Assignment 7

The due date for submitting this assignment has passed. As per our records you have not submitted this assignment.

Due on 2019-03-20, 23:59 IST.

Problem 1: Pollutant Uptake – Euler’s Method

Consider the following first-order model of pollutant uptake:

\[ \frac{dx}{dt} = W - Qx. \]

where, \( V = 10^6 \), \( W = 10^6 \), \( Q = 10^5 \).

Consider a lake with no pollutant initially (i.e., \( x(0) = 0 \)). First, divide the above equation throughout by \( V \) to write the equation in the form \( dx/dt = \) _________.

Use Euler’s explicit method with \( h = 0.2 \) to obtain value of \( x \) at time 10.

The true solution is given by: \( x(t) = 10 \left( 1 - e^{-0.1t} \right) \). Hence compute the error between Euler’s solution and true solution.

Repeat with \( h = 0.1 \) and 0.05 also.

1) Please report the solution \( x(10) \) at time \( t = 10 \) using Euler’s method with \( h = 0.2 \)

No, the answer is incorrect.

Score: 0

Accepted Answers:

(Type: Range) 6.34, 6.37

2) Please report the error at time \( t = 10 \) using Euler’s method with \( h = 0.1 \)

No, the answer is incorrect.
4) Please choose the order of accuracy in obtaining x(10) based on these results:

- \( O(h^1) \)
- \( O(h^2) \)
- \( O(h^3) \)
- \( O(h^4) \)
- \( O(h^5) \)

No, the answer is incorrect.
Score: 0
Accepted Answers:
\( O(h^1) \)

Problem 2: Midpoint Method

Please repeat Problem-1 with Midpoint Method. Here:

\[
\begin{align*}
y_{i+1} &= y_i + h k_2 \\
k_1 &= f(t_i, y_i) \\
k_2 &= f \left( t_i + \frac{h}{2}, y_i + \frac{hk_1}{2} \right)
\end{align*}
\]

5) Please report the solution x(10) at time t = 10 using Midpoint method with h=0.2

No, the answer is incorrect.
Score: 0
Accepted Answers:
\( (Type: Range) 6.31, 6.33 \)

6) Please report the error at time t = 10 using Midpoint method with h=0.1

No, the answer is incorrect.
Score: 0
Accepted Answers:
\( (Type: Range) 6e-5, 6.3e-5 \)

7) Please report the error at time t = 10 using Midpoint method with h=0.05
No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 1.4e-5, 1.7e-5

8) Please choose the order of accuracy in obtaining x(10) based on these results

\[ O(h^1) \]
\[ O(h^2) \]
\[ O(h^3) \]
\[ O(h^4) \]
\[ O(h^5) \]

No, the answer is incorrect.
Score: 0
Accepted Answers:
\[ O(h^2) \]

Problem 3: Higher-Order RK Method

The following is SPRK method:

\[ y_{i+1} = y_i + \frac{h}{6} (k_1 + k_2 + 4k_3) \]

where, \( k_1 = f(t_i, y_i) \), \( k_2 = f(t_i + h, y_i + hk_1) \), \( k_3 = f(t_i + \frac{h}{2}, y_i + \frac{hk_1}{2} + \frac{hk_2}{4}) \)

Repeat Problem-1 with SPRK method and report the following:

9) Please report the error at time \( t = 10 \) using SPRK method with \( h=0.1 \)

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 1.5e-7, 1.6e-7
(Type: Range) 1.2e-6, 1.3e-6

10) Please report the error at time \( t = 10 \) using SPRK method with \( h=0.05 \)

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 1.82e-8, 2e-8
(Type: Range) 1.5e-7, 1.6e-7

11) Please choose the order of accuracy in obtaining x(10) based on these results

None of the options are correct.
In Problem-2, you solved the ODE using Midpoint method using \( h = 0.05 \). Let us call that solution as \( x_{MP}(10) \). In this Problem, you solved ODE using SPRK method using the same \( h = 0.05 \), which we will call \( x_{SP}(10) \).

Compute the absolute difference between the two solutions, i.e., \( \Delta = |x_{MP}(10) - x_{SP}(10)| \).

Observe that \( \Delta \) is approximately equal to the error in RK-2 Midpoint Method.

Problem 4: Problem

A piece of metal, initially at 1200 K is allowed to cool down in air. Assuming heat is lost only due to radiation, the differential equation for the temperature of the ball is given by

\[
\frac{dT}{dt} = -2.2067 \times 10^{-12} (T^4 - 300^4), \quad T(0) = 1200
\]

Use ode45 to solve the above ODE and compute temperature at 240 and 480 seconds.

Please report the temperature at 240 s

No, the answer is incorrect.
Score: 0

Accepted Answers:
(Type: Range) 770,780

1 point

Please report the temperature at 480 s

No, the answer is incorrect.
Score: 0

Accepted Answers:
(Type: Range) 645,650

1 point