Module 5

DC to AC Converters

Lesson
34
Analysis of 1-Phase, Square - Wave Voltage Source Inverter

After completion of this lesson the reader will be able to:

(i) Explain the operating principle of a single-phase square wave inverter.
(ii) Compare the performance of single-phase half-bridge and full-bridge inverters.
(iii) Do harmonic analysis of load voltage and load current output by a single-phase inverter.
(iv) Decide on voltage and current ratings of inverter switches.

Voltage source inverters (VSI) have been introduced in Lesson-33. A single-phase square wave type voltage source inverter produces square shaped output voltage for a single-phase load. Such inverters have very simple control logic and the power switches need to operate at much lower frequencies compared to switches in some other types of inverters, discussed in later lessons. The first generation inverters, using thyristor switches, were almost invariably square wave inverters because thyristor switches could be switched on and off only a few hundred times in a second. In contrast, the present day switches like IGBTs are much faster and used at switching frequencies of several kilohertz. As pointed out in Lesson-26, single-phase inverters mostly use half bridge or full bridge topologies. Power circuits of these topologies are redrawn in Figs. 34.1(a) and 34.1(b) for further discussions.

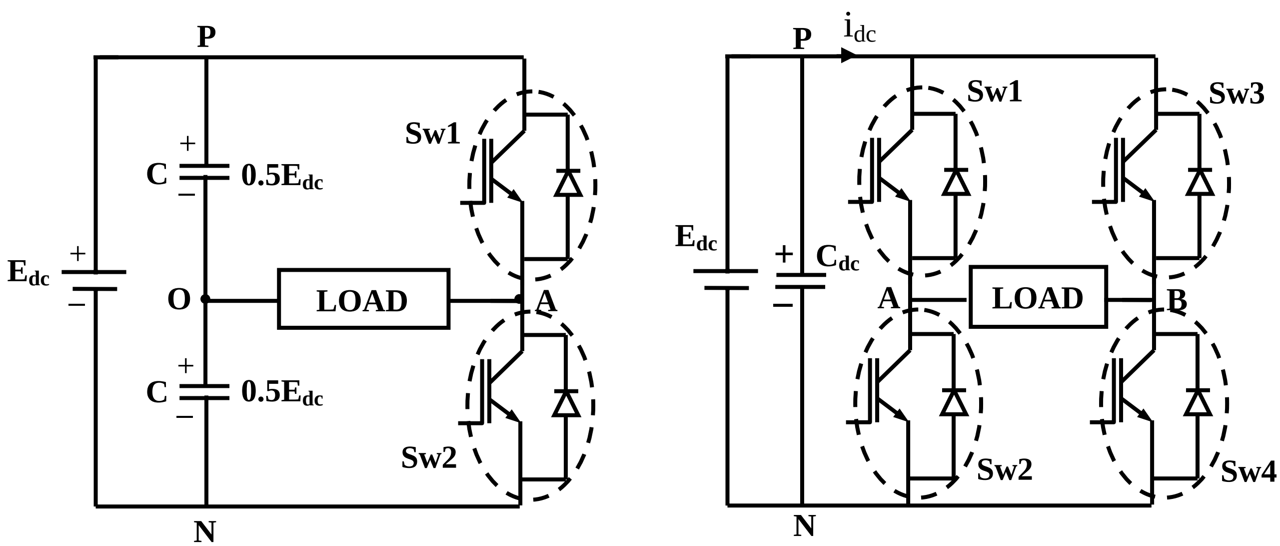

Fig. 34.1(a): A 1-phase half bridge VSI **Fig. 34.1(b): A 1-phase full-bridge VSI**

In this lesson, both the above topologies are analyzed under the assumption of ideal circuit conditions. Accordingly, it is assumed that the input dc voltage (E_{dc}) is constant and the switches are lossless. In half bridge topology the input dc voltage is split in two equal parts through an ideal and loss-less capacitive potential divider. The half bridge topology consists of one leg (one pole) of switches whereas the full bridge topology has two such legs. Each leg of the inverter consists of two series connected electronic switches shown within dotted lines in the figures. Each of these switches consists of an IGBT type controlled switch across which an uncontrolled diode is put in anti-parallel manner. These switches are capable of conducting bi-directional current but they need to block only one polarity of voltage. The junction point of the switches in each leg of the inverter serves as one output point for the load.

In half bridge topology the single-phase load is connected between the mid-point of the input dc supply and the junction point of the two switches (in Fig. 34.1(a) these points are marked as 'O' and 'A' respectively). For ease of understanding, the switches Sw1 and Sw2 may be assumed to be controlled mechanical switches that open and close in response to the switch control signal. In fact in lesson-33 (section 33.2) it has been shown that the actual electronic switches mimic the function of the mechanical switches. Now, if the switches Sw1 and Sw2 are turned on alternately with duty ratio of each switch kept equal to 0.5, the load voltage (V_{AO}) will be square wave with a peak-to-peak magnitude equal to input dc voltage (E_{dc}). Fig. 34.2(a) shows a typical load voltage waveform output by the half bridge inverter. V_{AO} acquires a magnitude of +0.5 E_{dc} when Sw1 is on and the magnitude reverses to -0.5 E_{dc} when Sw2 is turned on. Fig. 24.2 also shows the fundamental frequency component of the square wave voltage, its peak-to-peak magnitude being equal to $4/_{\pi} E_{dc}$. The two switches of the inverter leg are turned on in a complementary manner. For a general load, the switches should neither be simultaneously on nor be simultaneously off. Simultaneous turn-on of both the switches will amount to short circuit across the dc bus and will cause the switch currents to rise rapidly. For an inductive load, containing an inductance in series, one of the switches must always conduct to maintain continuity of load current. In Lesson-33 (section 33.2) a case of inductive load has been considered and it has been shown that the load current may not change abruptly even though the switching frequency is very high. Such a situation, as explained in lesson-33, demands that the switches must have bi-directional current carrying capability.

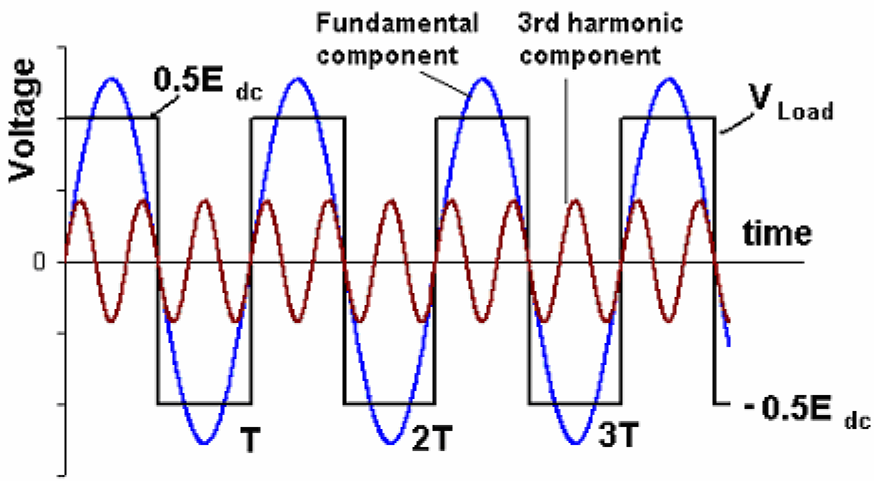

Fig. 34.2(a): Square wave load voltage output by half-bridge inverter

34.1 Harmonic Analysis of The Load Voltage And Load Current Waveforms

The load voltage waveform shown in Fig. 34.2(a) can be mathematically described in terms of its Fourier's components as:

$$V_{AO} = \sum_{n=1,3,5,7,...,\infty} \frac{2E_{dc}}{n\pi} \sin(nwt) \quad \ldots\ldots (34.1)$$

,where 'n' is the harmonic order and $w/_{2\pi}$ is the frequency ('f') of the square wave. 'f' also happens to be the switching frequency of the inverter switches. As can be seen from the expression of Eqn. 34.1, the square wave load voltage consists of all the odd harmonics and their magnitudes are inversely proportional to their harmonic order. Accordingly, the fundamental frequency component has a peak magnitude of $\frac{2}{\pi}E_{dc}$ and the nth harmonic voltage (n being odd integer) has a peak magnitude of $\frac{2}{n\pi}E_{dc}$. The magnitudes of very high order harmonic voltages become negligibly small. In most applications, only the fundamental component in load voltage is of practical use and the other higher order harmonics are undesirable distortions. Many of the practical loads are inductive with inherent low pass filter type characteristics. The current waveforms in such loads have less higher order harmonic distortion than the corresponding distortion in the square-wave voltage waveform. A simple time domain analysis of the load current for a series connected R-L load has been presented below to corroborate this fact. Later, for comparison, frequency domain analysis of the same load current has also been done.

34.1.1 Time Domain Analysis

The time domain analysis of the steady state current waveform for a R-L load has been presented here. Under steady state the load current waveform in a particular output cycle will repeat in successive cycles and hence only one square wave period has been considered. Let t=0 be the instant when the positive half cycle of the square wave starts and let I_0 be the load current at this instant. The negative half cycle of square wave starts at t=0.5T and extends up to T. The circuit equation valid during the positive half cycle of voltage can be written as below:

$$Ri + L\frac{di}{dt} = 0.5E_{dc}, \text{ for } 0 < t < 0.5T \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(34.2)$$

Similarly the equation for the negative half cycle can be written as

$$Ri + L\frac{di}{dt} = -0.5E_{dc}, \text{ for } 0.5T < t < T \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(34.3)$$

, where T (=1/f) is the time period of the square wave.

The instantaneous current 'i' during the first half of square wave may be obtained by solving Eqn.(34.2) and putting the initial value of current as I_0.

Accordingly, $i(t) = \frac{0.5E_{dc}}{R}(1 - e^{-t/\tau}) + I_0 e^{-t/\tau}$ for 0 < t < 0.5T ……………..(34.4)

, where τ= L/R is the time constant of the R-L load.

The current at the end of the positive half cycle becomes the starting current for the negative half cycle.

Thus the next half cycle starts with an initial current = $\frac{0.5E_{dc}}{R}(1 - e^{-T/2\tau}) + I_0 e^{-T/2\tau}$. The circuit equation for the next half cycle may now be written as

$$i(t) = -\frac{0.5E_{dc}}{R}(1 - e^{-(t-\frac{T}{2})/\tau}) + \left[\frac{0.5E_{dc}}{R}(1 - e^{-T/2\tau}) + I_0 e^{-T/2\tau}\right] e^{-(t-\frac{T}{2})/\tau} \quad \text{for } 0.5T < t < T$$

Simplifying the above equation one gets:

$$i(t) = -\frac{0.5E_{dc}}{R}(1+e^{-t/\tau}) + I_0 e^{-t/\tau} + \frac{E_{dc}}{R} e^{-(t-\frac{T}{2})/\tau}$$, for 0.5T < t < T …….(34.5)

Under steady state, the instantaneous magnitude of inductive load current at the end of a periodic cycle must equal the current at the start of the cycle. Thus putting t=T in Eqn. (34.5), one gets the expression for I_0 as,

$$I_0 = -\frac{0.5E_{dc}}{R}(1+e^{-T/\tau}) + I_0 e^{-T/\tau} + \frac{E_{dc}}{R} e^{-T/2\tau}$$

$$\text{or, } I_0\left(1-e^{-T/\tau}\right) = \frac{0.5E_{dc}}{R}(1-e^{-T/\tau}) + \frac{E_{dc}}{R}\left(e^{-T/2\tau}-1\right)$$

$$\text{or, } I_0 = \frac{0.5E_{dc}}{R} - \frac{E_{dc}}{R}\left(\frac{1}{1+e^{-T/2\tau}}\right) = -\frac{0.5E_{dc}}{R}\left[\frac{1-e^{-T/2\tau}}{1+e^{-T/2\tau}}\right]$$ ………….(34.6)

Substituting the above expression for I_0 in Eqn. (34.4) one gets,

$$i(t) = \frac{0.5E_{dc}}{R}\left[\frac{1+e^{-T/2\tau} - 2e^{-t/\tau}}{1+e^{-T/2\tau}}\right]$$, for 0 < t < 0.5T ………..………..(34.7)

It may be noted from Eqn. (34.7) that the load current at the end of the positive half cycle of square wave (at t=0.5T) simply turns out to be $-I_0$. This is expected from the symmetry of the load voltage waveform. Load current expression for the negative half cycle of square wave can similarly be calculated by substituting for I_0 in Eqn. (34.5). Accordingly,

$$i(t) = -\frac{0.5E_{dc}}{R} + \frac{E_{dc}}{R}\left[\frac{e^{-(t-\frac{T}{2})/\tau}}{\left(1+e^{-T/2\tau}\right)}\right]$$, for 0.5T < t < T

$$\text{or, } i(t) = -\frac{0.5E_{dc}}{R}\left[\frac{1+e^{-T/2\tau} - 2e^{-(t-\frac{T}{2})/\tau}}{1+e^{-T/2\tau}}\right]$$, for 0.5T < t < T .... (34.8)

The current expressions given by Eqns. (34.7) and (34.8) have been plotted in Figs. 34.2(b) to 34.2(e) for different time constants of the R-L load. The current waveforms have been normalized against a base current of $0.5\frac{E_{dc}}{R}$. The square wave voltage waveform, normalized against a base voltage of $0.5E_{dc}$ has also been plotted together with the current waveforms. It can be seen that the load current waveform repeats at fundamental frequency and the higher order harmonic distortions reduce as the load becomes more inductive. For L/R ratio of 2, the 3^{rd} order harmonic distortion in the load current together with its fundamental component has been shown in Fig. 34.2(e). In this case, it can be seen that the relative harmonic distortion in load current waveform is much lower than that of the voltage waveform shown in Fig. 34.2(a). The basis for calculating the magnitude of different harmonic components of load current waveform has been shown in the next subsection that deals with frequency domain analysis.

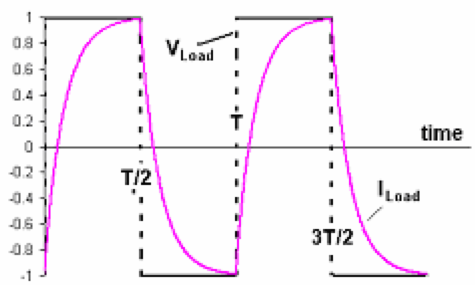

Fig. 34.2(b): Normalized load voltage and load current for L/R = 0.1T

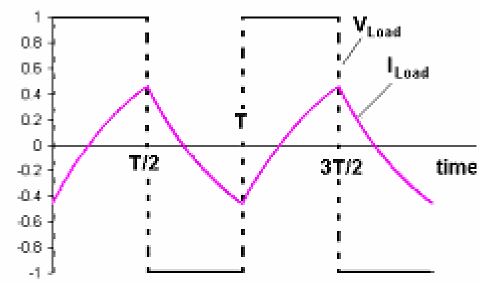

Fig. 34.2(c): Normalized load voltage and load current for L/R = 0.5T

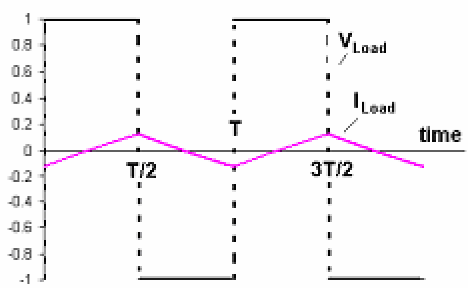

Fig. 34.2(d): Normalized load voltage and load current for L/R = 2T

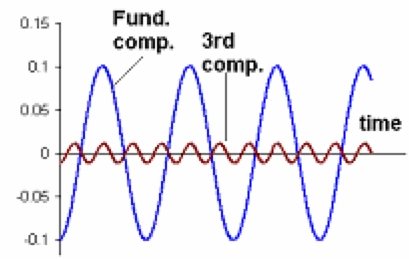

Fig. 34.2(e): Normalized fundamental and 3rd harmonic components of load current for L/R = 2T

34.1.2 Frequency Domain Analysis

The square shape load voltage may be taken as superposition of different harmonic voltages described by Eqn. 34.1. The load current may similarly be taken as superposition of harmonic currents produced by the different harmonic voltages. The load current may be expressed in terms of these harmonic currents. To illustrate this the series connected R-L load has once again been considered here. First the expressions for different harmonic components of load current are calculated in terms of load parameters: R and L/R (or τ) and inverter parameters: dc link voltage (E_{dc}) and time period of square wave (T).

For the fundamental harmonic frequency the load impedance (Z_1) and load power factor angle (ϕ_1) can be calculated to be

$$Z_1 = \sqrt{R^2 + (4\pi^2 L^2 / T^2)} \text{ and } \phi_1 = \tan^{-1}\left(\frac{2\pi L}{TR}\right) \ldots\ldots\ldots\ldots\ldots(34.9)$$

The load impedance and load power factor angle for the n[th] harmonic component (Z_n and ϕ_n respectively) will similarly be given by,

$$Z_n = \sqrt{R^2 + (4\pi^2 n^2 L^2 / T^2)} \text{ and } \phi_n = \tan^{-1}\left(\frac{2\pi nL}{TR}\right) \ldots\ldots\ldots\ldots(34.10)$$

The fundamental and n[th] harmonic component of load current, $(I_{load})_1$ and $(I_{load})_n$ respectively, can be found to be

$$(I_{load})_1 = \frac{2E_{dc}}{\pi Z_1}\sin(wt - \Phi_1) \text{ and } (I_{load})_n = \frac{2E_{dc}}{n\pi Z_n}\sin(nwt - \Phi_n) \ldots\ldots\ldots(34.11)$$

The algebraic summation of the individual harmonic components of current will result in the following expression for load current.

$$I_{Load} = \sum_{n=1,3,5,7,\ldots,\infty} \frac{2E_{dc}}{n\pi Z_n}\sin(nwt - \Phi_n) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(34.12)$$

From Eqns. 34.10 and 34.12 it may be seen that the contribution to load current from very higher order harmonics become negligible and hence the infinite series based expression for load current may be terminated beyond certain values of harmonic order 'n'. For L/R ratio = 2T, the individual harmonic components of load current normalized against a base current of $\frac{0.5E_{dc}}{R}$ have been calculated below:

$$(I_{load})_{1,normalized} = \frac{4}{\pi\sqrt{1+16\pi^2}}\sin(wt - \tan^{-1} 4\pi) = 0.1\sin(wt - 1.491)$$

$$(I_{load})_{3,normalized} = \frac{4}{3\pi\sqrt{1+144\pi^2}}\sin(3wt - \tan^{-1} 12\pi) = 0.011\sin(3wt - 1.544)$$

$$(I_{load})_{5,normalized} = \frac{4}{5\pi\sqrt{1+400\pi^2}}\sin(5wt - \tan^{-1} 20\pi) = 0.004\sin(5wt - 1.555)$$

$$(I_{load})_{7,normalized} = \frac{4}{7\pi\sqrt{1+784\pi^2}}\sin(7wt - \tan^{-1} 28\pi) = 0.002\sin(7wt - 1.559)$$

$$(I_{load})_{11,normalized} = \frac{4}{11\pi\sqrt{1+1936\pi^2}}\sin(11wt - \tan^{-1} 44\pi) = 0.0008\sin(11wt - 1.564)$$

It may be concluded that for L/R = 2T, the contribution to load current from 13th and higher order harmonics are less than 1% of the fundamental component and hence they may be neglected without any significant loss of accuracy.

Fig. 34.2(f) shows the load voltage and algebraic summation of the first five dominant harmonics (fundamental, 3rd, 5th, 7th and 11th) in the load current, the expressions for which have been given above. In Fig. 34.2(g) the load current waveforms of Fig. 34.2(e) and 34.2(f) have been superimposed for comparison. It may be seen that the load current waveform of Fig. 34.2(f) calculated using truncated series of the frequency domain analysis very nearly matches with the exact waveform of Fig. 34.2(e), calculated using time domain analysis.

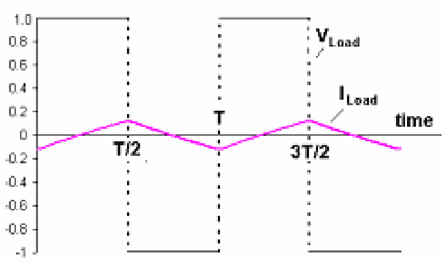

Fig. 34.2(f): Normalized load voltage and load current for L/R = 2T

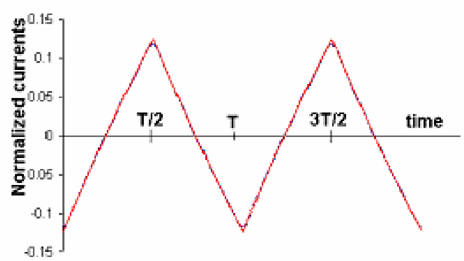

Fig. 34.2(g): Superimposed normalized load current waveforms for L/R = 2T

34.2 Analysis Of The Single-Phase Full Bridge Inverter

Single-phase half bridge inverter has already been described above. The single-phase full bridge circuit (Fig. 34.1(b)) can be thought of as two half bridge circuits sharing the same dc bus. The full bridge circuit will have two pole-voltages (V_{AO} and V_{BO}), which are similar to the pole voltage V_{AO} of the half bridge circuit. Both V_{AO} and V_{BO} of the full bridge circuit are square waves but they will, in general, have some phase difference. Fig. 34.3 shows these pole voltages staggered in time by 't' seconds. It may be more convenient to talk in terms of the phase displacement angle 'Φ' defined as below:

$$\Phi = (2\pi)\frac{t}{T} \text{ Radians} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(34.13)$$

, where 't' is the time by which the two pole voltages are staggered and 'T' is the time period of the square wave pole voltages.

The pole voltage V_{AO} of the full bridge inverter may again be written as in Eqn. 34.1, used earlier for the half bridge inverter. Taking the phase shift angle 'Φ' into account, the pole-B voltage may be written as

$$V_{BO} = \sum_{n=1,3,5,7,\ldots,\infty} \frac{2E_{dc}}{n\pi} \sin n(wt - \Phi) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(34.14)$$

Difference of V_{AO} and V_{BO} gives the line voltage V_{AB}. In full bridge inverter the single phase load is connected between points 'A' and 'B' and the voltage of interest is the load voltage V_{AB}. Taking difference of the voltage expressions given by Eqns. 34.1 and 34.14, one gets

$$V_{AB} = \sum_{n=1,3,5,7,\ldots,\infty} \frac{2E_{dc}}{n\pi} \left[\sin nwt - \sin n(wt - \Phi)\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(34.15)$$

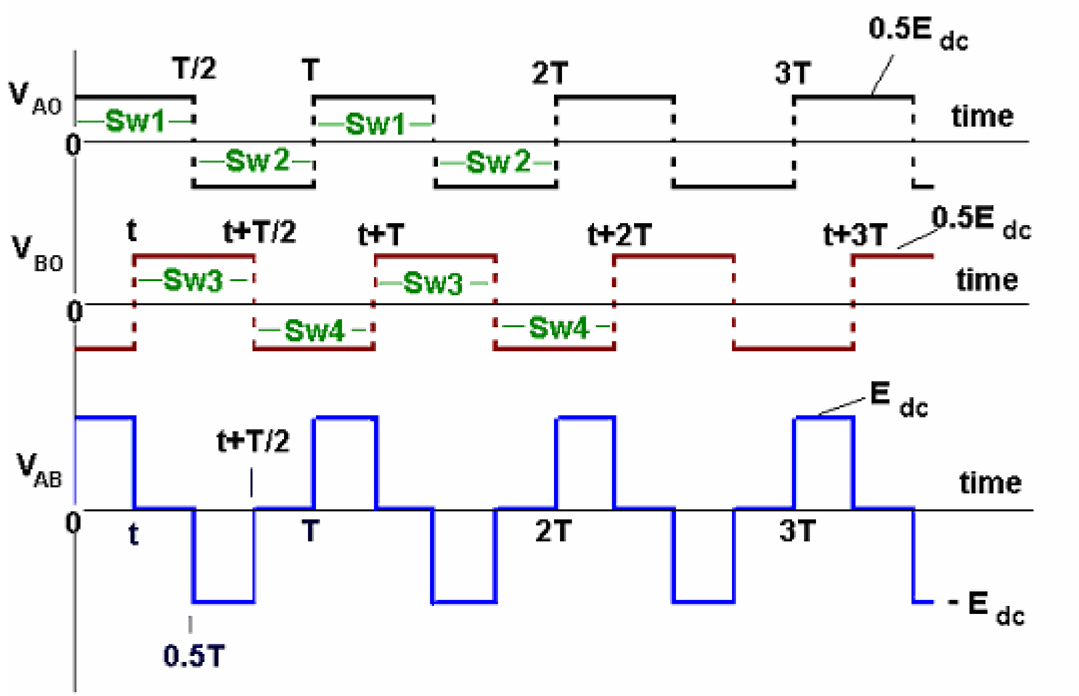

Fig. 34.3: Pole and line voltages output by 1-ph. full bridge inverter

The fundamental component of V_{AB} may be written as

$$V_{AB,1} = \frac{2E_{dc}}{\pi}\left[\sin wt - \sin(wt - \Phi)\right] = \frac{4E_{dc}}{\pi}\cos(wt - \frac{\Phi}{2})\sin\frac{\Phi}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(34.16)$$

The n^{th} harmonic component in V_{AB} may similarly be written as

$$V_{AB,n} = \frac{2E_{dc}}{n\pi}\left[\sin nwt - \sin n(wt - \Phi)\right] = \frac{4E_{dc}}{n\pi}\cos n(wt - \frac{\Phi}{2})\sin\frac{n\Phi}{2} \quad \ldots\ldots\ldots\ldots\ldots(34.17)$$

From Eqn. 34.16, the rms magnitude of the fundamental component of load voltage may be written as

$$(V_{AB,1})_{rms} = 0.9E_{dc}\sin\frac{\Phi}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(34.18)$$

The rms magnitude of load voltage can be changed from zero to a peak magnitude of $0.9E_{dc}$. The peak load voltage magnitude corresponds to Φ = 180 degrees and the load voltage will be zero for Φ = 0^0. For Φ = 180 degrees, the load voltage waveform is once again square wave of time period T and instantaneous magnitude E.

As the phase shift angle changes from zero to 180^0 the width of voltage pulse in the load voltage waveform increases. Thus the fundamental voltage magnitude is controlled by pulse-width modulation.
Also, from Eqns. 34.17 and 34.1 it may be seen that the line voltage distortion due to higher order harmonics for pulse width modulated waveform (except for Φ = 180^0) is less than the corresponding distortion in the square wave pole voltage. In fact, for some values of phase shift angle (Φ) many of the harmonic voltage magnitudes will drastically reduce or may even get eliminated from the load voltage. For example, for Φ = 60^0 the load voltage will be free from 3^{rd} and multiples of third harmonic.

34.3 Voltage And Current Ratings Of Inverter Switches

Switches in each leg of the inverter operate in a complementary manner. When upper switch of a leg is on the lower switch will need to block the entire dc bus voltage and vice versa. Thus the switches must be rated to block the worst-case instantaneous magnitude of dc bus voltage. In practical inverters the switch voltage ratings are taken to be somewhat higher than the worst-case dc voltage to account for stray voltages produced across stray inductances, the turn-on transient voltage of a power diode etc. For a well laid out circuit a 50% margin over the dc-bus voltage may be the optimum switch voltage rating. Each switch of the inverter carries load current during half of the current cycle. Hence the switches must be rated to withstand the peak magnitude of instantaneous load current. The semiconductor switches have very small thermal time constant and they cannot withstand overheating for more than a few milli seconds. Thus even though the load current passes through the switches only in alternate half cycles, the thermal limit may be reached during half cycle of current itself. It may be pointed out that each inverter switch consists of a controlled switch in anti-parallel with a diode. The distribution of current between the diode and the controlled switch will depend on the load power factor at the operating frequency. In general both diode as well as the controlled switch should be rated to carry the peak load current.

34.4 Applications Of Square Wave Inverter

The square wave voltage-source inverter discussed in this lesson finds application in many low cost ac motor drives, uninterruptible power supply units and in circuits utilizing electrical resonance between an inductor and a capacitor. Some examples of circuits utilizing resonance phenomenon are induction heating units and electronic ballasts for fluorescent lamps.

Quiz Problems

1. A single-phase full bridge inverter with square wave pole voltages is connected to a dc input voltage of 600 volts. What maximum rms load voltage can be output by the inverter? How much will be the corresponding rms magnitude of 3^{rd} harmonic voltage

   (a) Approximately 270 volts of fundamental and 30 volts of 3^{rd} harmonic voltage
   (b) Approx. 480 volts fundamental and 160 volts of 3^{rd} harmonic voltage
   (c) Approx. 540 volts fundamental and 180 volts of 3^{rd} harmonic voltage
   (d) Approx. 270 volts fundamental and 90 volts of 3^{rd} harmonic voltage

2. How does the output power handling capacity of a single-phase half bridge inverter compare with that of a single-phase full bridge inverter when they are connected to same dc bus voltage and the peak current capability of the inverter switches is also same. Also compare their costs.

   (a) The half bridge inverter can output double power but cost also doubles.
   (b) The half bridge inverter can output only half the power but cost is less.
   (c) The half bridge inverter can output only half the power but cost is nearly same
   (d) The output power capability is same but half bridge inverter costs less.

3. A single-phase full bridge inverter is connected to a purely resistive load. Each inverter switch consists of an IGBT in anti-parallel with a diode. For this load how does the diode conduction loss compare with the IGBT conduction loss?

(a) Diode and IGBT will have nearly same conduction loss
(b) Diode conduction loss will be nearly half of the IGBT loss
(c) Diode will have no conduction loss
(d) IGBT will have no conduction loss

4. Using frequency domain analysis estimate the ratio of 5^{th} and 7^{th} harmonic currents in a purely inductive load that is connected to the output of a single phase half bridge inverter with square wave pole voltages.

   (a) 5^{th} harmonic current will be nearly double of the 7^{th} harmonic current
   (b) 5^{th} harmonic current will be 40% more than the 7^{th} harmonic current
   (c) 5^{th} harmonic current will be zero while 7^{th} harmonic current will be present
   (d) Both 5^{th} and 7^{th} harmonic currents will be zero

(Answers to the quiz problems: 1-d, 2-b, 3-c, 4-a)