Assignment 8

The due date for submitting this assignment has passed. As per our records you have not submitted this assignment.

Due on 2019-03-27, 23:59 IST.

Problem 1: Continuous Reactor in Pharma Industry

The model for continuous reactor is given by the following equations:

\[
\begin{align*}
\frac{dS}{dt} &= D(S_{in} - S) - r \\
\frac{dX}{dt} &= -DX + Y_x r \\
\frac{dP}{dt} &= -DP + Y_p r
\end{align*}
\]

In the above equations, \(S\) is concentration of sugar, \(X\) is concentration of biomass and \(P\) is concentration of pharmaceutical product. The rate of reaction is given by:

\[r = 25 \frac{SX}{15+S}\]

The values of various parameters are: dilution rate \(D = 4\); inlet sugar concentration \(S_{in} = 5\); and yield coefficients \(Y_x = 0.7\) and \(Y_p = 0.3\).

Initially, at \(t = 0\), the concentrations are \([5.0; 0.1; 0]\). Use ode15s to solve the above system of ODEs and obtain the concentrations at \(t = 1\) and 10.

1) What is the product concentration at \(t = 1\)?

No, the answer is incorrect.
Score: 0

Accepted Answers:
(Type: Range) 0.056, 0.057

2) What is the sugar concentration at \(t = 10\)?

...
3) What is the biomass concentration at $t = 10$?

No, the answer is incorrect.

Score: 0

Accepted Answers:

(Type: Range) 0.36, 0.37

1 point

4) What is the product concentration at $t = 10$?

No, the answer is incorrect.

Score: 0

Accepted Answers:

(Type: Range) 0.15, 0.16

1 point

Problem 2: Heat Conduction – Method of Lines

Use the method of lines to solve the following problem of heat conduction:

$$\frac{dT}{dt} = \alpha \frac{\partial^2 T}{\partial x^2} - \beta (T - T_a)$$

where $T$ is the temperature in a metal rod (in degrees C). The domain in the figure above is discretized in ten intervals with $h = 0.1$. Thus, the resulting set of ODEs are:

$$\frac{dT_i}{dt} = \frac{\alpha}{h^2} \left(T_{i+1} - 2T_i + T_{i-1} \right) - \beta (T_i - T_a), \ \forall i = 1 \ to \ 9$$

(2)

The following parameter values and conditions are given:

$T_0 = 80, \ T_{10} = 40, \ \alpha = 2, \ \beta = 0.04, \ T_a = 20$

Since both the boundary temperatures are given, only the internal nodes are solved for using ode45. At the initial time, the temperature in the rod is uniformly at 80 deg C. Thus, define the initial condition as the entire solution vector, $[T] = 80$.

Solve the set of ODEs in Eq. (2) using ode45 to obtain temperatures from $t=0$ to $5$.

5) Please report the temperature $T_3$ at time $t = 5$

No, the answer is incorrect.

Score: 0

Accepted Answers:

(Type: Range) 67, 69

1 point

6) Please report the temperature $T_5$ at time $t = 5$
Problem 3: System with Sinusoidal Forcing

A system such as mass-spring-damper with sinusoidal forcing is given by:

\[ \frac{d^2x}{dt^2} + 3 \frac{dx}{dt} + 2x + 3 \sin(\omega t) = 0. \]

Consider \( \omega = 0.5 \). With initial conditions \( x(0) = 5 \) and \( dx/dt(0) = 0 \), use ode45 to compute the solution at times \( t = 2 \) and \( 5 \).

9) Report the value of \( x(2) \)

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Year: Range) 0.73, 0.74

10) Report the value of \( x(5) \)

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Year: Range) -1.3, -1.2

Problem 4: Dynamics of Infections
Infections such as chicken pox give immunity to those who are infected. Let $N$ be the number of “normal” individuals, $I$ be the number of “infected” individuals and $R$ be the number of recovered individuals. The simplest model governing dynamics of the population is:

$$
N' = -aNI \\
I' = aNI - rI \\
R' = rI
$$

Let the “normal” population in a village be 1000. At time $t = 0$, one additional infected individual enters the village. We now want to model the evolution of $N$, $I$ and $R$ members of a population, given the value of $a=0.0004$ and $r=0.16$, and time is in number of days.

Solve the above system of equations using ode45 and report the answers at $t=25$ days.

**Note:** ODE solution will give you fractions (e.g., $R = 15.213$); this behavior is expected. Please report the results as nearest integer by solving the original ODEs.

11) Report the normal population at 25 days (please report integer values only)

No, the answer is incorrect.
Score: 0

Accepted Answers:
(Type: Range) 638, 641

1 point

12) Report the infected population at 25 days (please report integer values only)

No, the answer is incorrect.
Score: 0

Accepted Answers:
(Type: Range) 181, 184

1 point