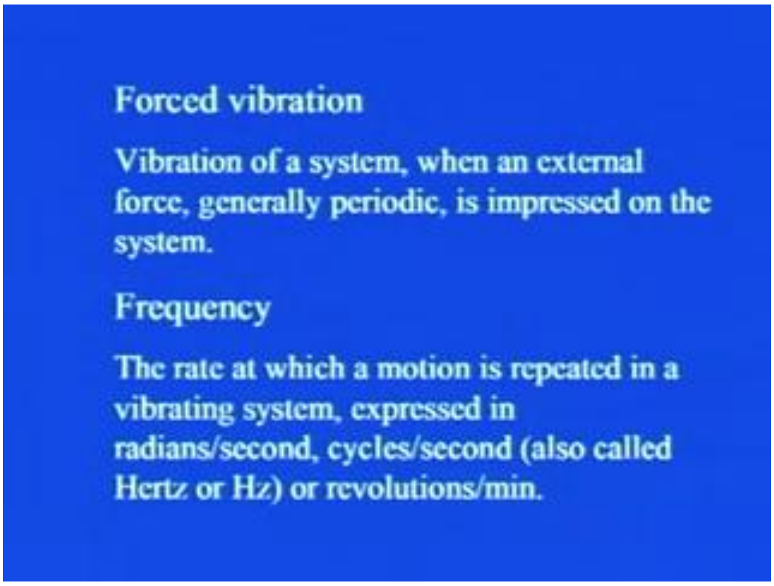

Then, after free vibration, forced vibration, vibration of a, system when an external force, generally periodic, is impressed on the system. So, let us say, that a system, you see a swing kind of thing, a person, who is sitting behind that, he just pushes that, and then it vibrates, so the person is applying that external force, so that kind of vibration is called forced vibration.

Then frequency, the rate, at which a motion is repeated, in a vibrating system, expressed in radians per second or cycles per second, which is also called as Hertz, and represented as Hz, or revolutions per minute. This is an, very important term, which we will be using, in subsequent slides again and again, so you must, be clear about the definition of these terms.

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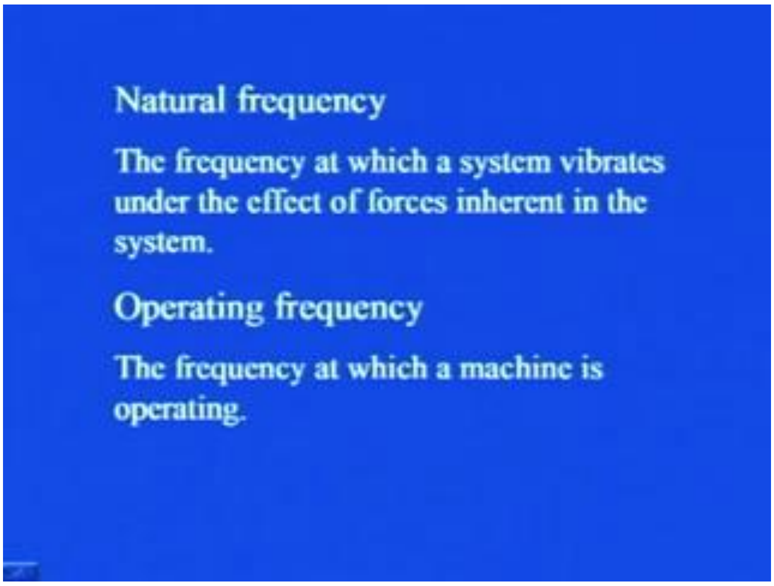

Then, we will be using one more term that is natural frequency, it is the frequency, at which a system vibrates, under the effect of forces, which are inherent in the system. So, whatever is the force, which is inherent in the system, because of that, when the body is vibrating, so that, particular frequency is called as natural frequency. Then, operating frequency, the frequency, at which a machine is operating, is called as operating frequency.

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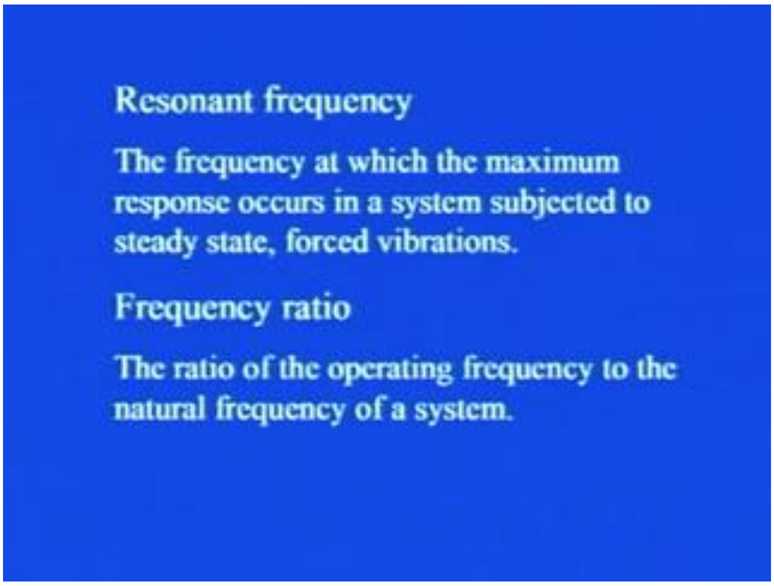

Then resonant frequency, the frequency at which the maximum response occurs in a system, subjected to steady state or forced vibration. So, whenever, the maximum amplitude is occurring, or maximum response is occurring, corresponding to that maximum response, whatever is the frequency, that frequency is called as resonant frequency. Then frequency ratio of the system, see natural frequency, is the property of any system, however, the operating frequency is the manufactured one, that at, what frequency the machine is operating, so the ratio is known as frequency ratio.

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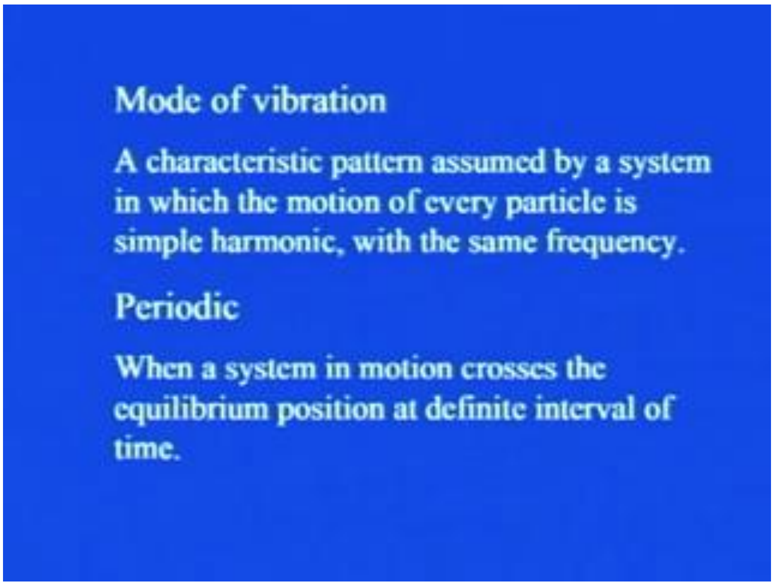

Then, mode of vibration, a characteristic pattern, assumed by a system, in which the motion of every particle is, simple harmonic with the same frequency that is known as mode of vibration. That is, a particular pattern, in which the anybody, is vibrating, or it is every particle of, that particular body will be vibrating, that is periodic, when a system in motion, crosses the equilibrium position at, definite interval of time that kind of motion is called as periodic motion.

That means, at particular interval the time, the motion will be crossing its, equilibrium position, let us say, if you take, the case of a pendulum. You, we you can, find out that, when it is crossing the vertical position, which is the position of its equilibrium, so that particular, if it is following that particular, or it is coming or crossing to that equilibrium position, in that particular time, then, that kind of motion is known as periodic motion.

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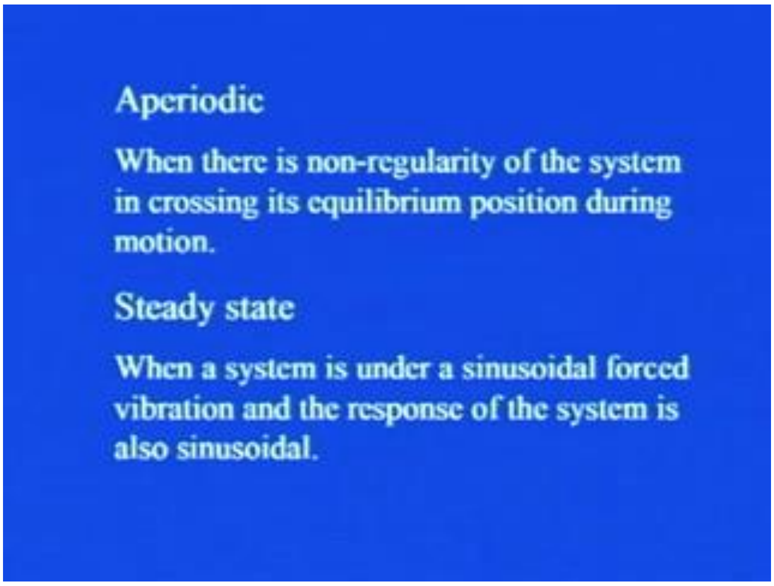

Then aperiodic motion, when there is non-regularity of the system, in crossing it is equilibrium position, during motion that is called aperiodic. That is, it is not, that only after this much particular, interval of the time, it will be crossing it is, equilibrium position, the crossing of the equilibrium position, may happen at, irregular time intervals in a periodic motion. Steady state, when a system is under a sinusoidal force, vibration and the response of the system is also sinusoidal, in that case, it the state of the system is known as steady state.

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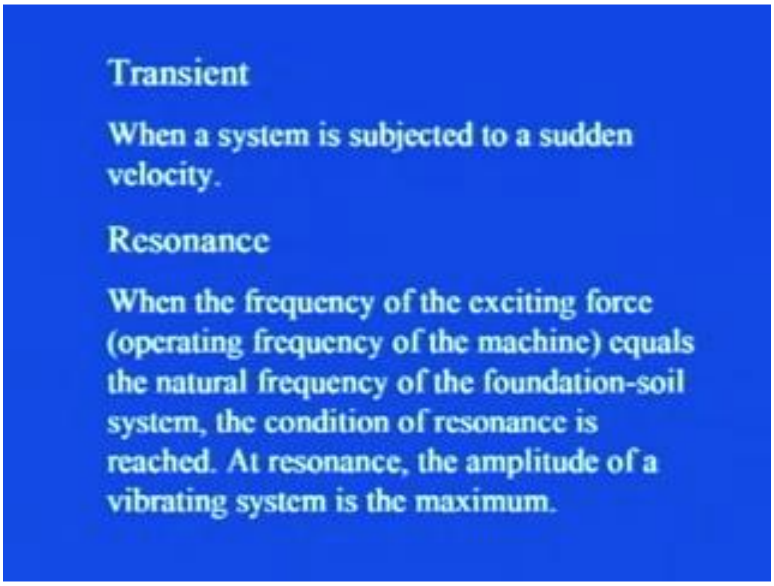

Then, transient state, when a system is subjected to a sudden velocity, then in that case, the response will be known as transient response. Then, there is one very important term, which is known as resonance, what exactly does it mean, let us try to see, at when the frequency of the exciting force, that is the operating frequency, in case of machine, equals the natural frequency of the foundation soil system, the condition of resonance is reached.

That means, that whatever is the frequency, at which the machine is operating, then it becomes, equals to the natural frequency of the foundation soil system, in that particular case, the resonance condition is reached. At resonance, the amplitude of the vibrating system is the maximum, as I told you, when we were discussing about the resonant frequency, it was that maximum response of a system, you get at resonant frequency. So, similarly, at the response condition or when the response is reached, during that time, the amplitude of the system will be the maximum.

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Now, after knowing, all these terminology let us try to have a look that what exactly is the design criteria, for any machine foundation, to perform satisfactorily. Because, you have seen that, when the resonance is reached, the amplitude is maximum, obviously, we would not like to, reach to the resonance condition. So, all these things, we have to keep in mind, there are other conditions along with this resonance condition, let us try to see, that what exactly are those criteria.

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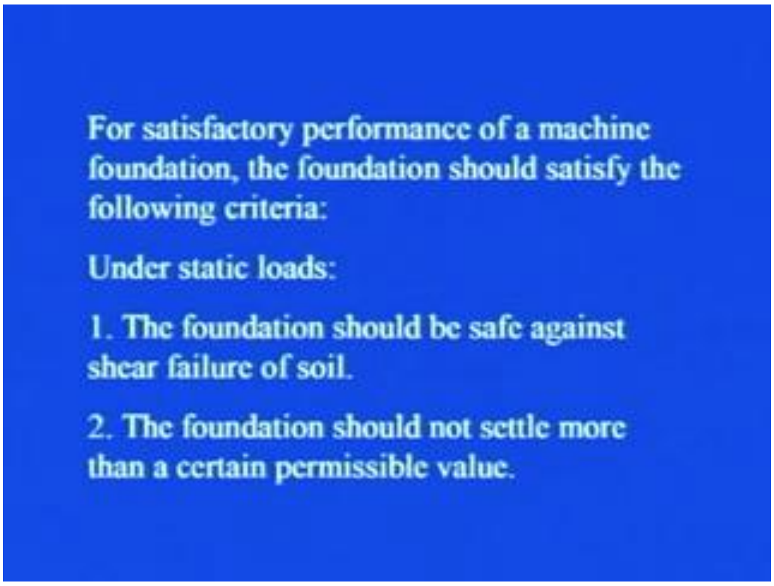

For, satisfactory performance of a machine, foundation the foundation should satisfy the following criteria. As you know, that in case of machine foundation, it is subjected to static as well as dynamic load, and you have seen, the analysis in case of, static loads, when you were studying, shallow foundations and de foundation or pile foundation. However, in case of machine foundation, additional dynamic loads are also there, so first let us, try to have a look that, under static load, what should be the criteria, that machine foundation must satisfy.

That is, that the foundation should be safe against, shear failure of soil, and another one is foundation should not settle, more than a certain permissible value. See, all these things, you have already studied in shallow foundation and pile foundation chapter. Exactly, on the similar lines, for machine foundation, since it is subjected to a combination of static loads, and dynamic loads. So, we have to, take care of both the type of loads, and such that, it is not failing under static loads, as well as, it not failing under dynamic load.

So, for static load, it should be safe against safety that sorry, safe against shear failure of soil, plus it should be serviceable. And, how you confirm this serviceability condition, is that the settlement of the foundation should not exceed the permissible settlement of that, particular type of foundation.

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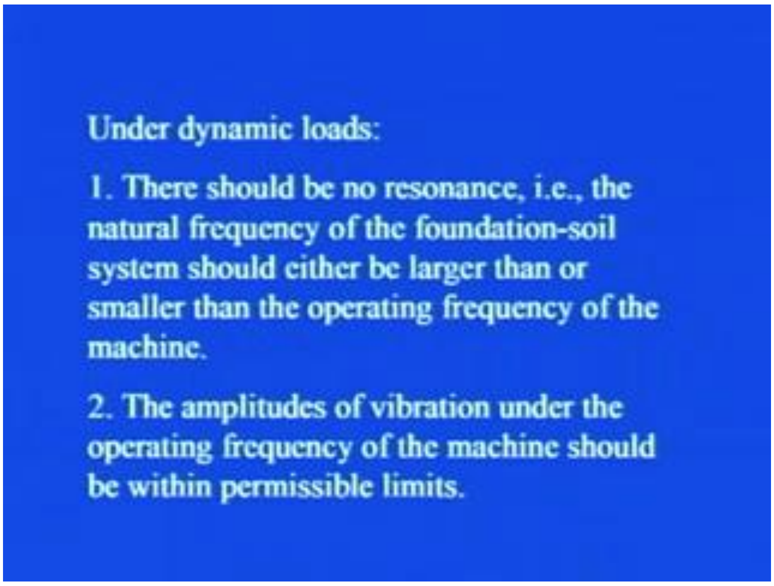

Now, let us try to see, that what are the conditions, under dynamic loads, there should be no resonance, that is, the natural frequency of the foundation soil system, should either be large or smaller than the operating frequency of machine. You have seen the definition of resonance, that at resonance the operating frequency of the machine, and the natural frequency of foundation soil system, they are same, and at resonance, the response of the system or the amplitude of the system, is maximum.

Obviously, we would not like to reach this condition, because when the deformation or the displacement is maximum that is an undesirable condition, as far as the foundation is concerned, so the one condition is, that there should be no resonance. Then, the amplitudes of vibration, under the operating frequency of the machine, should be within permissible limit, as it was there, for case of static loads, that the settlements should be within permissible limit, under dynamic loads, the amplitude of vibration, under operating frequency of the machine should be, within permissible limit.

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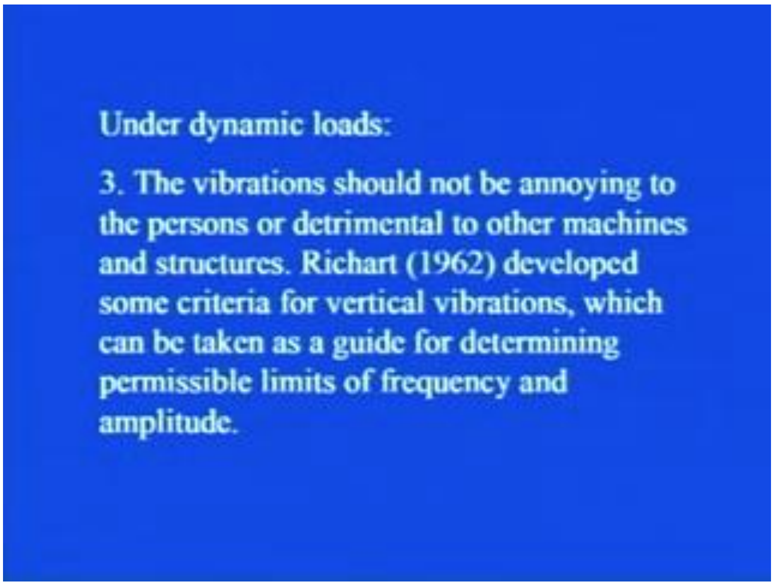

Then, the vibration should not be annoying to the persons, or detrimental to other machines or structures. You see, wherever you will be, operating that particular machine, there will be somewhere near by foundation or nearby structure or some persons will be, working nearby area. So, condition of the machine or the vibration of the foundation, should not be such that, that it annoys the persons, who are working around, that particular foundation or the it should not be, hazardous or detrimental to other machine foundations, which are in nearby area.

Then, to avoid this kind of thing, that Richart 1962 developed some criteria for vertical vibrations, which can be taken as a guide, for determining permissible limits of frequency and amplitude. So, these are the standard things, which are available in any of the standard text book, for the scope of this particular subject, as far as your level is concerned, it is beyond the scope of our study, however, one should always, know that such kind of thing exist.

So, three criteria are there, for machine foundation to work satisfactorily, or to such that its design is satisfactory, first is that there should not be any resonance, resonance condition should not be reached. The second one is, that amplitude, on with operating speed of the machine, should not be more, than it should be less than permissible limit, and then the vibration should not be, annoying to persons or detrimental to the other machines, which are there in nearby area.

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Then, let us try to introduce this theory of linear weightless spring, now you must be wondering, that from where exactly. this thing is coming, all of a sudden. You see, we are talking of foundation and soil system, so we first have to have a look, that how we can represent, this soil foundation system, in quantitative measure. So, you see here, that any simple continuous system, can be represented by an equivalent spring, so that is why, we are trying to convert or we are trying to model, the soil foundation system, with the help of an equivalent spring.

So, that, we can get, or we can quantify various things, because right now, when we are taking of, that resonance or natural frequency, we are not taking in any of the numeric values. So, how we can say, that whether then, this resonance condition has been reached or not, we have to, see to it that, how we can model this. So, that is why to model the soil foundation system, you can a replace, the soil by equivalent spring and some mass attached to it, will represent the foundation.

So, that is why, before going to that advance thing, first we should try to understand, the basic theory of linear weightless spring, and how it represents the soil and the mass, which is connected to it as foundation. So, first we talk of, single degree freedom system, that is free vibration, free vibration we have already seen, that in the absence of any external force or the vibration of the system, due to the forces, which are inherent in the system, there is no external force.

And, degree of freedom, also you have seen, that whatever is the coordinates, required to define any particular type of system, under vibration.

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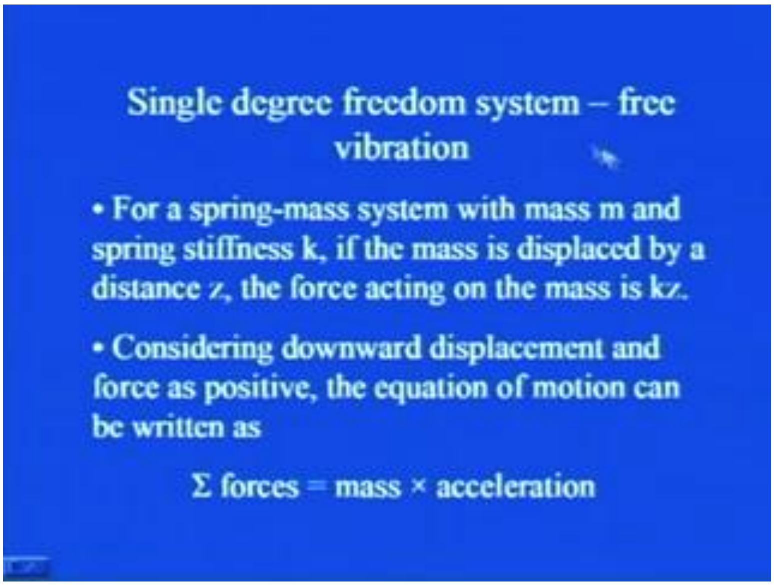

So, here, when I say single degree freedom system that means the degree of freedom is one, that is one particular coordinate is sufficient enough. To define, the location of the system, which is vibrating, with under this condition, that is free vibration is taking place, there is no external force. So, let us, see that, how we develop this model, for a spring mass system with mass m, and spring stiffness k, if the mass is displaced by a distance z, the force acting on the mass will be k into z.

That we know, from the very basic principle of mechanics, we can write that force, which will be acting on the masses k into z, because you know that, for this spring, this is very well known thing. Then, you consider, downward displacement and force as positive, the equation of motion, can be written as, that summation of all the forces should be equal to the inertia force, which is mass into acceleration. Now, let us, try to see, with the help of a figure, that what exactly do, we mean by, this single, this is a single degree freedom system, with free vibration and this spring mass system.

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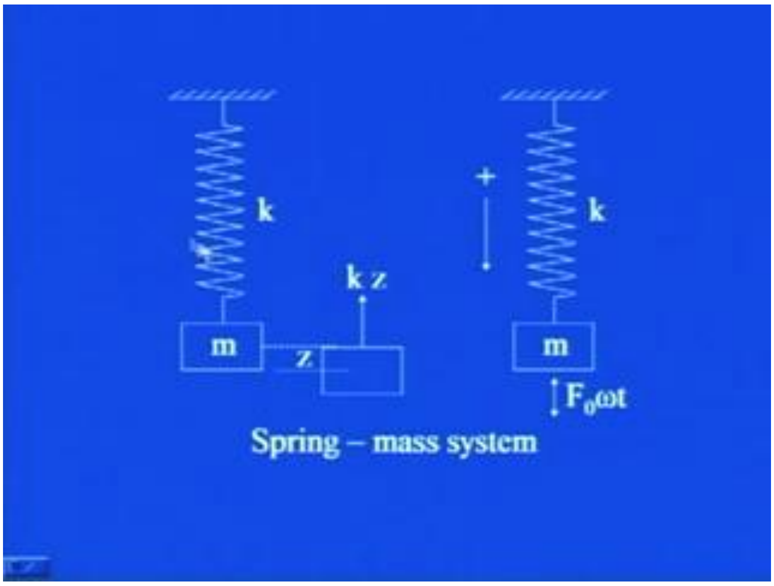

You see here, the soil here has been replaced by, this spring, equivalent spring of stiffness k, and a mass m is attached to it. This mass has been displaced let us say, that this dotted line is showing, the, it is equilibrium position, it has been displaced, by this amounts, that is z from it is equilibrium position, then, the force, will be generated in this particular spring as it is reaction, and that will be in this particular direction, and it is magnitude will be k into z, where k is the stiffness of the spring, and z is the displacement of this mass, from its equilibrium position.

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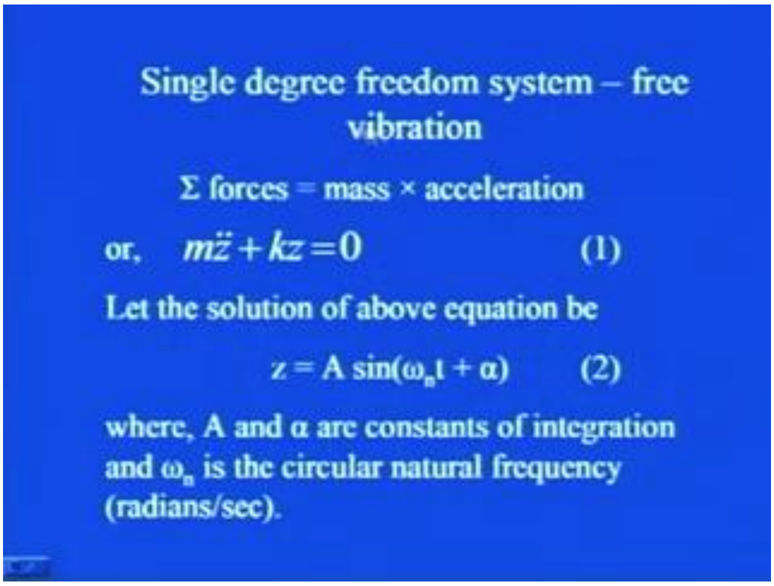

So, you see here, that, this one is under free vibration, then, summation of forces is equal to mass into acceleration, this we have seen. From there, if you see that, mass into acceleration, this z double dot, I am showing that, it is the derivative of, this displacement z, with respect to time, twice the double derivative. So, that is m into z double dot, plus k z, forces we have seen that only, k z is there, that is equal to 0, so this equation is resulting into this particular equation, that is m z double dot, plus k z is equal to 0.

Now, we assume that the solution of this particular equation is represented by z is equal to A sin omega n t plus alpha. Where, A and, alpha are constants of integration and w n is circular natural frequency, we have already seen, that what do we mean by this natural frequency, and it is dimension are, it is dimension is radians per second.

Then, we have this particular equation, you have z is equal to A sin omega n t, plus alpha. So, if you differentiate with differentiate this z, with respect to t once, you will be getting the velocity, and if you differentiate with respect to t twice, then you will be getting the acceleration. So, see, what exactly happens, because this z double dot, we can get from, this particular expression, because we need to find out this A and alpha, because they are constant of integration, we do not know about them, so we have to find them out.

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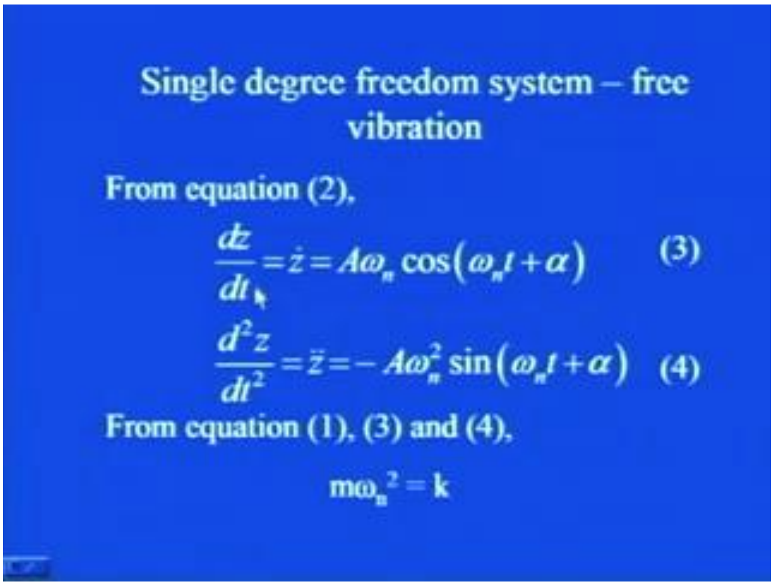

So, from the previous equation, you can get that, differentiating it once, that the d z d t is z dot, is equal A w n sorry, omega n cos omega n t plus alpha. Then, if you further differentiate this particular expression, that is d 2 z d t 2, will become z double dot, and this you know that cos alpha, when you differentiate that, becomes minus sin alpha. So, that is what is here, and this w n term will come, so that will make is omega n square, sin omega n t plus alpha.

So, if you combine, these two equations, with equation number 1 that is if you, put the expression of this z double dot, in that particular expression, you we will get, this particular equation, that is m omega n square, is equal to k.

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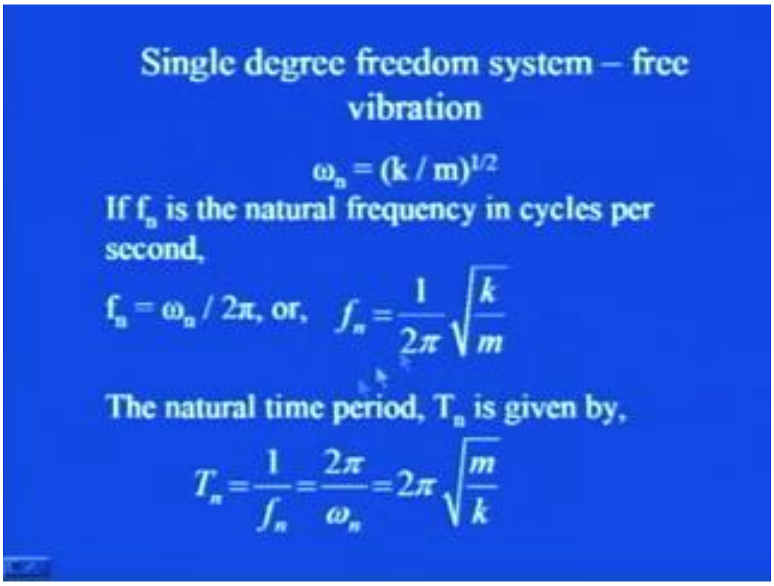

So, from here, I can find out, this omega n, which is equal to, you see here ((Refer time: 29:44)), omega n square will be k by m, and if you take this square root of, that you will be getting omega n, that is omega n is, square root of k by m. So, if say, that f n is the natural frequency in cycles per second, then we know that from fundamentals, f n is defined as omega n by 2 pi, so omega n, we know that, is it is square root of k by m, so fn that is natural frequency will be, 1 by 2 pi square root of k by m.

Then, the natural time period T n is given by, T n is equal to 1 by f n, that is 2 pi by omega n, and you can see here, from this particular expression, if you substitute here, you will get 2 pi square root of m by k. So, this is how, we can evaluate the natural frequency, because you know that, the soil property k you can find out, and then mass, that also, you can get the mass of the foundation, and so, you can get the natural frequency, and then the natural time period.

Now, that was all about, the free vibration, in case, there is the presence of some forced, external force, in that case, what we can do.

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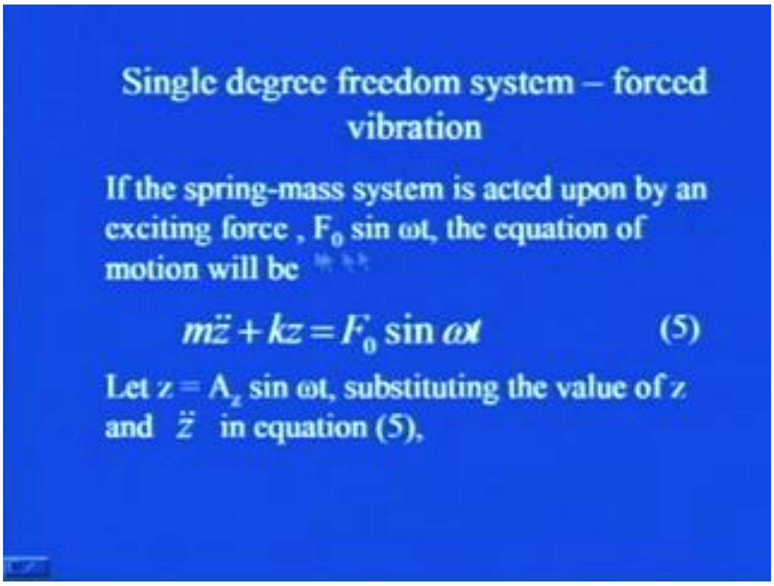

So, you have seen in pervious figure, that if the spring mass system is acted upon, by an exciting force of magnitude F naught sin omega t, the equation of the motion will be, again you have to use, that particular equation, that summation of all the forces is equal to mass into acceleration. So, here, that external force will be acting, in the opposite direction of this k z, so that will result, into this particular equation, that is m z double dot, plus k z is equal to, F naught sin omega t.

Now, let us say that its, equation is being given by, z is equal to A z sin omega t, then if you substitute the value of this z, and z dot the sorry, double dot. So, you see here, that is you differentiate it once, you will get z dot, you differentiate it again, you will get z double dot. So, from here, you can get that Z double dot, and you can simply substitute it here, as we did, in the case of free vibration.

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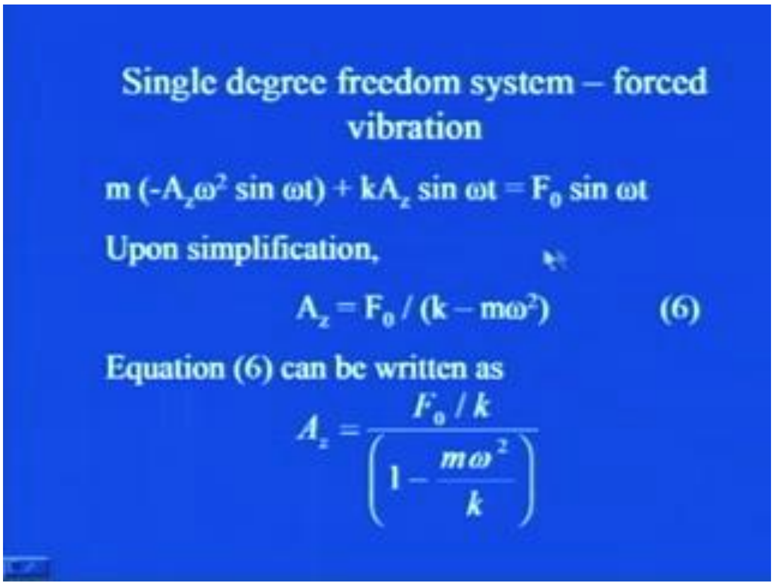

What it results into is, this particular equation, you differentiate that expression you will be ending up with this expression. So, n z double dot, plus k z, that is A z sin omega t, which is equal to F naught sin omega t. And, if you simplify this one, this expression, so sin omega t, you see is common. So, it will get cancelled out, as it is not equal to 0. So, simply, you will be left, with minus m A z omega square, plus k A z is equal to F naught, and A z if you take common, so you will be left with, k minus m omega square, that is equal to F naught.

And, then you can subsequently, get this A z is equal to F naught divided by k minus m omega square. Or, this equation can be rewritten as, you take k common, that is F naught by k, divided by 1 minus m omega square by k, you take k common out of this bracket. So, that will become A z, in this particular form, that is F naught by k divided by 1 minus m omega square by k.

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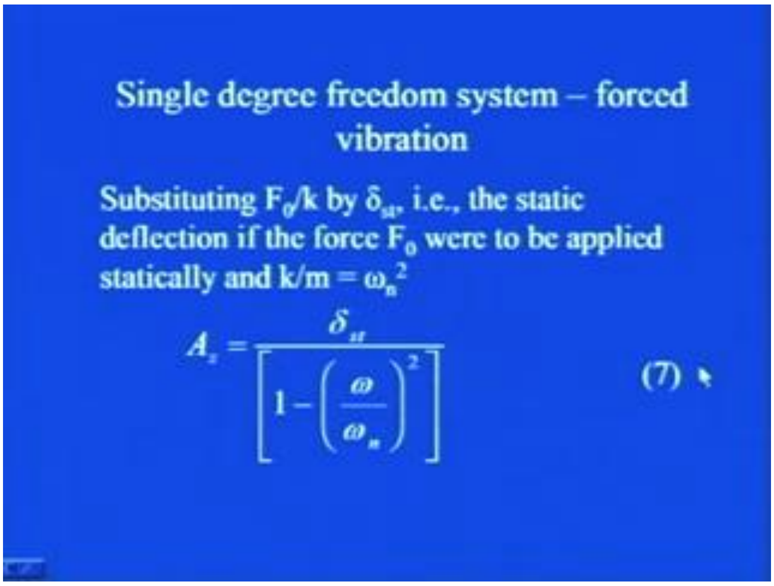

So, in the previous expression, you substitute this F naught by k by delta s t, now what is this delta s t, is that, the static deflection, if the force F naught were to be applied statically. You see, although it is subjected that to, that F naught sin omega t, but you imagine a case, that in case, you apply this F naught statically, that is no dynamic load, static load F naught, and then whatever is the corresponding deflection, that is delta s t. So, you see here, in this expression ((Refer Time: 34:04)), if you substitute as, delta s t then and then, we have just now found out, that omega n was square root of k by m.

So, omega n square, we can substitute here, you see here this, m by k term is coming, so if I take this m to be in denominator, so that will become k by m. And that k by m, we can write as omega n square. So, that by this substitution, the pervious expression will result into this particular simpler expression, which is A z is equal to delta s t, divided by 1 minus omega by, omega n whole square, where omega n is natural frequency.

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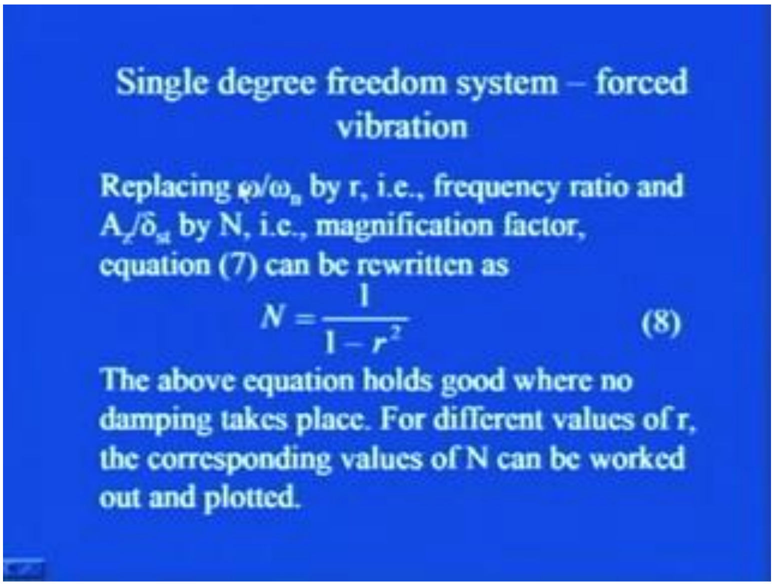

So, replacing omega by omega n by r, now, I am introducing this r, which I have already explained you, that it was the frequency ratio, that it is the ratio of operating frequency of the machine, to the natural frequency of the system. So, you see, omega is any operating frequency of the machine, and omega n is the natural frequency, so I am calling, this particular ratio by r, which is the frequency ratio. And, A z by delta s t by N, and that N, I am calling as magnification factor, delta s t is your static deflection.

So, in that case, this equation, that is this you see, A s t by delta z, and this delta s t, if you bring to other side, it will come in denominator. And, if you substitute this A z, by delta s t by N, so that will result into N. And here, omega by omega n, I am representing by r, so that way, simply become, N is equal to 1 upon 1 minus r square, so you see, we started with such a complicated expression.

And, we are getting, such a simple one, that is N is equal to 1 upon 1 minus r square, where N is the magnification factor, defined as the ratio of A z and delta s t, r is frequency ratio, which is defined as the ratio of operating frequency of the machine, and natural frequency of foundation soil system. The above equation holds good, where no damping takes place, because when we were, taking that summation of the forces is equal to mass into acceleration, the very first basic equation there we did not consider any force due to the damping.

So, this equation is valid for, the case when the damping is absent in the system. So, for different values of r, the corresponding values of N can be, worked out and can be plotted, so how it looks like, let us try to have a look here, in this particular figure.

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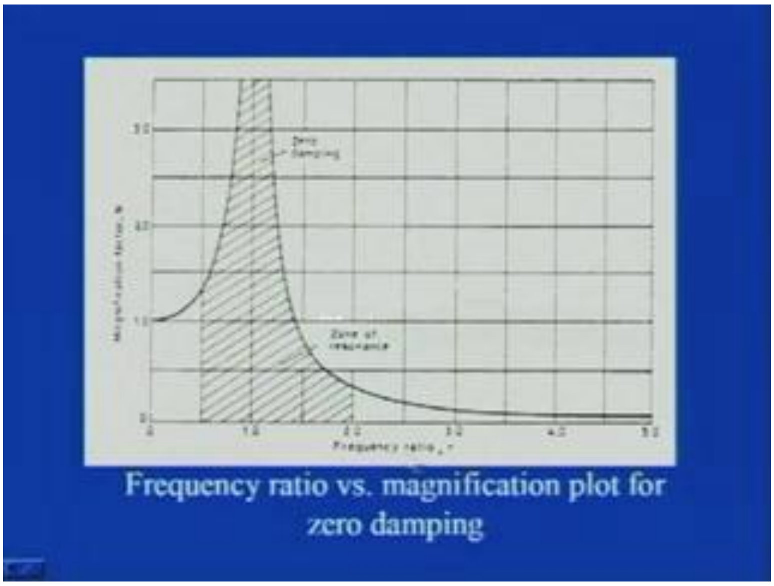

That on x axis, it is the frequency ratio, on y axis, it is magnification factor, so incase, the frequency ratio becomes 1. So, you see, if r is becoming 1, then 1 minus r is square will become 0, and the n, which is defined as 1 upon, 1 minus r square will tend to infinity. So, you can see here, that incase, when the r is equal to 1, the curves they are becoming, ((Refer Time: 37:35)) here, in this particular region, and they are tending towards infinity.

So, when r is less than 1, this is the curve for N, that is capital N, which is magnification factor, and for frequency ratio more than 1, this becomes the curve for N, that is from this one, you can get, the value of this magnification factor. However, this hash zone, represents the zone of resonance, that during this, that is beyond this, in the vicinity of r is equal to 1, as such r is equal to 1, is the exactly the position, where the resonance will be reaching.

But, in the vicinity of r is equal to 1, also it approaches to the resonance condition, so that is why, that nearby zone is called as the zone of resonance, so this hashed zone, is zone of resonance. You, have to keep this thing in mind, that this plot is for 0 damping, that is when the damping is absent, we have assumed, that there is no damping in the system, then only, you will be getting such kind of curves. Now, let us try to see, because you know that in nature, there is no system which exists, without the damping.

Otherwise, everything or every system, which is vibrating, would have been vibrating, for infinite time. So, what it is very important factor, and we must know, that what, exactly is the effect of damping, on any vibrating system. so let us try to see, that what, exactly is the effect of damping on the system.

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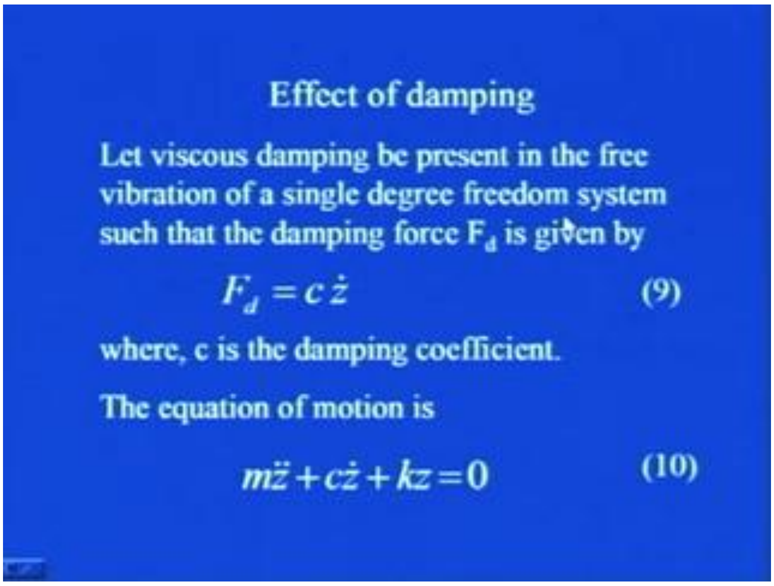

Now, I assume that, the viscous damping is present in the system, so first we talk of, free vibration of single degree freedom system, as we did, in earlier case, when the damping was absent we, analyze for free vibration and then we went for, the forced vibration. So, likewise in, when we are studying, this effect of damping then also, we first considered, that it is free vibration of a single degree freedom system, such that, the damping force F d, d this subscript, d stands for this damping.

F d is given by, F d is equal to c into z dot, where C is the damping coefficient, so this is what is the definition of this damping force, that it is defined as, the damping coefficient multiplied by the velocity, of the system, that is Z dot. This we can get, by differentiating Z, with respect to time once, so if you again consider, that fundamental equation, that summation of the forces is equal to mass into acceleration, so in earlier case, the force due to the damping force that was absent.

However, in this case, when we are considering the damping, this damping force F d will also be present, and that will make, this equation of motion as, m z double dot, plus c z dot plus k z to be equal to 0. Since, it is free vibration of a single degree freedom system that is why here, on right hand side, you are getting a 0.

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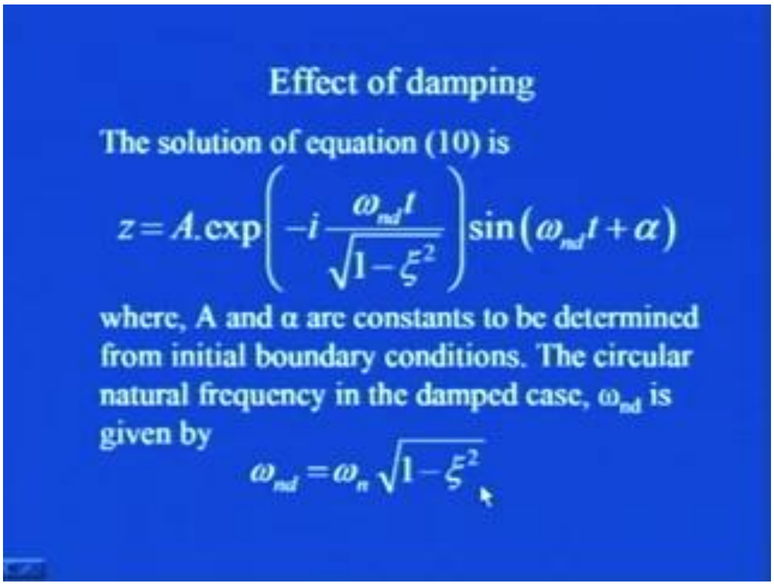

Then, it is solution, although you can derive this solution, but that is beyond the scope of your, this particular course. So, its equation, you can simply get that as, z is equal to A into exponential minus i omega n d, n d stands for natural frequency of the damped system, into t divided by square root of 1 minus zeta square, sin omega n d t plus alpha. Now, this A and alpha they are constants, to be determined from initial boundary conditions.

You, see here, we are talking in terms of this time is coming into picture, so that is why, initial condition come, because at time t is equal to 0, if system is at rest, so you will have that, at time t is equal to 0, the velocity is 0, that is z dot is equal to 0. So, all those condition, they help in, evaluating these two constants, that is A and alpha, the circular natural frequency, in the damped case, that is omega n d is given by, omega n d is equal to omega n square root of 1 minus zeta square.

This is an important expression and easy to remember, you do not need to remember, this particular expression, however, this is a very important one, you must remember this, that omega n d is, omega n square root of 1 minus zeta square.

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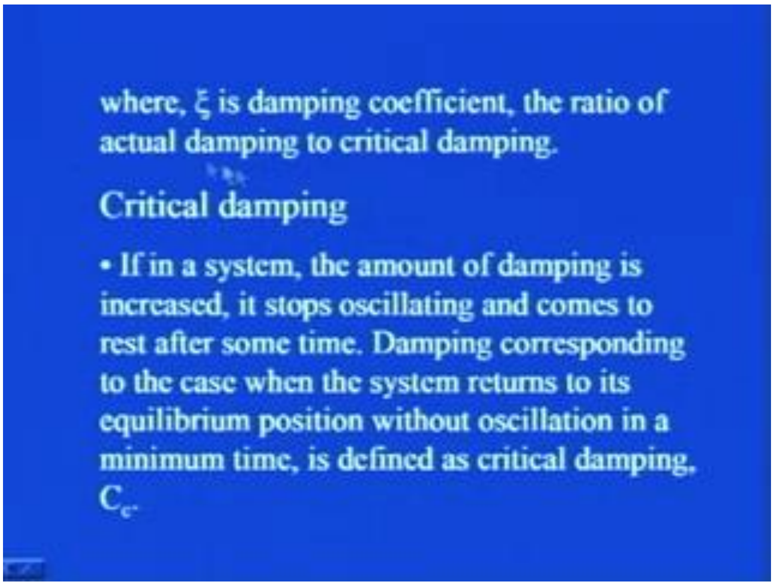

Now, what is this zeta, zeta is the damping coefficient, and it is, defined as the ratio of actual damping to the critical damping. Now, again, here I am introducing, another term as critical damping, let us try to understand, that what exactly, do we mean by this critical damping term So, if in a system, the amount of damping is increased, it stops oscillating and comes to rest after sometime, it is a universal law, that if the damping is more the resistance, to vibration is more, then obviously, the it will the body, which is vibrating, will stop oscillating and will come to rest, after some time.

And, once it has started vibrating slowly, slowly it is amplitude will go on reducing, and a situation will come, a condition will come, when that amplitude will become 0, and at that particular point of time, the body will be in rest. Damping corresponding to the case, when the system returns to its equilibrium position without oscillation, in a minimum time is defined as critical damping. So, whenever, I use this critical damping term, you must understand, that what exactly do we, mean by this critical damping.

That is, it is the damping, corresponding to the case that when, the system is coming to it is equilibrium position without, oscillating in one particular minimum time. Because, you see, every time when it is oscillating, it will come to its equilibrium position, because it is vibrating, but we want that particular case, when it is coming to it is equilibrium position in the minimum time, and whenever, that is occurring, the damping corresponding to that is called as critical damping.

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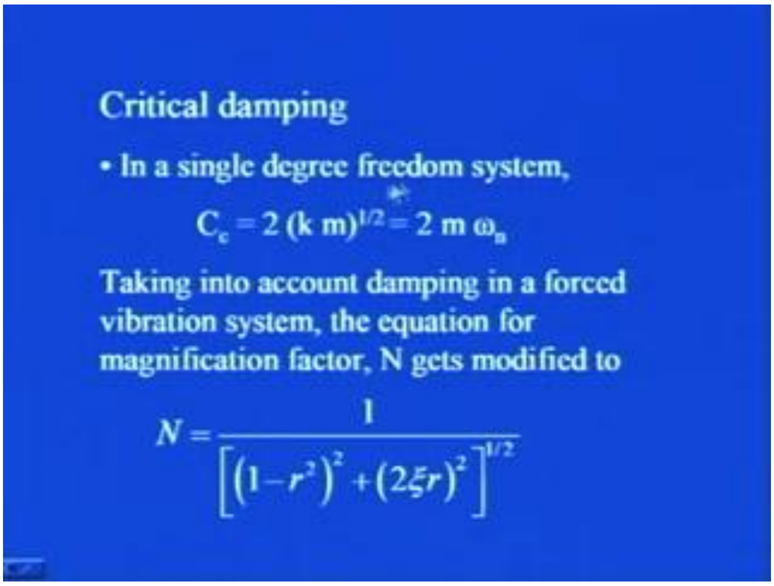

Then, how you can define it, that in a single degree freedom system, the critical damping is defined as, twice square root of k into m, and you know, that omega n square was equal to that k by m. So, you substitute it here, so you will be simply getting, here that omega n square, if you substitute here, so you will be getting, the 2 into m into omega n, taking into account, damping in a first vibration system, see right now, we were talking, in terms of free vibration now.

Let us say, that the system is vibrating, with the influence of some external force, so in that case, that will be subjected to forced vibration, and then if we take into account, the damping in this forced vibration system, the equation for magnification factor, gets modified to N is equal to, 1 upon 1 minus r square whole square plus 2 zeta r whole square and this whole to the power half. However, in case of, free vibration system we saw, that it was, 1 upon 1 minus r square, in case the damping was absent.

However, if the damping is there and the system is forced system, then in that case, you have this particular expression. This all these expressions, you can derive, but that require lot of mathematics so, that is how, why we are not discussing it here, and it is beyond the scope of your course. So, you see here, that earlier we discussed one plot, which was for 0 damping case.

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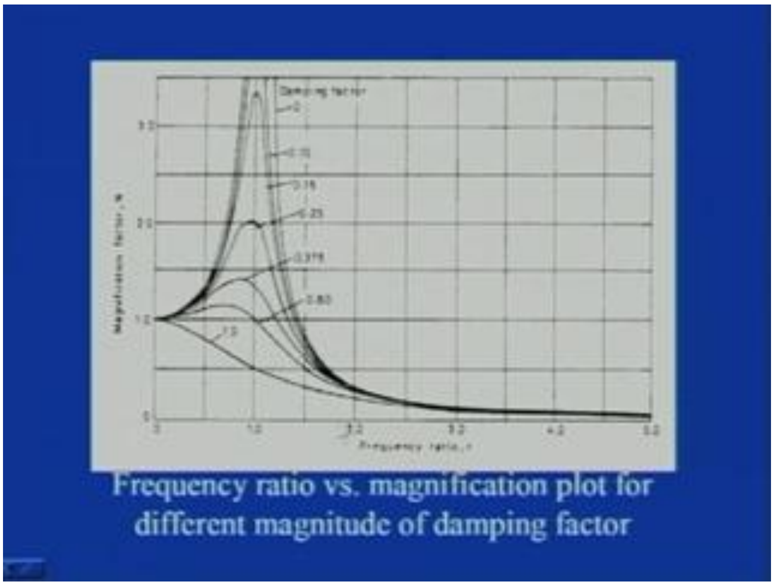

And in case, you have the damping, here is the curve, which is showing that frequency ratio verses magnification plot for different magnitude of damping factor. So, here, on x axis, we have frequency ratio r, on y axis we have this magnification factor, so and then, this damping factor is there, damping factor is your zeta. So, as your zeta is changing from 1 to 0, you see corresponding to 0, you see this line, you please follow the cursor, the last line this is this 1, that is, this corresponding to zeta is equal to 0, so in case, this is equal to 0, this is becoming, almost asymptotic and tending to infinity.

And, what does this mean that in case, when this frequency ratio is tending to, or this magnification factor is tending to infinity, in that case that represent the condition of resonance, so which is, not at all desirable. So, as you can see that as this, damping factor goes on reducing from 1 to 0.5 to 0.375, 0.25, 0.15, 0.1 and 0, the peak is more and more, and the peak is shifting towards right, you see its peak was somewhere here, then when it got, reduced from 0.5 to 0.375 the peak gets, shifted towards right here.

Then, when it is getting further reduced to 0.25, the, peak is further shifted to the right side, and then so on. So, it is peak, the magnitude, of this magnification factor, corresponding to the lesser value of this damping factor, is go on increasing and it becomes infinity, at this damping factor to be equal to 0.

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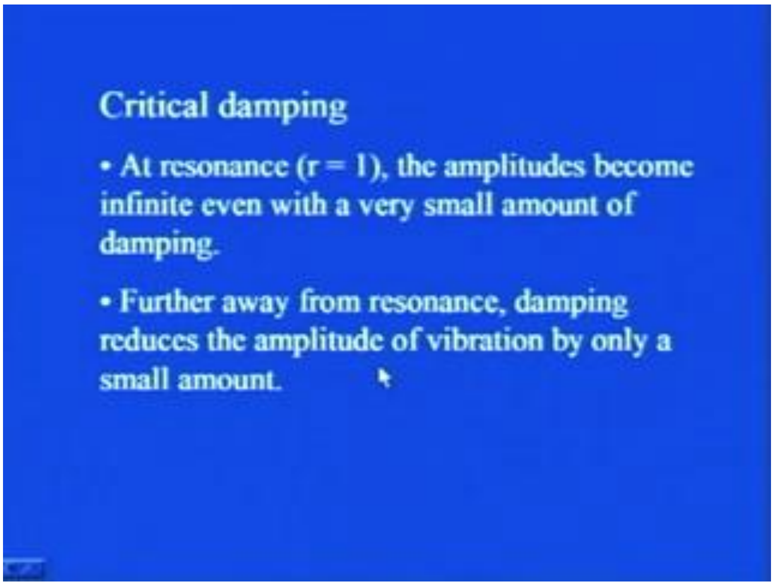

Then, there are some of the points, salient points, that at resonance, which is r is equal to 1, the amplitudes becomes infinite, even with a very small amount of damping. So, you can see, that although, the damping factor is very small, that is 0, you see here, that point 1, for that also, it is almost tending to infinity. So, near this frequency ratio, which is equal to 1, which represent, the condition of resonance, the magnitude of the amplitude, becomes almost infinite, even, the damping is very small.

So, further away, from resonance damping reduces the amplitude of vibration by, only a small amount. Again, ((Refer Time; 49:23)) this is evident from this particular figure, that you see, even though this frequency ratio is high. You see, when then it is quiet high, all the curves, they are merging to 1, that is, there is very less difference, in these curves depending on the damping factor. So, if the damping factor is very small, then also, you have magnification factor, very less for, large frequency ratio.

And then, if the damping factor is high, say 1 in this case, there also you will get less magnification factor for higher frequency ratio. So, further away from the resonance, this damp damping reduces the amplitude of vibration, just by small amount. So, today, in this lecture, we saw, that what exactly, do we mean by, machine foundation, why it is required. Why, it is different from the normal type of foundation that is any type of shallow or deep foundation that, it is subjected to dynamic loads, in addition to static loads.

Then, we saw, that there are three, type of machines, that is reciprocating rotary, and then the third one was impact type of machine. Depending on the subjected to they are subjected to different type of loading, these have been categorized, in these three groups, then they, have their own characteristics, so that also we saw. Then, we switched over to, the linear spring mass system that I told you that, why exactly it is necessary, in the subsequent lectures, you will be appreciating this aspect more.

When we will be, dealing with the analysis of different type of foundation, which you have seen that block foundation, box foundation and wall foundation. And, then we saw, that various terminology, like vibration, natural frequency, damping resonance etcetera. And then, we saw that, what are the various design criteria, that a foundation should work satisfactorily, obviously, the criteria, which you studied for static load, remain as it is, and in addition to that, the design criteria, such that it behaves satisfactorily.

Under, dynamic loads also were there, that there should not be, any resonance condition the amplitude of the system, must be within permissible limit. And then, the vibration of the machine should not be annoying to the person or detrimental to the structures, which are near, by. And then, we switched over to, the analysis of spring mass system, there we saw two cases, that, it is free vibration and forced vibration.

In free vibration, it was, in the absence of any external force, that is the vibration of the system was there, due to the forces, which are inherent in the system, however, in case of forced vibration, the vibration was there, due to, the external forces. There, we further divided into two parts, that is one in the absence of damping and another ((Refer Time: 52:44)) the effect of damping. Suppose, development of the equations, governing equations we say, then we define, that, what exactly is the natural frequency, in all the cases.

And then, after that we saw, that what, is the effect of damping, in case of, free vibration and in case of forced vibration. And then, we after that particular point, we talked of critical damping. Now, in the subsequent lectures, we will be seeing that, how you can analyze, by taking the help of, all this knowledge, that you acquired in this particular lecture, that how you can analyze, the different type of foundation, and how you can get the response of them, that we will see, in the next class.

Thank you.