

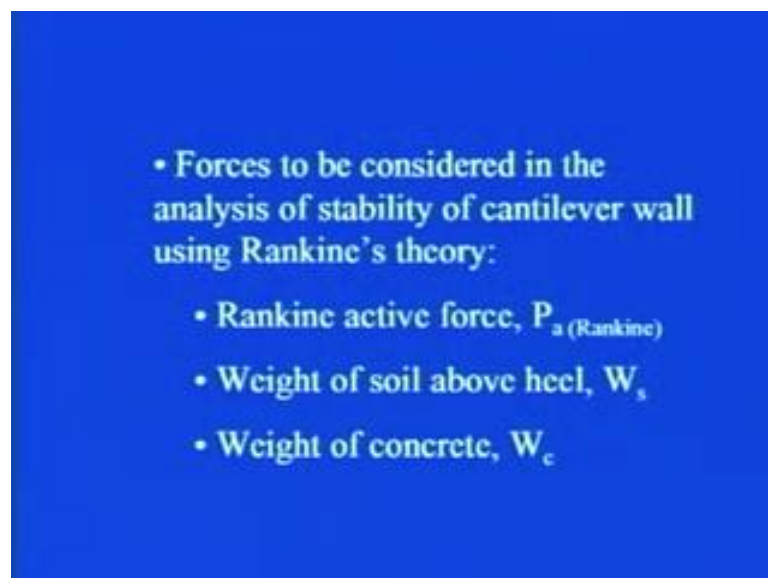
Foundation Engineering
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Module - 02
Lecture - 04
Lateral Earth Pressure Theories and Retaining Walls – 4

We were discussing about the Retaining Walls, in the last class we discussed about various types of retaining walls that is gravity walls, semi gravity walls, cantilever walls and then we talked of counter fort retaining walls. And then we started with that how the Rankine or Coulombs active earth pressure theory can be applicable to estimate the lateral earth forces in case of these retaining walls. So, we were discussing about the Rankine's theory and retaining walls.

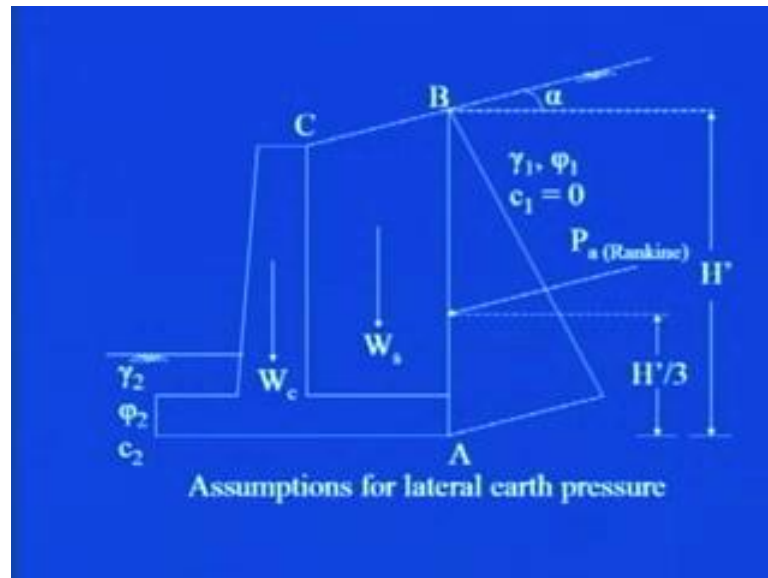
So, let us try to see, that what are the various aspects of this kind of analysis? If you remember we made an assumption that the failure is taking place at a vertical plane, which I indicated in the last class as A B. So, various forces, which will be acting on that particular wall they are Rankine active force.

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That is I am representing by P_a Rankine, weight of the soil above heel which is W_s and then weight of the concrete, this concrete weight is from which the wall is made up of that is W_c . You see in this figure how all these three forces they are acting on the wall.

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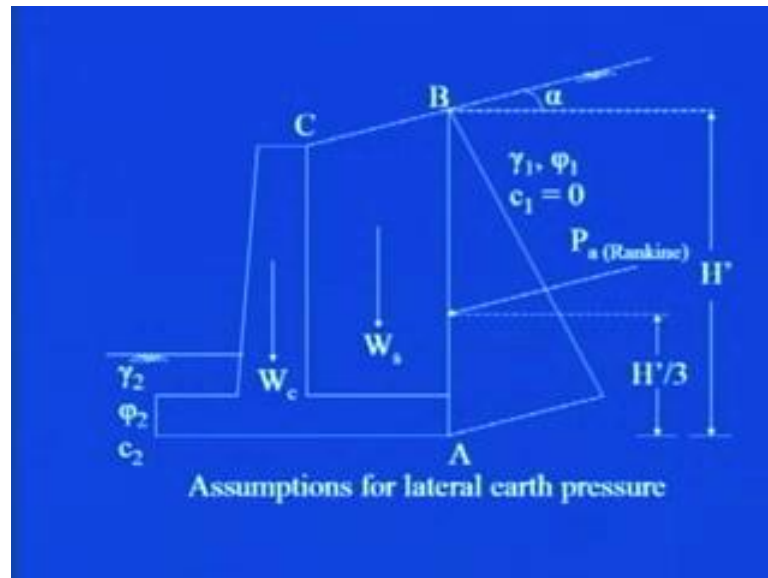


Since the backfill is inclined and as you know as, we have already discussed in Rankine's theory that this Rankine force will be acting parallel to the inclination of the backfill. So, here it is P_a , which is acting at an angle α from the horizontal, this is what is your weight of the soil above the heel that is W_s and this wall is made up of concrete, so the weight of concrete is acting over here. Now, this if H be the vertical height of the wall, we are considering this H' , you see here this H' is the length of the face AB you can see here in this figure that this AB is H' .

And then this P_a Rankine is acting at $H/3$ from the bottom of the wall, we have assumed backfill to be frictionless and therefore, cohesion value is equal to 0 you just have angle of internal friction and unit weight of the soil. And the properties of the soil which is lying below the slab that is, this base slab they are c_2 and ϕ_2 they are shear resistant parameters of that soil and γ_2 is the unit weight of that soil.

This was in the case of cantilever retaining wall, what happens in case of gravity wall the situation remaining the same as far as, Rankine's theory is concerned only three forces which were acting on the cantilever retaining wall they act on gravity wall also. You can see here that three forces ((Refer Time: 03:29)), first one is Rankine active force P_a Rankine, weight of soil above heel that is W_s and then weight of concrete W_c .

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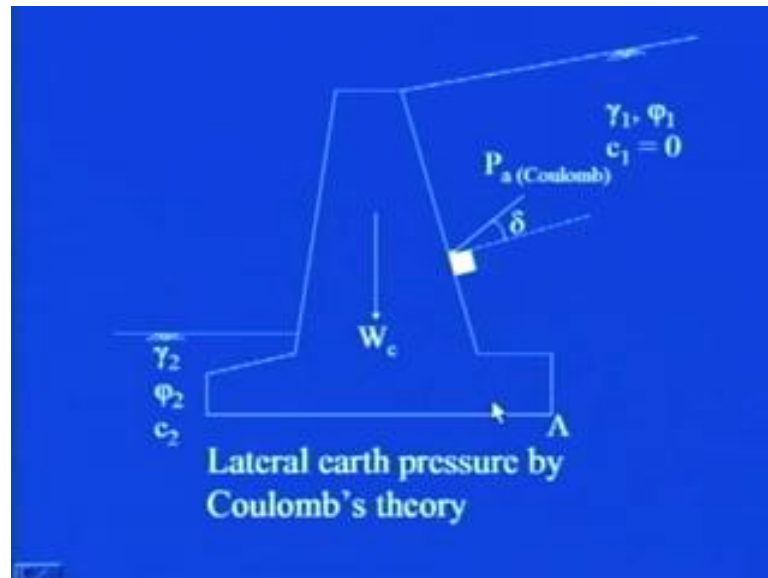
Exactly, on the similar lines earlier it was for cantilever wall now it is for gravity wall. You can see that in this case also it has been assumed that the soil or the this trial wedge is failing along this vertical face which is A B, P_a is parallel to this acting at a distance of H' by 3, where H' is the length of this face A B. Backfill is frictionless in this case also, so $c_1 = 0$. γ_1 and ϕ_1 are the properties of soil whereas, γ_2 , ϕ_2 and c_2 they are the properties of soil which is lying below the base slab of wall.

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- Forces to be considered in the analysis of stability of gravity wall using Coulomb's theory :
 - Coulomb's active force, P_a (Coulomb)
 - Weight of wall, W_c

Then if you consider Coulombs theory, this was the case that 2 cases that we discussed was for where for Rankine's theory, if you discuss with respect to the Coulombs theory then there is little difference in the cases. Only two forces will be acting in that case that is Coulombs active force P_a Coulomb and then weight of the wall W_c , How it will be acting that is shown in this figure.

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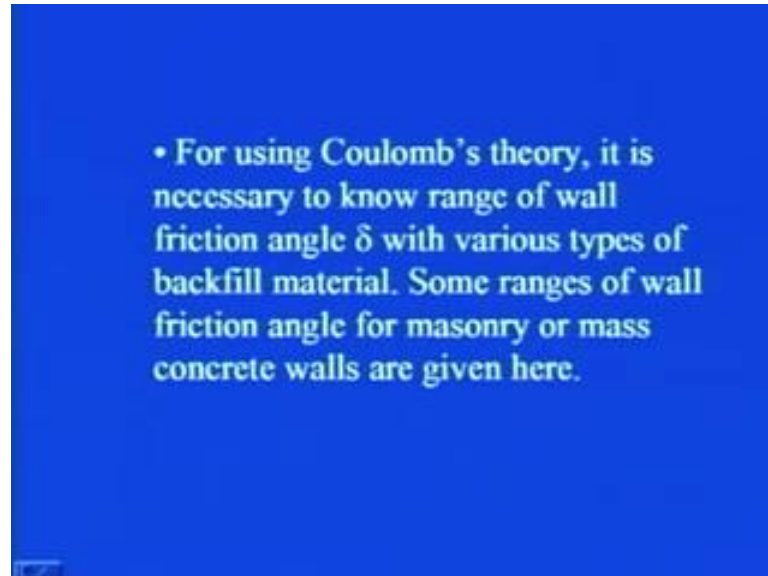
Since you know that Coulombs theory can take into account wall friction, so you see here this P_a Coulomb will be acting at an angle of δ to the normal drawn to the back face of the wall. You see this is the back face of the soil, this is the normal drawn to this particular surface of the wall and then you draw a line, which is making an angle δ from this normal and that will represent the line of action of P_a Coulomb, where δ is angle of wall friction.

See in this case we are not assuming the failure to occur along a vertical plane that is A B which was there in earlier case in case of Rankine's theory. In this case also the backfill has been assumed to be frictionless c_1 is 0 and so, you are left with γ_1 and ϕ_1 as the property of backfill soil however, for the soil which is lying below the base slab of the wall the properties are ϕ_2 c_2 and γ_2 .

So, what is the difference between the analysis using Rankine's theory and Coulombs theory is that that the Rankine's theory assume that the failure is taking place along the vertical face which we represented by line A B in the previous figure. However, in

Coulombs theory no such assumption have been made and this Pa Coulomb that is active force is directly acting on the face of the wall.

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So, for using Coulombs theory it is necessary to know the range of wall friction angle with various types of backfill material. Some ranges of the wall friction angle have been shown in the subsequent slide which is.

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Backfill material	Range of δ (deg)
Gravel	27-30
Coarse sand	20-28
Fine sand	15-25
Stiff clay	15-20
Silty clay	12-16

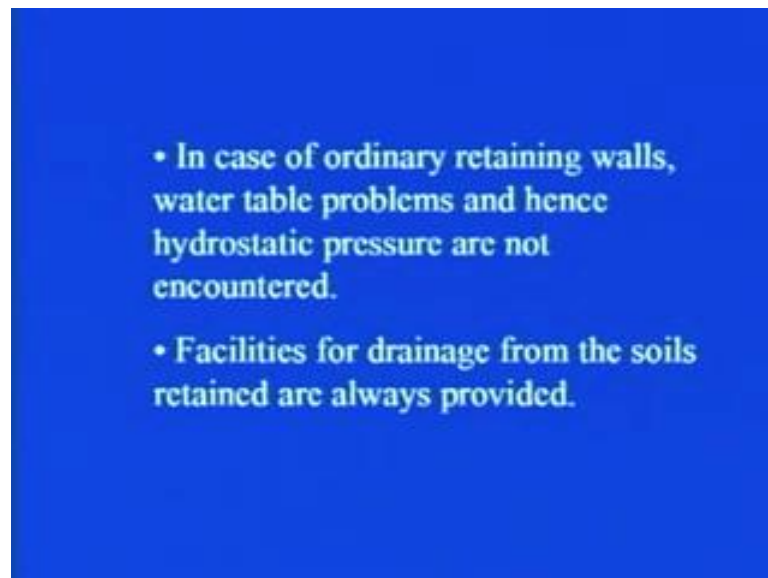
That first column is representing backfill material and the next column is representing the range of delta in degrees, you see it is not a very hard and past rule that you have to

pick any particular value from this has come from the experience. So, in case the backfill material is gravel in nature then the range of wall friction angle can be of the order of 27 degree to 30 degree whereas, in case of coarse sand it can vary between 20 to 28 degree.

For fine sand it is 15 to 25 degree, for silty clay and stiff clay they it is 12 to 16 degree and the range for stiff clay is 15 to 20 degrees. So, this gives you rough idea, while you can decide upon that what should be the value of delta depending on the type of backfill material. So, backfill material you will be knowing before hand before going for the construction obviously, that soil you are retaining, so you are knowing about that particular soil that what type of soil is that.

And then accordingly from these guidelines you can pick the value of delta and go ahead in the design procedure.

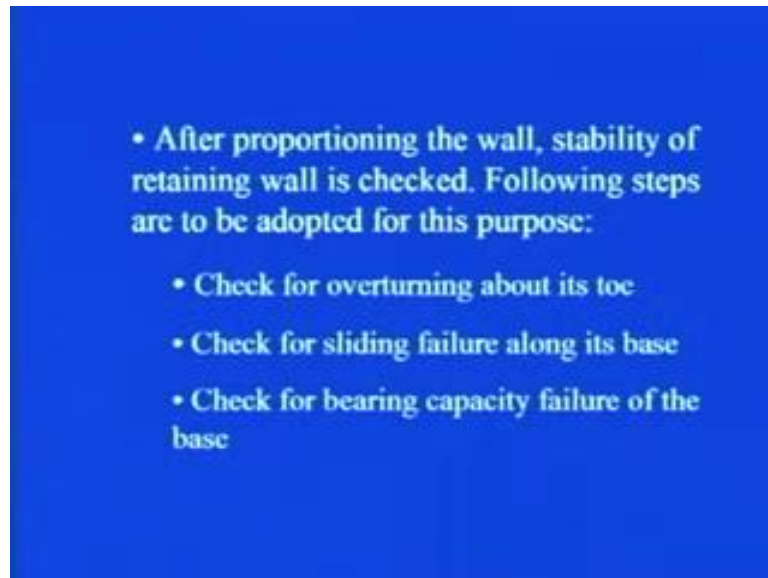
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Now, in case of ordinary retaining walls water table problems and hence hydrostatic pressure are not encountered. Facilities for drainage from the soils retained are always provided, why I am mentioning this these points here because, otherwise you will be wondering that why we are not at all taking into account the presence of or the probability of the presence of water table in the analysis. We will be discussing how this drainage takes into account in the wall towards the end of this chapter.

So, for the time being you just remember that right now, we are not taking into account any hydrostatic pressure which can be present there in practical field. Now, last time we saw how the proportioning can be done, so once the proportioning is done as I mentioned you that you have to provide the check for overturning sliding and bearing capacity failure.

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So, after proportioning the wall a stability of retaining wall is checked the following steps are to be adopted for this purpose, first is check for overturning about it is toe. See all the forces which are acting they have some of the forces they have the tendencies to make the wall overturn above the toe and some of the forces which resist this overturning. So, we have to identify these various forces, which are disturbing or which are resisting and with the help of these forces we should be able to find out that factor of safety against overturning that we will see in subsequent slides.

The second point is check for sliding failure along the base and then third one is check for bearing capacity failure of the base. So, let us see one by one, first I will tell you that how we can provide the check for overturning.

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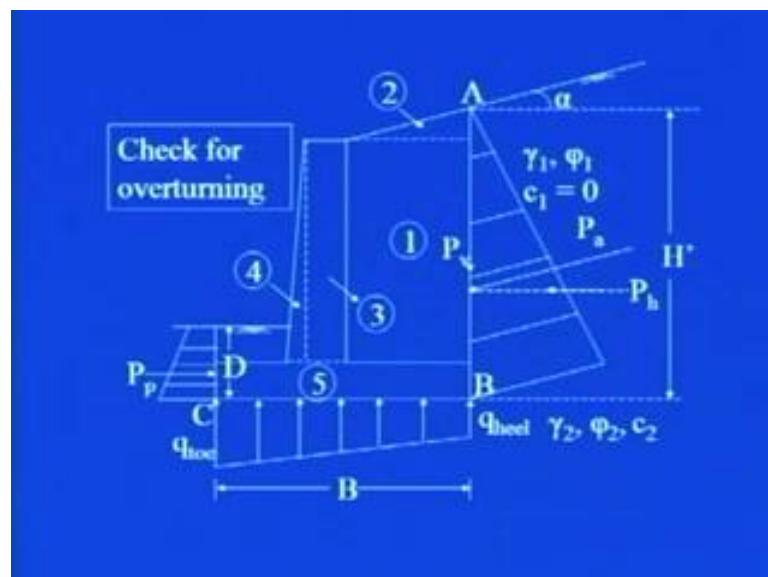
Check for overturning

- It has been assumed that Rankine active pressure is acting along a vertical plane AB drawn through the heel.
- P_p is the Rankine passive pressure and its magnitude is

$$P_p = \frac{1}{2} K_p \gamma_2 D^2 + 2c_2 \sqrt{K_p} D$$

It has been assumed that Rankine active pressure is acting along a vertical plane A B drawn through the heel we have seen this already. And your passive pressure you have already seen it in the few previous lectures, that it is Rankine's passive pressure and its magnitude is half $K_p \gamma_2 D^2$ plus $2c_2 \sqrt{K_p} D$, where your D is the depth of placement of base slab of the wall below the ground surface.

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You can see here in this particular figure, that this is the inclined back fill at an inclination of α from the horizontal, this is what is the depth D. So, you see as soon

as, it is the wall will start moving let us say in this direction in due to the presence of this backfill what will happen that this side of the wall will be moving away from the soil, so here active conditions will be generated.

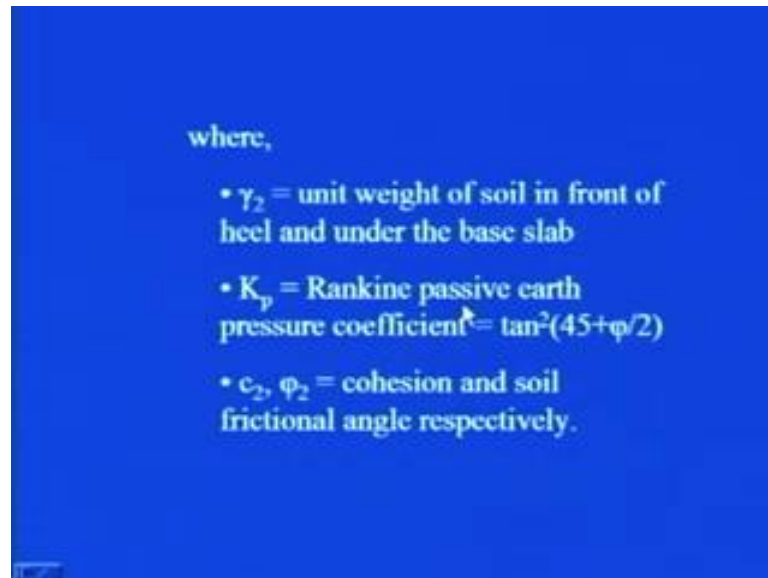
However, on the other side of the wall if the moment of the wall is in this direction then in this area what will happen is the wall will be pushing towards the soil and so, the passive conditions will be generated and therefore, the passive pressure will be generated over this much depth. And then you can see that how this active pressure is getting generated its line of action is parallel to the inclination of this backfill, so you can see here this is what is your active force which is acting it is having a horizontal component as P_h and a vertical component as P_v .

Again here you see in this area passive condition is developed, so passive pressure will be developed and subsequently passive force. So, that will be generated in this particular manner over the depth D , then what will happen since, the wall base slab of the wall has been placed on the soil, soil will put its reaction on the base of the wall and that reaction here I am showing by q_{heel} and q_{toe} . It may happen that I mean depending on the type of the soil this variation here I am assuming is it to be linear variation, but depending on the type of the soil this variation may or may not be linear, but for the analysis purpose we assume this to be linear.

So, at heel part I am calling this ordinate to be q_{heel} and at toe part I am calling this ordinate to be q_{toe} , B is your width of base slab of wall then you see here below the base slab of the wall your soil properties are γ ϕ c τ . However, the backfill which we have assumed to be frictionless $c = 0$, so only shear strength parameter ϕ will be there in the picture and unit weight of the soil.

And here you see this we we have already assumed that the failure is taking place along this $A B$ face. The length of this 1 or the depth of the this $A B$ face is H' and then this P_a will be acting at $H/3$ from the bottom of the wall.

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As I told you γ_2 is unit weight of the soil in front of heel and under the base slab, in front of the heel that means, here and below the base slab, so this part is γ_2 the unit weight of the soil is γ_2 . K_p is Rankine's passive earth pressure coefficient which is $\tan^2(45 + \phi/2)$, $c_2 \phi_2$ is the cohesion and soil frictional angle respectively.

Now, how can we take into account the analysis for overturning, as I told you that there will be some forces which will be trying to overturn the wall along its toe. This is toe this is heel, so along its toe there will be few forces which will be trying to destabilize the wall; however, there will be few forces which will be trying to stabilize the wall, let us try to see one by one.

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The factor of safety against overturning about the toe, i.e., about point C, may be expressed as:

$$FS_{(\text{overturning})} = \frac{\sum M_R}{\sum M_O}$$

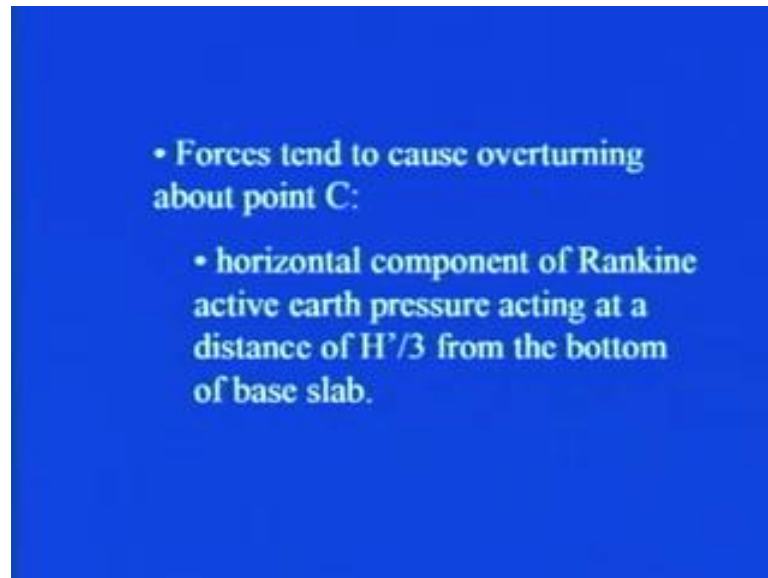
where, $\sum M_O$ = sum of moments of forces tending to overturn about point C
 $\sum M_R$ = sum of moments of forces tending to resist overturning

That how we can find out these various forces and the moments. The factor of safety against overturning about the toe that is about point C. (Refer Slide Time: 14:56) You see here in this one, this is the toe, this is what is the point C, so about this point C we will check for overturning, this may be expressed as factor of safety overturning is equal to summation of all the moments which are trying to overturn and then they are the resisting.

So, these are the resisting moments and this is which will try to make the wall overturn. So, where your summation M_O is sum of moments of forces tending to overturn about the point C. And summation of M_R is the sum of moments of forces tending to resist the overturning, so before knowing the these moments, first we have to identify that what all are the forces which are acting on the wall and what is the lever arm of those forces with respect to the point C.

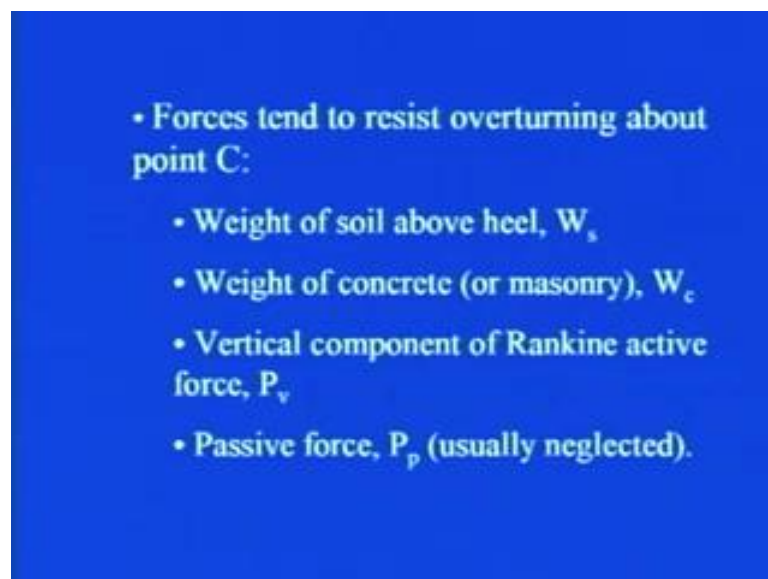
Once you know the force and its lever arm with respect to point C if you multiply these 2 values you will be getting, the moment about the point C and accordingly, you can go ahead in the design procedure.

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So, forces which tend to cause overturning about the point C what are they, horizontal component of Rankine active at earth pressure acting at a distance of H prime by 3 from the bottom of a base slab.

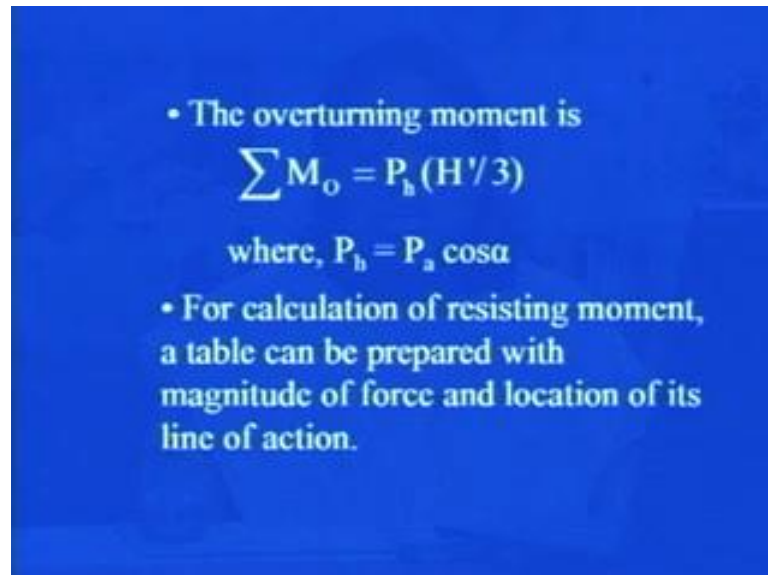
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Then forces tend to resist overturning about the point c, they are weight of the soil above heel that is W_s , Weight of concrete or masonry W_c , Vertical component of Rankine active force. Then passive force, see the thing is that the amount of or the magnitude of this passive force is quite small that is why for all the practical purposes it has been

neglected in the analysis obviously, by neglecting this we are committing a mistake towards safe side that is why we can ignore this particular force.

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- The overturning moment is
$$\sum M_o = P_h (H'/3)$$
where, $P_h = P_a \cos\alpha$
- For calculation of resisting moment, a table can be prepared with magnitude of force and location of its line of action.

Then overturning moment, overturning turning forces just the horizontal component of the active force which is P_h as I showed you in previous slides. So, your summation of M naught which is equal to P_h into H prime by 3, because P_h is the only force which is causing overturning about the point C, P_h is equal to $P_a \cos \alpha$ is the horizontal component of this active force P_a .

Now, for calculation of since you have seen that there are many forces which are resisting the overturning. So, we prepare a sort of table which facilitate the calculation and clarity, so that table is prepared for the magnitude of force and location of its line of action, so we prepare the whole thing in a form of a table which makes the thing easier for us.

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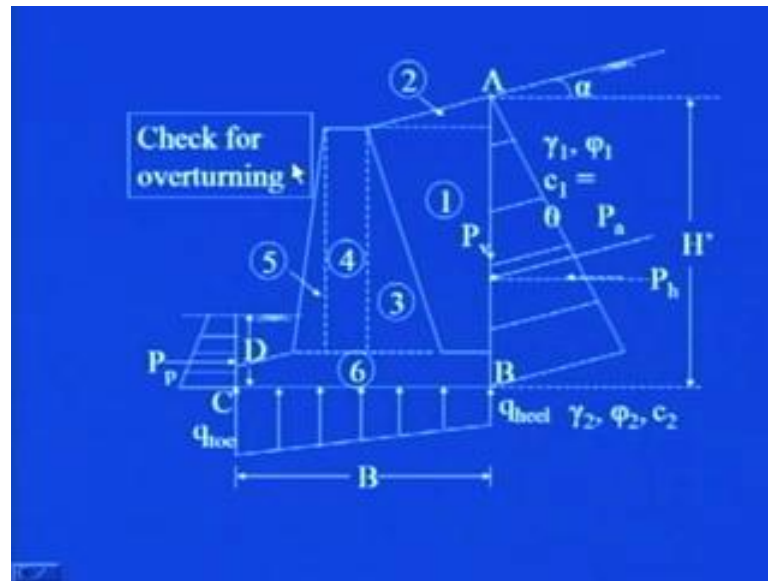
Procedure for calculation of $\sum M_R$

Section	Area	Weight /unit length of wall	Moment arm measured from C	Moment about C
1	A_1	$W_1 = \gamma_1 \times A_1$	X_1	M_1
2	A_2	$W_2 = \gamma_2 \times A_2$	X_2	M_2
3	A_3	$W_3 = \gamma_c \times A_3$	X_3	M_3

So, you see in this manner you can make these various sections, that is different sections 1, 2, 3 their area, because once you have drawn lateral earth pressure diagram then you can simply take the area or let us say if you have to find out the weight. So, where what exactly is the area, you multiply by the unit rate of the soil over there and then you will be getting, the weight of that particular area per unit length of the wall.

So, simply you see here γ_1 into A_1 will give you this one and then moment arm which is measured from C see moment about C are M_1 , M_2 , M_3 , for A_1 , A_2 , A_3 areas respectively. Let us try to have a look that how we can calculate the area corresponding to particular section and then, how we can find the weight per unit length of the wall and then subsequently the moment about point C.

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If you see here this is a case of gravity retaining wall which I am considering and I have divided the whole thing into 6 sections, first, Second is this triangular region, third which is the which is a part of the weight of this wall that is this triangular region, fourth is this rectangular one, fifth is this triangular one and then sixth is this base area. So, for the first one knowing this area we can get, the weight per unit length of the wall wal by multiplying this area to the unit weight of the soil, that is unit weight of the backfill which is equal to gamma 1.

So, if you see here W_1 is equal to gamma 1 into A_1 , A_1 you can find out see this is trapezoidal kind of thing, so you know this dimension as you have proportioned this proportioned this wall. And then, you know this dimension you know this height you can find out this area of the section 1 simply, multiply that by gamma 1 which will give you the weight of the soil, which is lying in this particular area per unit length of the wall.

Similarly, for this section number 2 this is again the area of the soil, so you take the area multiply with gamma 1 and you will get the weight of this particular area. You see sec section number 2 second area and then W_2 is equal to gamma 2 into A_2 then, you see if I take this triangular area where its line where its will be lying, is here at a distance of whatever is this distance 1 third of this distance it will be acting right here, so lever arm I can find out correspondingly.

I know all these dimensions, I know this dimension of the base, I know this and then further I can add this much amount and I can find out this X_1 . So, once I draw this figure to the scale simply, what I can do I can measure these distances from this point C and by multiplying this W_1 with this X_1 , I can get the value of this moment about C. Similarly, if I consider this third area let us try to see that what about this third area.

So, you see this third area is the concrete area, so you simply take the area of this triangular portion you multiply with the unit weight of the concrete, so you will be getting the weight of the wall of this particular sectional area 3, and then you can get correspondingly, the lever arm that is at what point its ((Refer Time: 22:11)) will be lying. So, the weight will weight of this particular area will be acting at that particular point and you can find out the distance of that point from this point C.

$W_3 \times \gamma_c$ into area 3 into X_3 , if you multiply W_3 into X_3 you will get this moment M_3 . So, we completed for section 1 section 2 section 3.

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Procedure for calculation of $\sum M_R$

Section	Area	Weight /unit length of wall	Moment arm measured from C	Moment about C
4	A_4	$W_4 = \gamma_c \times A_4$	X_4	M_4
5	A_5	$W_5 = \gamma_c \times A_5$	X_5	M_5
6	A_6	$W_6 = \gamma_c \times A_6$	X_6	M_6
		P_v	B	M_v
		$\sum V$		$\sum M_R$

Similarly, for section 4, 5 and 6 we can find out the area you see here section 4 is rectangular part, it is area we can find out we have already proportioned the thickness of the stem at the top of the wall. We know this particular height simply, multiply these 2 dimensions you will be getting the area of this part of the wall or area of the section 4 multiply that by gamma of concrete if it is made up of concrete.

If it is made up of stone masonry you have to multiply the gamma of stone masonry, so accordingly, whatever material that you are using simply multiply the area by the unit weight of that particular material and that, will result into the weight of the wall of that particular sectional area. Then its weight will be acting midway of this particular thickness, so we know this particular width as we have proportioned and then half of this particular width which will be giving you X_4 that is this one this X_4 , you multiply W_4 into X_4 , X_4 you will be getting M_4 .

Similarly, for area 5 area 6 you can find out correspondingly, W_5 W_6 then X_5 X_6 and subsequently M_5 M_6 by multiplying W_5 and X_5 , and M_6 you can get as W_6 into X_6 . Then apart from this weight of the wall and weight of the soil there is vertical component of active force, which is also resisting this overturning. So, see here I am adding this particular force or this particular force per unit length of the wall that is P_v and this P_v you see here this is what is your P_a .

So, its horizontal component is P_h vertical component is P_v and this P_v will be acting at a distance of B from this point C . So, its lever arm will be B and if you multiply the magnitude of this P_v by B you will be getting, the moment which will be generated due to this force P_v that I am naming as M_v . So, if I sum all the forces or all the this weight per unit length of the wall plus this P_v , so W_1 plus W_2 plus W_3 plus W_4 plus W_5 plus W_6 plus P_v this will get result into this particular term, which is summation of C that is summation all vertical forces.

Why we will require you will realize little later, for the time being just you just remember that you have to find out the total vertical force. And then the total moment that is, resisting moment you can get by summing up all the moments that is from M_1 to M_6 plus M_v , this will result into summation of M_R which is sum of all the resisting moment for overturning.

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where, $P_v = P_a \sin \alpha$

The moment of force P_v about C is M_v
 $= P_v B = P_a \sin \alpha B$

where, B = width of base slab.

Once $\sum M_R$ is known, the factor of safety can be obtained.

Where from the table if you see, your P_v is the vertical component of active force that is P_a and that you can find out using this expression that is P_v is equal to $P_a \sin \alpha$. The moment of force P_v about point C is M_v , and as I have explained you in the previous figure that this P_v is will be acting at a distance of B from the point C about, which we are considering the moments of all the forces.

So, this moment due to this P_v force will be equal to P_v into B which will result into $P_a \sin \alpha$ into B where your B is width of base slab. So, once this summation of M_R is known using the expression for factor of safety you can evaluate the same.

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$$FS_{(\text{overturning})} = \frac{M_1 + M_2 + M_3 + M_4 + M_5 + M_6 + M_v}{P_a \cos \alpha (H/3)}$$

• The usual minimum desirable factor of safety with respect to overturning is 2 to 3.

• Some designers prefer to determine factor of safety against overturning with

$$FS_{(\text{overturning})} = \frac{M_1 + M_2 + M_3 + M_4 + M_5 + M_6}{P_a \cos \alpha (H/3) - M_v}$$

Factor of safety how you can do that, you can have a look in this slide, that factor of safety against overturning is equal to, summation of all the moments which are resisting the overturning divided by the force which is causing the overturning. So, you see here $M_1 + M_2 + M_3 + M_4 + M_5 + M_6 + M_v$ divided by horizontal component of active force into $H' / 3$, what is the horizontal component of active force which is equal to $P_a \cos \alpha$.

So, accordingly here in denominator you have this $P_a \cos \alpha$ into $H' / 3$, the usual minimum desirable factor of safety with respect to overturning is taken as 2 to 3. Now, some designers prefer to determine the factor of safety by the following expression, here you can see that M_v is causing the resistance to the overturning. Now, if I put this M_v with a negative sign in the denominator the things are going to be same because opposite of that M_v will be causing overturning.

So, some of the designers they prefer this expression as, they sum all the moment which are generated due the weight of either soil or weight of the wall. And then they, divide it by this expression that is $P_a \cos \alpha H' / 3$ minus, the moment which is caused by the vertical component of active force. Now, it was all about the factor of safety determination of factor of safety against overturning.

Now, what will be the case for sliding along the base, again as if you have seen that. Factor of safety is always taken the ratio of resisting 1 to the driving 1 that whatever is, the disturbing force or moment whatever it is.

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Check for sliding along the base

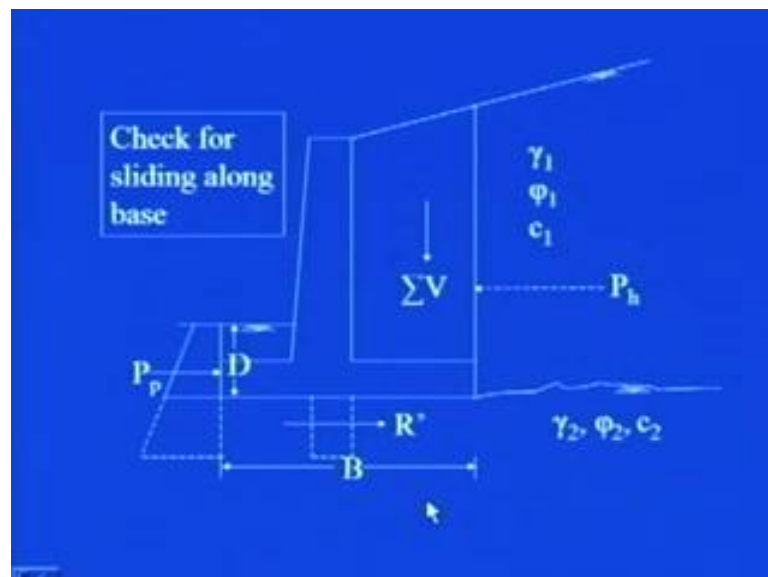
- The factor of safety against sliding may be expressed by the equation:

$$FS_{(sliding)} = \frac{\sum F_{R'}}{\sum F_d}$$

where, $\sum F_{R'}$ = sum of horizontal resisting forces
 $\sum F_d$ = sum of horizontal driving forces

So, the factor of safety against sliding may be expressed by the equation that is F S sliding is F summation of FR prime and then summation of F d, where F R prime is sum of horizontal resisting forces and F d is sum of horizontal driving forces.

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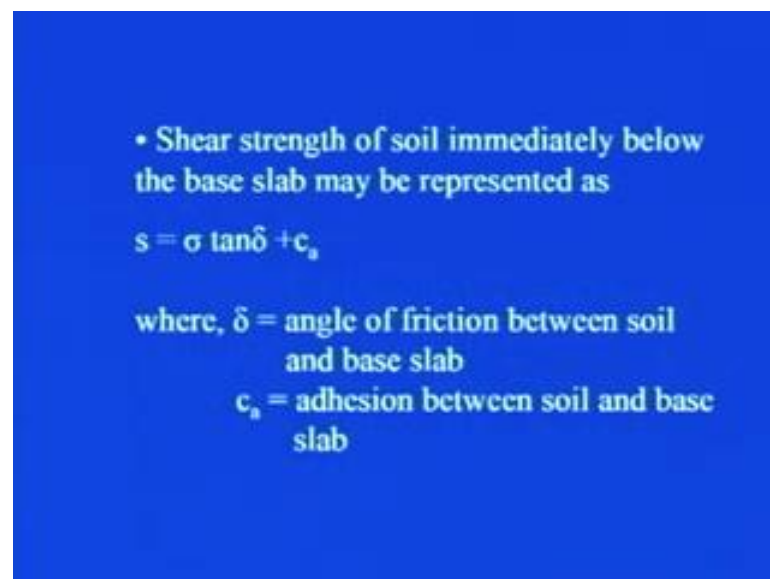
You see what are the various horizontal forces, because sliding will be taking place as the horizontal 1 which is above along the base. So, there will be few forces which will be causing the base to slide over the soil some of the forces they will be resisting it, so the first one is the horizontal component of this active force which is your P h. Your total

vertical force let us say summation V as you have seen in the earlier slides that I have mentioned this summation V it is the summation of all the vertical forces then, your P p that is active force sorry passive force which is acting over this depth D.

So, if the wall is moving towards this 1 this towards this direction then what will happen, your P h is the force which is causing the wall to slide however, this force Pp which is against the motion of direction of motion of the wall, so it will be trying to resist the sliding. Then there will be some friction between this wall and the soil that is this base of the wall and the soil, so that force I am calling to be R prime.

You can see here the property of the backfill soil is $\gamma_1 \phi_1 C_1$ and for this 1 this soil here that is below the base is $\gamma_2 \phi_2$ and C 2. Base of the sorry the base width is B that is base width of the base slab.

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• Shear strength of soil immediately below the base slab may be represented as

$$s = \sigma \tan \delta + c_a$$

where, δ = angle of friction between soil and base slab
 c_a = adhesion between soil and base slab

You see shear strength of soil immediately below the base slab it is represented as s is equal to sigma tan delta plus c a, where delta is angle of friction between the soil and the base slab. And c a is the adhesion between soil and base slab and that is the shear strength which will be acting to resist the sliding.

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Thus the maximum resisting force that can be derived from the soil per unit length of wall along the bottom of base slab is

$$R' = s (\text{area of cross section}) = s (B \times 1) \\ = B \sigma \tan \delta + B c_a$$

However, $B\sigma = \text{sum of vertical forces} = \sum V$
and therefore,

$$R' = \sum V \tan \delta + B c_a$$

So, thus the maximum resisting force that can be derived from soil per unit length of wall along the bottom of base slab is which I showed you in earlier figure as R prime, that will be equal to since, s is the shear strength of the soil and that will be providing the resistance to the wall from sliding. So, that s if you simply multiply that by area of the cross section then, that will give you the total force.

So, in the longitudinal direction of the wall I am considering unit length, that is why the area of the cross section on which the shear strength is acting that acting to be, which we are considering that it is acting on that particular area, that will be your base width multiplied by the unit length in the longitudinal direction of the wall, so that is B into 1. So, accordingly as you have seen the expression for s which is sigma tan delta plus c a simply, substitute this expression over here in this particular expression for R prime, you will be getting s is equal to B sorry R prime is equal to s into B.

And then s is your sigma tan delta plus ca simply multiply by B in both the terms you will be getting the magnitude of this force which is resisting the sliding. However, this B into sigma which is sum of the vertical forces is equal to summation V and therefore, your R prime will result into summation V tan delta plus B c a, I hope that now you are able to appreciate for ,that is why we determine the summation of all the vertical forces. You see once the table is prepared as, I explained you earlier you simply have to pick

that particular value and substitute it here and you will be getting the value of this R prime directly.

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• The passive force P_p is also a horizontal resisting force. Therefore,

$$R' = \sum V \tan \delta + B c_a + P_p$$

• The only horizontal driving force which will tend to slide the wall along base is horizontal component of active force, P_a . Therefore,

$$\sum F_d = P_a \cos \alpha$$

The passive force is also horizontal resisting force, so that total R prime that is total resisting force will be equal to the resistance which is offered by the soil which is lying at the base of the wall, plus the force due to passive pressure that is P p. So, R prime will result into summation V tan delta plus B into ca plus Pp that is passive force, the only horizontal driving force, which will tend to slide the wall along the base is horizontal component of active force P a, which we are representing by P h that is equal to P a into cos alpha.

Therefore, your summation of F d because this term summation F d you are using in the expression of factor of safety this will simply, become P a into cos of alpha. So, your resulting expression for factor of safety will be, that is factor of safety against sliding along the base that will be.

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- The resulting expression for factor of safety against sliding along base will be

$$FS_{(sliding)} = \frac{(\sum V) \tan \delta + Bc_a + P_p}{P_a \cos \alpha}$$

- The minimum factor of safety of 1.5 is required against sliding along base.

Factor of safety against sliding is equal to summation V tan delta plus B c a plus passive force that is, P p divided by the horizontal component of active force that is Pa cos alpha. In case of overturning I told you that usually factor of safety between 2 to 3 is adopted however, in case of factor of safety against sliding along the base this factor of safety of 1.5 is required.

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- Many cases, the passive force P_p is ignored for calculation of factor of safety with respect to sliding.
- In general, $\delta = k_1 \phi_2$ and $c_a = k_2 c_2$
- In most cases, k_1 and k_2 are in the range of $\frac{1}{2}$ to $\frac{2}{3}$.

$$FS_{(sliding)} = \frac{(\sum V) \tan (k_1 \phi_2) + B(k_2 c_2) + P_p}{P_a \cos \alpha}$$

Now, in many cases the passive force P p is ignored for the calculation of factor of safety with respect to sliding, and then in general this delta is taken to be some factor of phi 2,

ϕ_2 is the angle of internal friction of the soil which is lying in front of heel and below the base slab. So, k_1 is any factor you simply multiply this factor k_1 to the value of ϕ_2 and you will be getting the value of δ , that is the angle of friction between the base slab and the wall.

And the adhesion between the base of the wall base slab of the wall and the soil is equal to a factor k_2 , when you multiply it by the cohesion of the soil which is lying below the base slab that will result into c_a . In most of the cases this k_1 and k_2 are in the range of half to 2 by 3, so usually either it will be given to you provided by the consultant or whoever is coming to you for the design of retaining wall, or from your experience slowly you will get to know that what should be these values, but usually they are to be taken between the range 1 by 2 to 2 by 3.

So, if you substitute this value of δ and c_a in the previous expression of factor of safety you will result into this expression, that is factor of safety against sliding along the base of the wall will be equal to summation of $V \tan \delta$. And now, δ I am substituting by this quantity that is $k_1 \phi_2 + B$ into c_a , c_a you substitute here as $k_2 c_2 + P_p$ divided by $P_a \cos \alpha$, $P_a \cos \alpha$ is nothing, but the horizontal component of the active force, which is the driving force for the sliding along the base of the wall.

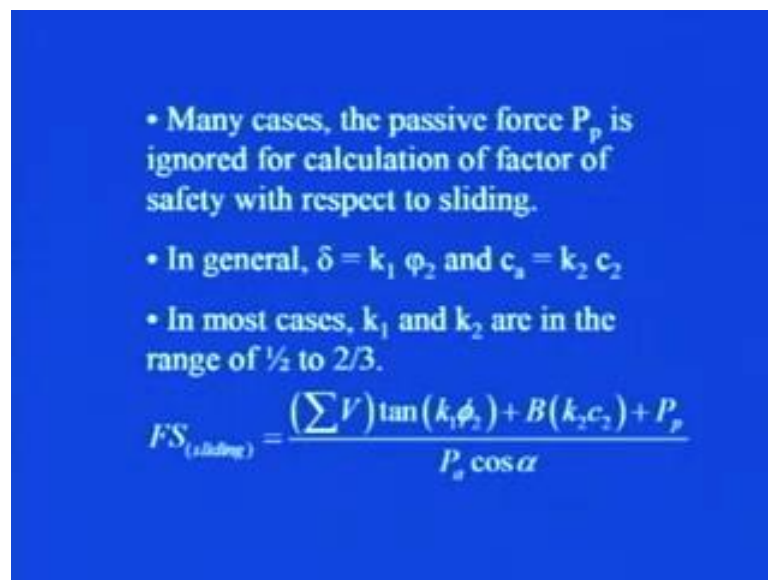
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- In some instances, certain walls may not yield a desired factor of safety of 1.5.
- To increase their resistance to sliding, a base key may be used.
- It helps in increasing the passive resistance and hence factor of safety.

In some instances certain wall may not need a desired factor of safety of 1.5 that is as I explained you in the previous class, that many a times it may happen that a wall, which is safe against overturning or it is safe against bearing capacity failure it may happen that it can fail in sliding. So, in that in case I told you that you have to re-proportion the whole thing.

Now, the second option which you can adopt such that the wall can become safe against sliding is that we are going to talk about. So, to increase its resistance in sliding a base key may be used I will show you that what exactly do I mean by this base key, it helps in increasing the passive resistance and as the passive resistance is increasing.

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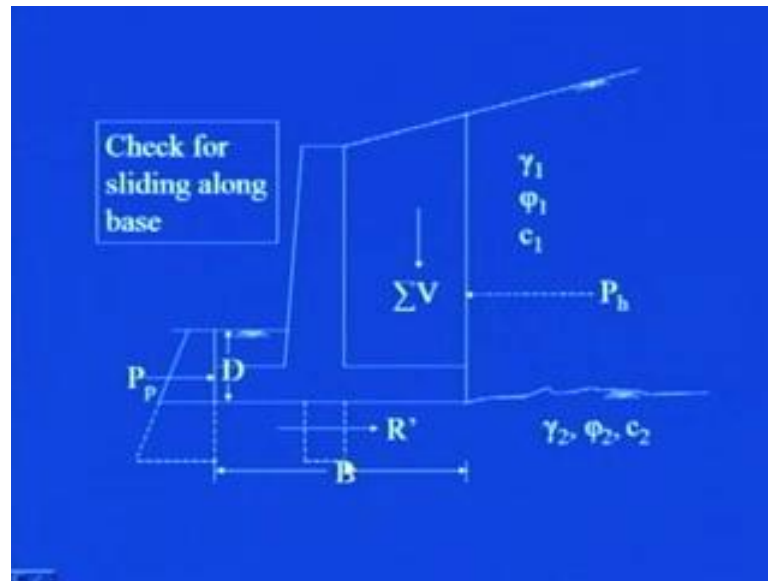


- Many cases, the passive force P_p is ignored for calculation of factor of safety with respect to sliding.
- In general, $\delta = k_1 \phi_2$ and $c_s = k_2 c_2$
- In most cases, k_1 and k_2 are in the range of $1/2$ to $2/3$.

$$FS_{(sliding)} = \frac{(\sum V) \tan(k_1 \phi_2) + B(k_2 c_2) + P_p}{P_a \cos \alpha}$$

You can see here, in this expression that as the passive resistance is increasing the factor of safety value will also increase. And as soon as you achieve the value of 1.5 as factor of safety against the sliding the your wall you can consider to be safe against sliding along its base.

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You see here this is a kind of key that is provided, this is additional one, so while your construction is not that it is an additional unit what happens it is, that the monolithic construction is done of this particular key with the base slab. So, that the resistance which is developed due to this should be over this particular depth, you see here with the presence of this key what will happen is that, when the soil will be pushing towards. When the wall will be pushing towards the soil, the passive force which is getting generated only upto a depth of D in the absence of this key will be developed over this particular depth if the key is provided.

Please, see it again that in case the key is provided the passive force or passive resistance which is getting generated over this depth D earlier when the key was not present. So, as soon as this key is introduced this depth get increased to this extent see from here to here. And once this depth is increased the pressure is increased and simultaneously, if you find out the area the force will also be increased, and as the passive force is increased since, it is acting as a resistance against sliding of the base of the wall, your factor of safety against the sliding is increasing.

So, instead of re-proportioning the wall, let us say there is a case where the wall is safe against overturning the wall is safe against bearing capacity failure the wall is just failing in sliding. So, instead of proportioning or re-proportioning the wall and doing all the steps altogether again we try with this base key and then we check whether by providing

this base key if we are safe or the wall is safe against sliding or not. Some of the key points related to this aspect, because it is a very important aspect that you must keep in your mind.

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- The passive force at the toe without the key is

$$P_p = \frac{1}{2} K_p \gamma_2 D^2 + 2c_2 \sqrt{K_p} D$$

- If the key is included, the passive force per unit length of the wall becomes,

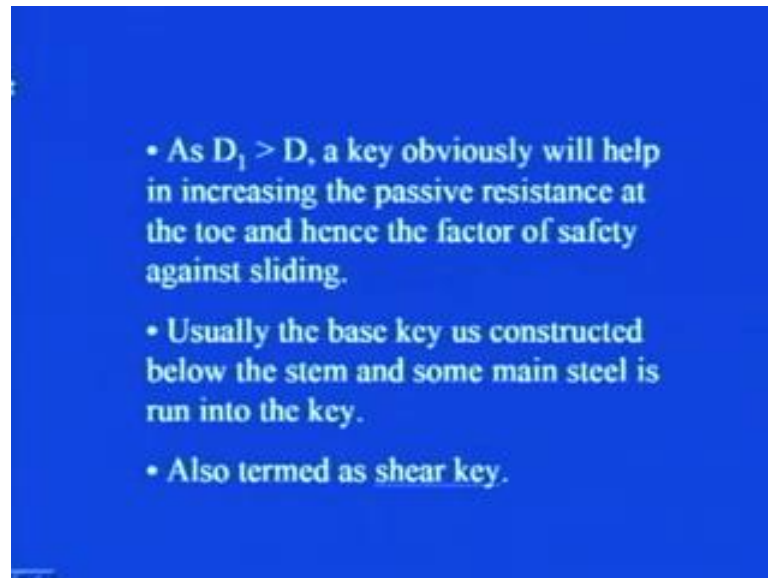
$$P_p = \frac{1}{2} K_p \gamma_2 D_1^2 + 2c_2 \sqrt{K_p} D_1$$

where, $K_p = \tan^2 (45 + \phi_2/2)$

So, let us try to put in words and some of these are some of the points related to this, the passive force at the toe without the key is you see the passive force in case the key is not present, the passive force is getting generated over this depth D only. So, the expression for passive force is half K p gamma 2 D square plus 2 c 2 square root of K p into D. Now, the if the key is included the passive force will become, you see here if the this 1 is introduced what will happen, the passive force will be generated over this step.

Let us say that I am calling this step to be D 1, so in that case your passive force will become half K p gamma 2 D 1 square plus 2 c 2 square root of K p into D 1. Where Kp is tan square 45 plus phi by 2 phi 2 by 2, where phi 2 is the angle of friction of the soil which is lying below the base slab and in front of heel. Since with the inclusion of this base key the depth is getting increased.

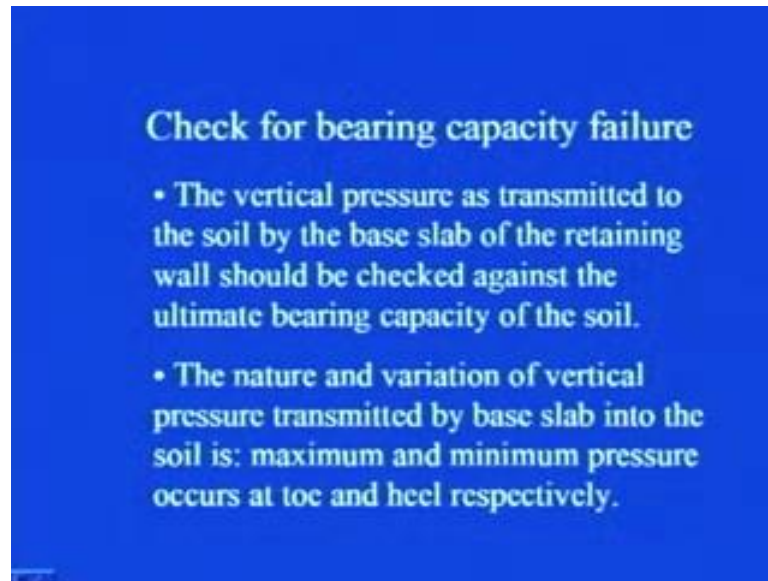
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That is in all the possibility or in all the cases your D_1 is going to be always greater than D , so as D_1 is greater than D a key obviously, help in increasing the passive resistance at the toe and hence the factor of safety against sliding. You have seen that this passive force comes in the numerator in the expression of factor of safety, so as soon as the this passive force is getting increased obviously, the factor of safety will get increased.

So, usually the base key is used in constructed below the stem and some main steel is run into the key this is also termed as shear key. Now, this was all about check for overturning and check for sliding along the base of the wall, let us try to see the third one, which is check for bearing capacity failure, some of the points let us first discuss about that and then we will go into mathematical things about this particular check.

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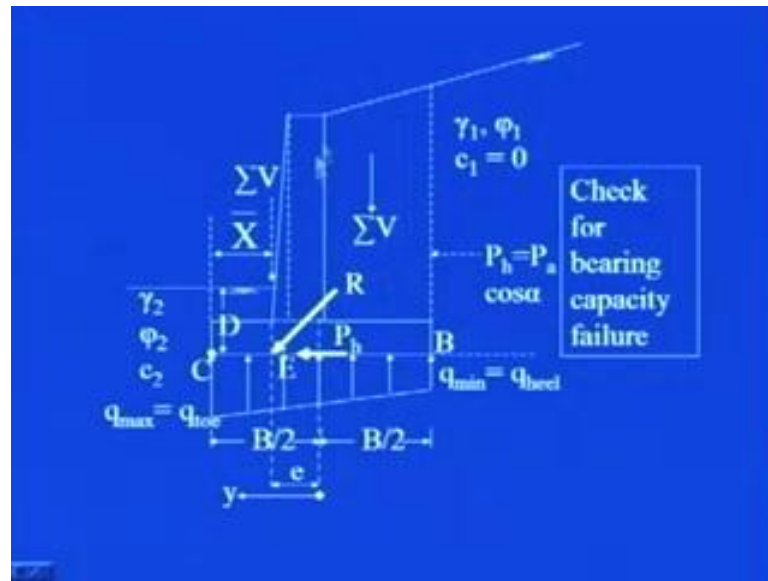


The vertical pressure as transmitted to the soil by the base slab of the retaining wall that should be checked against ultimate bearing capacity of the soil. You see how this vertical pressure which is coming into the picture, the weight of the soil and whatever, let us say in case the surcharge is present all these will transfer the pressure to the soil. You have already studied in the shallow foundation that how you can determine the bearing capacity of the soil, that bearing capacity is a kind of strength of the soil that is that much load the soil can stand without failing.

So, if the total pressure which is getting transmitted from the wall to the soil, if that is less than the bearing capacity of the soil, the soil will not fail in the bearing capacity. So, here the disturbing force is going to be the vertical pressure, which is getting transmitted from the wall to the soil and the resisting force will be the ultimate bearing capacity of the soil.

The nature and variation of vertical pressure that is transmitted by base slab into the soil is maximum and minimum pressure occurs at toe and heel respectively. Now, let us try to see that how this mechanism takes place.

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This is some of all the vertical forces in which this P_v is also coming into picture and that is why I am not showing this P_v separately over here simply, P_h which is equal to $P_a \cos \alpha$ that is the horizontal component of active force. The backfill is frictionless, so cohesion is 0 that is $c_1 = 0$ γ_1 is the unit weight of the backfill soil ϕ_1 is angle of friction of this backfill soil.

Then here at heel the pressure which is getting transmitted from, the wall to the soil is minimum which I am naming as q_{heel} and here it is maximum which is q_{toe} , γ_2 is the unit weight of the soil which is lying below the base slab and in front of heel. ϕ_2 and c_2 they are shear strength parameters of the soil lying below the base slab, in the absence of any shear key, this wall base of the wall is placed at a depth of D from this ground surface.

So, you see here again we have to find out that what exactly are the forces and ultimate capacity of this soil which is lying below the base. So, first of all for that thing we really need to find out that what exactly is the magnitude of q_{toe} and q_{heel} . So, let us try to see the steps which are to be followed for the determination of q_{toe} and q_{heel} .

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Determination of q_{ice} and q_{snow}

- Sum of vertical forces acting on base slab (Table showing details of forces and moments) = $\sum V$
- The horizontal force = $P_a \cos \alpha$
- Let resultant force = R , or

$$\vec{R} = \sum \vec{V} + (\overline{P_a \cos \alpha})$$

For that we have some of vertical forces acting on base slabs I have already shown you table, which was showing the details of all the forces and the moment when we were finding out the factor of safety against overturning. So, you have this summation of V already with you, then the total horizontal force you know it is the horizontal component of this active force, which is the P_a and that horizontal component is $P_a \cos$ of alpha.

Now, let the resultant of these 2 V R , you see here in this place, that is this summation of V and this P_h it is resultant is R , ((Refer Time: 48:14)) which is the vectorial summation of this summation V and $P_a \cos$ of alpha. See this arrow I am showing that it is the vectorial summation.

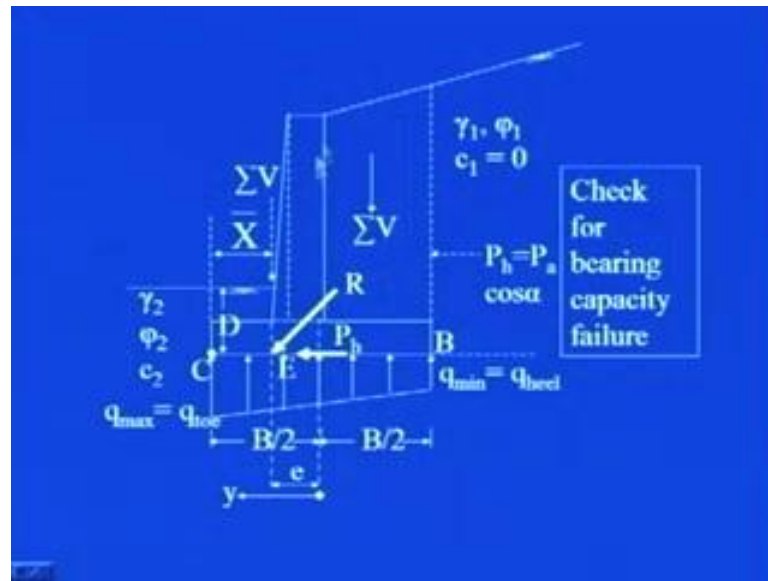
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- The net moment of these forces about point C is $M_{net} = \sum M_R - \sum M_O$
- $\sum M_R$ and $\sum M_O$ are already known (Table giving details of forces and moments)
- Let the line of action of resultant, R, intersect the base slab at E, then distance is $\overline{CE} = \bar{X} = \frac{M_{net}}{\sum V}$

Then the net moment of these forces about the point C is, if I call that as M net that will be equal to summation of all the moment, which are resisting the overturning minus summation of all the moments which are causing the overturning. In notational form it is going to be summation M R minus summation M O as written here, and I have already shown you we once we prepared that table half of the work is done, in that table summation V was known summation M R was known and summation M O is known.

So, simply you pick all those value from that particular table and here you will be able to find out this M net. Now, I have to find out the line of action of that resultant also, so let us say that the line of action of this resultant R, intersect the base slab at E, then this distance C E will become let us say if I name it to be X bar this will be equal to M net by summation V.

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So, this C distance you can see in this figure is that this distance that wherever, this resultant R will be acting at this particular point. You see here this dotted line is coming vertical down, now if I want to find out it is eccentricity from the centre of the base of this base slab. So, how I can find out this eccentricity e is that, if I substitute subtract this C distance from B by 2 then I will be getting this eccentricity e .

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- The eccentricity of the resultant, R may be expressed as,

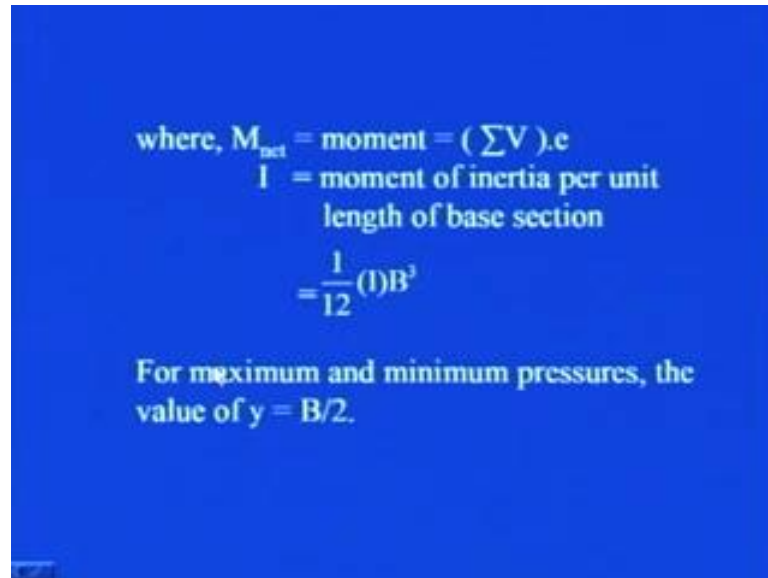
$$e = \frac{B}{2} - \overline{CE}$$
- The pressure distribution under the base slab may be determined by using the simple principles of mechanics of materials as:

$$q = \frac{\Sigma V}{A} + \frac{M_{net} y}{I}$$

So, you see here the eccentricity of the resultant R it can be expressed as, that is B by 2 minus the distance $C E$. And then the pressure distribution under the base slab may be

determined by using the simple principles of mechanics of materials. As your q is equal to summation of V by A plus minus M net y by I now what are all these things, Summation V you have already seen is the sum of all the vertical forces.

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where, $M_{net} = \text{moment} = (\sum V) \cdot e$
 $I = \text{moment of inertia per unit length of base section}$
 $= \frac{1}{12} (1) B^3$

For maximum and minimum pressures, the value of $y = B/2$.

M_{net} is the moment that you have seen either, you take summation of M_R minus summation of M_O or what you can do is you can if you know this value of e you can find out summation of V into e , I is moment of inertia per unit length of base section, that is 1 by twelve into 1 into B cube. Why we are taking 1 because in the longitudinal direction of the wall we are considering the unit length and y value will be that is for maximum and minimum pressure this value of y will be become B by 2. So, in this expression we know this, we know this, we know this and this, so we can find out this value of q , y is your B by 2 for maximum and minimum pressures.

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$$q_{\max} = q_{\text{toe}} = \frac{\sum V}{(B)(1)} + \frac{e(\sum V) \frac{B}{2}}{\frac{1}{12}(B^3)}$$
$$= \frac{\sum V}{B} \left(1 + \frac{6e}{B} \right)$$

Similarly,

$$q_{\min} = q_{\text{heel}} = \frac{\sum V}{B} \left(1 - \frac{6e}{B} \right)$$

So, you see here, q_{\max} which is occurring at toe that is equal to summation of V by B into 1, that is the area which you are considering plus e into summation of V into B by 2 divided by I which is equal to 1 by twelve B cube into 1. So, this will result into this expression that is summation V by B 1 plus $6e$ by B similarly, we can find out q_{\min} which will be at the heel part of the wall, that is q_{heel} is equal to summation V upon B 1 minus $6e$ upon B .

So, you have seen that how we can find out the value of e and then accordingly we can calculate the maximum and minimum value of this q . So, you see this is how we can find out the maximum and minimum pressure at the base of the wall, that is the maximum 1 will be occurring at toe and the minimum 1 will be occurring at heel. Now, what are the various aspects of the this analysis and how we can complete is this is we are in between half way, so may be we will be we discussing this in the next class.

So, today what exactly we saw is that, that how you can provide the check for overturning and sliding once the proportioning is done. And then we saw that what do you mean by shear key, how it is useful in increasing the value of factor of safety against sliding along the base. And then we started with the check for bearing capacity failure, we found out the maximum and minimum pressure at toe and heel respectively, after this we will be seeing all the aspects in the next class.

Thank you.