

# Module 10

## Measuring Instruments

# Lesson 43

## Study of Electro-Dynamic Type Instruments

## Objectives

- To understand the basic construction of a dynamometer instrument.
- Explain basic operation and development of torque expressions for ammeter, voltmeter and wattmeter.
- Study of ammeter, voltmeter and wattmeter connections.
- To investigate the errors involve in wattmeter readings and its compensation.
- Understanding the effect of inductance of voltage coil (moving coil) on the wattmeter readings.

### L.43.1 Introduction

Electrodynamic type instruments are similar to the PMMC-type elements except that the magnet is replaced by two serially connected fixed coils that produce the magnetic field when energized (see Fig.43.1). The fixed coils are spaced far enough apart to allow passage of the shaft of the movable coil. The movable coil carries a pointer, which is balanced by counter weights. Its rotation is controlled by springs. The motor torque is proportional to the product of the currents in the moving and fixed coils. If the current is reversed, the field polarity and the polarity of the moving coil reverse at the same time, and the turning force continues in the original direction. Since the reversing the current direction does not reverse the turning force, this type of instruments can be used to measure AC or DC current, voltage, or its major application as a wattmeter for power measurement. In the first two cases, the moving and fixed are serially connected. For power measurement, one of the coils (usually the fixed coils) passes the load current and other coil passes a current proportional to the load voltage. Air friction damping is employed for these instruments and is provided by a pair of Aluminum-vanes attached to the spindle at the bottom. These vanes move in a sector shaped chamber. Cost and performance compared with the other types of instruments restrict the use of this design to AC or DC power measurement. Electro-dynamic meters are typically expensive but have the advantage of being more accurate than moving coil and moving iron instrument but its sensitivity is low. Similar to moving iron vane instruments, the electro dynamic instruments are true RMS responding meters. When electro dynamic instruments used for power measurement its scale is linear because it predicts the average power delivered to the load and it is calibrated in average values for AC. Voltage, current and power can all be measured if the fixed and moving coils are connected appropriately. Other parts of the instruments are described briefly below:

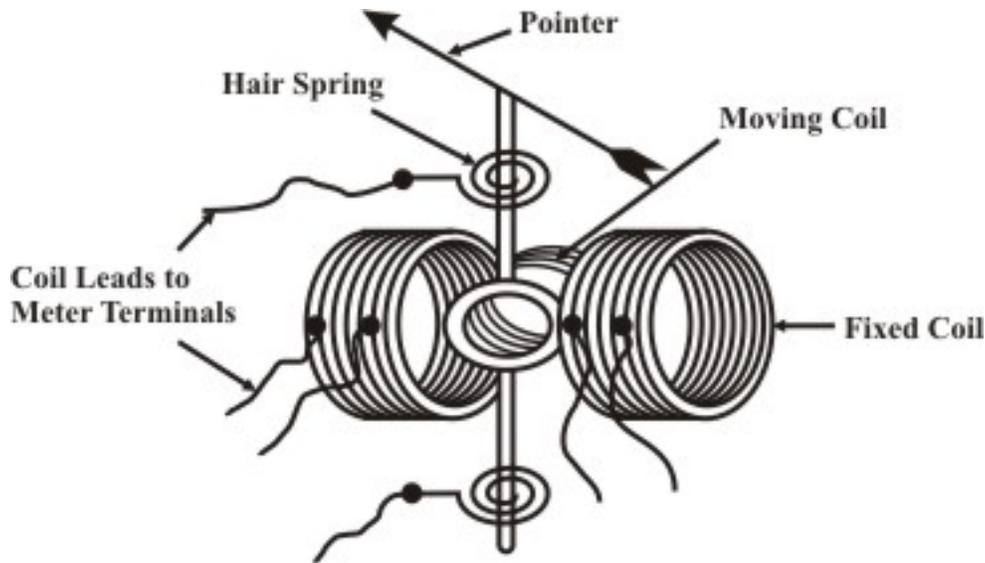


Fig. 43.1(a)

### Electro dynamic (or Dynamometer) type Instruments:

**Fixed coil:** The magnetic field is produced by the fixed coil which is divided into two sections to give more uniform field near the centre and to allow passage of the instrument shaft.

**Moving coil:** The moving coil is wound either as a self-sustaining coil or else on a non-magnetic former. A metallic former cannot be used, as eddy currents would be induced in it by alternating field. Light but rigid construction is used for the moving coil. It should be noted that both fixed and moving coils are air cored.

**Springs:** The controlling torque is provided by two control springs. These hairsprings also act as leads of current to the moving coil.

**Dampers:** Air friction damping is employed for these instruments and is provided by a pair of Aluminum-vanes attached to the spindle at the bottom. These vanes move in a sector shaped chamber.

**Shielding:** Since the magnetic field produced by fixed coils is weaker than that in other types of instruments, these meters need a special magnetic shielding. Electro-dynamic instruments are effectively shielded from the effects of external magnetic fields by enclosing the mechanism in a laminated iron hollow cylinder with closed ends.

### L.43.2 Operating Principle

Let us consider the currents in the fixed and moving coils are  $i_f$  and  $i_m$  respectively. The action of electrodynamic instrument depends upon the force exerted between fixed and moving coils carrying current. The flux density  $B$  ( $wb/m^2$ ) produced by the fixed coil is proportional to  $i_f$

(fixed coil current). The force on the conductors of the moving coil, for a given strength field, will be proportional to  $i_m$  (moving coil current) and the number of turns '  $N$  ' of the moving coil. In case of ammeter and voltmeter fixed and moving coils are connected in series and the developed torque is due to the interaction of the magnetic fields produced by currents in the fixed and moving coils and thus it will be proportional to  $i^2$  ( $i_f = i_m = i$ ). Thus, dynamic instruments can be used for dc and ac measurements.

### Expression for developed torque:

#### Case-a: Torque expression based on energy concept

Let us assume that the fixed and moving coils having self-inductances  $L_f$  and  $L_m$  respectively. Further it is assumed that the mutual inductance between the fixed and movable coils is  $M$ .

Total energy stored in the magnetic field of the coils is given by

$$W = \frac{1}{2} L_f i_f^2 + \frac{1}{2} L_m i_m^2 + M i_f i_m \quad (43.1)$$

where  $i_f$  and  $i_m$  are the currents through the fixed and moving coils. From equation (43.1) one can write the expression for torque developed as

$$T_d = \frac{dW}{d\theta} = i_f i_m \frac{dM}{d\theta} \quad (43.2)$$

Note  $L_f$  and  $L_m$  are not functions of  $\theta$  but the mutual inductance '  $M$  ' between the coils is a function of the deflection  $\theta$  (i.e relative position of moving coil). The equivalent inductance between fixed and moving coils can be found out as

$L_{eq} = L_f + L_m + 2M$  (cumulative manner) and from this one can find the mutual inductance between them as

$$M = \frac{1}{2} [L_{eq} - (L_f + L_m)]$$

With all deflection type instruments, however, the mutual inductance varies with the relative positions of the moving and fixed coils. The maximum value  $M_{max}$  of the mutual inductance occurs when the axes of the moving and fixed coils are aligned with  $\theta = 180^\circ$ , as this position gives the maximum flux linkage between coils. When  $\theta = 0^\circ$ ,  $M = - M_{max}$ . If the plane of the moving coil is at an angle  $\theta$  with the direction of B that produced by the fixed coil, then the mutual inductance  $M$  is expressed by

$$M = - M_{max} \cos \theta \quad (43.3)$$

**D.C operation:** Expression (43.2) for the developed torque is rewritten by setting  $i_f = I_f(d.c)$  and  $i_m = I_m(d.c)$

$$T_d = I_f I_m \frac{dM}{d\theta} = I_f I_m M_{max} \sin \theta \quad (43.4)$$

If the control is due to spiral springs, the controlling torque is proportional to the angle of deflection  $\theta$ .

$$\text{Controlling torque } T_c = k_s \theta \quad (43.5)$$

where  $k_s$  is the spring constant.

Note that  $T_d = T_c$  at steady deflection, i.e.,

$$\begin{aligned} I_f I_m \frac{dM}{d\theta} &= k_s \theta \\ \therefore \theta &= \frac{I_f I_m}{k_s} \left( \frac{dM}{d\theta} \right) \end{aligned} \quad (43.6)$$

**A.C operation:** The dynamometer instrument is used to measure alternating current or voltage, the moving coil—due to its inertia takes up a position where the average deflecting torque over a complete cycle is balanced by the restoring torque of the spiral spring. The deflecting torque is proportional to the mean value of the square of the current or voltage (note both coils are connected in series for ammeters or voltmeters), and the instrument scale can therefore be calibrated to read r.m.s values of alternating current or voltage.

$$\text{Average deflecting torque } T_{d,av} = \frac{1}{T} \int_0^T i_f(t) i_m(t) \frac{dM}{d\theta} dt$$

Let  $i_f(t) = I_{\max,f} \sin \omega t$ ,  $i_m(t) = I_{\max,m} \sin(\omega t - \alpha)$  where ' $\alpha$ ' is the phase-angle between two currents

$$T_{d,av} = I_1 I_2 \cos \alpha \frac{dM}{d\theta} \quad (43.7)$$

where  $I_1$  and  $I_2$  are the r.m.s values of fixed and moving coil currents. In steady state condition of deflection,

$$\begin{aligned} T_{d,av} &= T_c \\ I_1 I_2 \cos \alpha \frac{dM}{d\theta} &= k_s \theta \\ \therefore \theta &= \frac{I_1 I_2 \cos \alpha}{k_s} \frac{dM}{d\theta} \end{aligned} \quad (43.8)$$

In case of ammeter or voltmeter, both the coils are connected in series and the same current is flowing through the coils. Equation (43.8) can be written as

$$\therefore \theta = \frac{I^2}{k_s} \frac{dM}{d\theta} \quad (43.9)$$

where  $I_1 = I_2 = I$  and  $\alpha = 0^\circ$ .

### Case-b: Torque expression based on electro-magnetic force concept

Fig. 43.1(b) shows that the torque exerted in all electro-dynamic instruments depend upon the current  $i_m$  flowing through the moving coil and the magnetic flux density  $B$  which is directly proportional to the current  $i_f$  through the fixed coil.



Fig. 43.1(b): Magnetic - field and torque developed in moving coil of an electro dynamoter.

The torque acting upon the moving coil can be easily calculated if the flux density  $B$  is assumed constant throughout the space occupied by the moving coil.

Consider the length of the coil is  $l$  and the width is ' $2r$ ', then when a current  $i_m$  flows through it, the force acting on the top portion of the moving coil of  $N$  turns is  $f = N B l i_m$  and hence the resulting torque  $T_d$  is  $f = 2 N B l i_m r$ . If the plane of the moving coil is at an angle  $\phi$  with the direction of  $B$ , then resulting clockwise torque developed on the moving coil is given by

$$T_d = 2 N B l i_m r \cos \phi = 2 N k_f i_f l i_m r \cos \phi$$

where, the flux density  $B$  is directly proportion to the fixed coil current i.e  $B = k_f i_f$ . At steady state condition of deflection, the resulting torque  $T_d$  developed by the moving coil is balanced by the spring restraining torque  $T_c = k_s \theta$ , one can obtain the following relationship.

$$T_d = 2 N k_f i_f l i_m r \cos \phi = k_s \theta \Rightarrow \theta = \left( 2 N \frac{k_f}{k_s} l r \cos \phi \right) i_f i_m$$

We see that the angle of deflection  $\theta$  depends on the product of the moving coil current  $i_m$  and the fixed coil current  $i_f$ . When the both coils carry alternating currents, say  $i_f = i_{\max,f} \cos(\omega t + \alpha_1)$  and  $i_m = i_{\max,m} \cos(\omega t + \alpha_2)$ , the average value of deflection-angle  $\theta$  expression has the form

$$\theta = k_\phi I_{\max,f} I_{\max,m} \cos \alpha$$

where the value of  $k_\phi \left( = N \frac{k_f}{k_s} l r \cos \phi \right)$  varies as the value of  $\phi$  varies. Note that the average value of the product of two instantaneous current signals is expressed as  $I_{\max,f} I_{\max,m} \cos \alpha$  where  $\alpha = \alpha_1 - \alpha_2$ .

## L.43.2.1 More about ammeters

### Ammeters

Fig. 43.2(a) shows that fixed coils and moving coil of a dynamometer instrument connected in series and assumed the current through moving coil does not exceed a certain the upper limit depending on its construction.

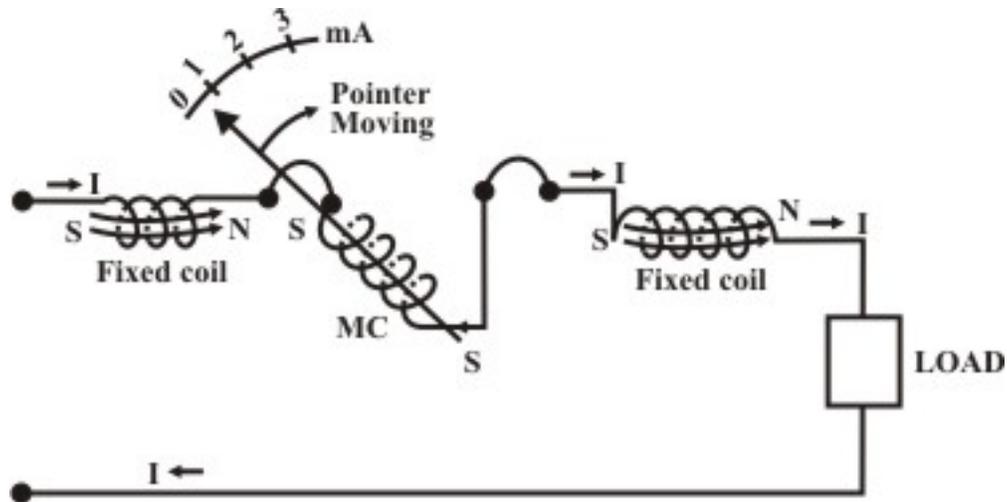


Fig. 43.2(a): Torque produces positive deflection (clockwise)

The flux direction through the fixed and movable coils due to current is shown in Fig. 43.2 (a). it can be noted that the  $N$ -pole of the moving coil flux is reflected from the adjacent  $N$ -pole of the fixed coil and on the other side adjacent  $S$ -poles are also repelled each other. This results the pointer to move clockwise direction from 'zero position' to a steady position depending upon the magnitude of current flowing through the coils. Fig.43.2(b) illustrate the effect of reversing the direction of the current through the coils and shows that the deflecting torque produces movement of the pointer in the same direction. This means that the dynamometer instrument suitable for both dc and ac measurements of current and voltage. The dynamic instrument when uses as a voltmeter, the fixed coils wounded with thin wire are connected in series with the moving coil and a non-inductive resistance (see Fig.43.5). For ammeter application the fixed coils are connected in parallel with the moving coil, and in parallel with a shunt, as required (see Fig.43.4).

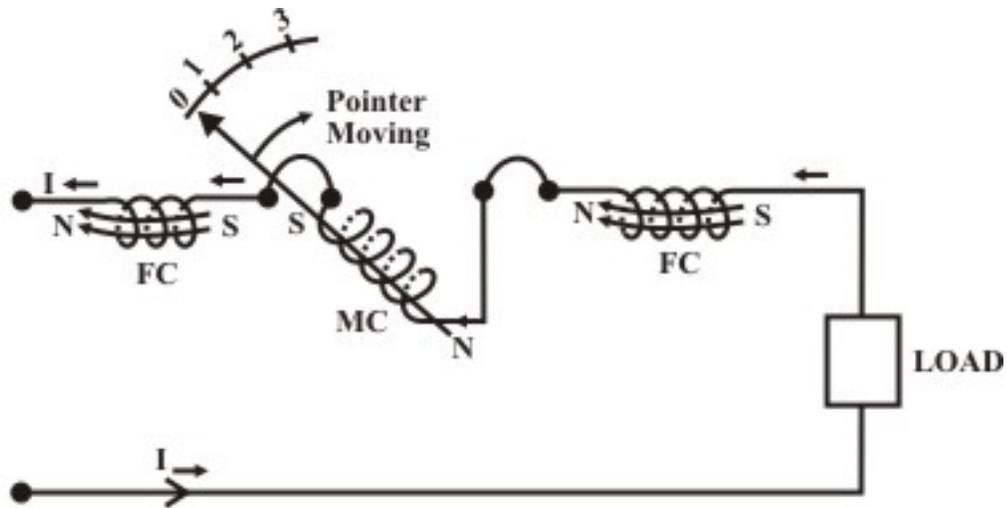


Fig. 43.2(b): Current flowing right to left produces positive deflection again.

**Remarks:** The scale of the instrument can be conveniently calibrated on dc and then used to measure ac.

### L.43.2.2 Ranges of Ammeters and Voltmeters

#### Ammeters

A given size of instruments requires a definite number of ampere-turns to be supplied by the fixed and moving coils to obtain a full-scale deflection. Ammeter ranges are altered by changing the number of turns and size of conductor in the fixed and moving coils. A double range instrument may easily be obtained by connecting different coil sections either in series or in parallel. The internal connections are shown in Fig.43.3. The maximum range for which ammeters are usually constructed is dependent on its application. For ammeter use in which only fraction of rated current (say 200 ma) is carried by the moving coil to alter its range by changing the mode of connection of the fixed coils.

**Voltmeters:** With voltmeters the ranges is altered by changing the number of turns in the coils and the value of series resistances, but the range of a given instrument may be increased by connecting additional resistances in series with it. For example, the range of a given voltmeter may be doubled while connecting in series with it a non-inductive resistance equal in value to the original resistance of the instrument.

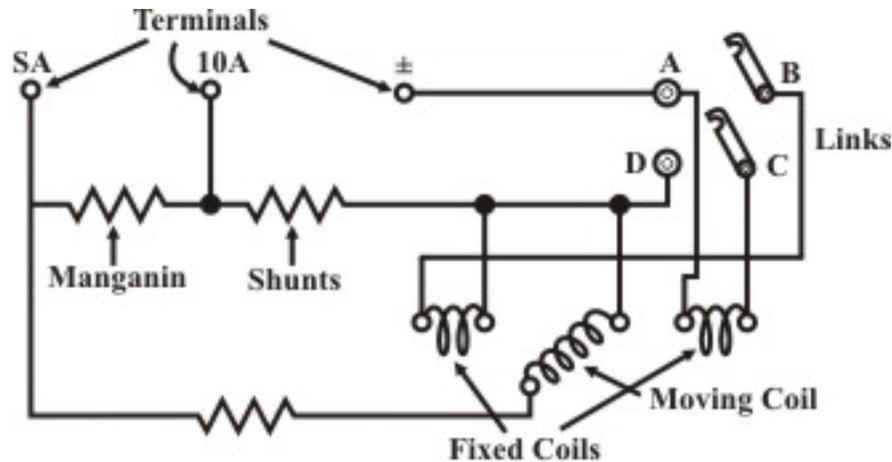


Fig. 43.3: Connections of double-range electro-dynamic ammeter. (For higher range, Fixed coils are in parallel; for lower range, fixed coils are in series).

## L. 43.3 Connections for ammeter, voltmeter and wattmeter

### Ammeter

When ammeters for ranges above about  $250\text{ mA}$ , the moving coil cannot be connected in series with the fixed coil (note the control spring is unsuitable for currents above about  $250\text{ mA}$ ). Therefore, the moving coil must be connected in parallel with the fixed coils as shown in Fig 43.4.

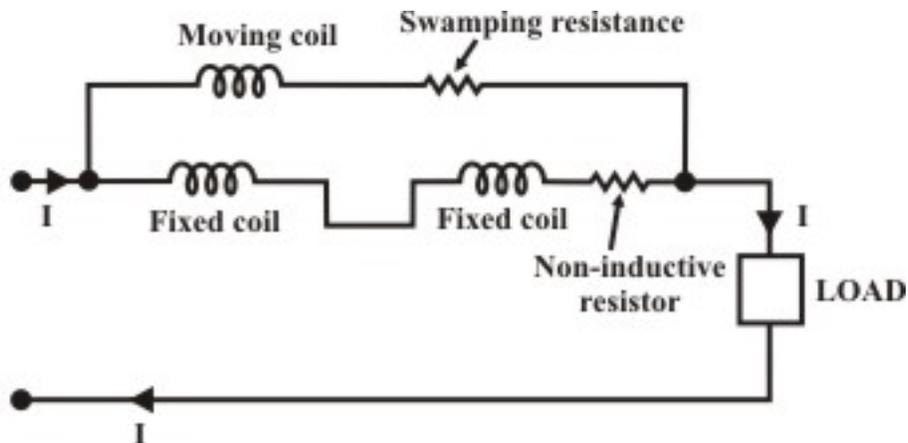


Fig. 43.4: Electro dynamic Ammeter (above 250mA) connection

Here the moving coil current is kept within  $200\text{ mA}$  and the rest of current is passed through the fixed coil. Moving coil carries a small fraction of measured current through the moving coil. For extreme accuracy the connection shown in Fig. 43.4 must fulfill the following conditions.

- The resistance/reactance ratio must have the same value (i.e time constant of moving coil = time constant of fixed coil) for each branch.

- The percentage change of resistance with temperature must be the same for the two branches.

**Voltmeters:** The connection for use as a voltmeter is shown in Fig. 43.5, in which fixed and moving coils are connected in series with a high series resistance having “zero resistivity coefficients”.

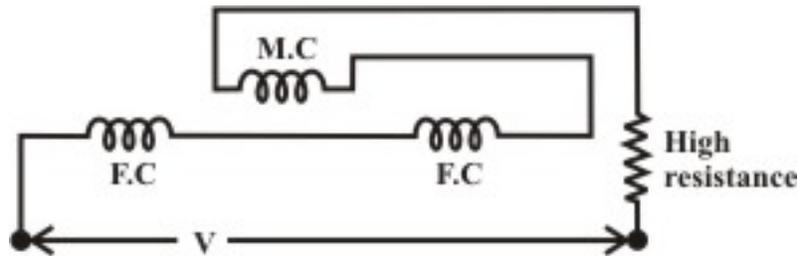


Fig. 43.5: Electro-dynamic voltmeter connection

This combination is connected across the voltage source or across the load terminals whose voltage is to be measured. The deflecting torque is given by

$$T_d = i_f i_m \frac{dM}{d\theta} = \frac{V}{Z} \frac{V}{Z} \frac{dM}{d\theta}$$

where  $Z$  is the magnitude of total impedance of the voltmeter circuit. At steady state condition of deflection

$$\begin{aligned} T_d &= T_c \\ \therefore k_s \theta &= \frac{V^2}{Z^2} \frac{dM}{d\theta} \\ \therefore \theta &= \frac{V^2}{Z^2 k_s} \frac{dM}{d\theta} \end{aligned} \quad (43.10)$$

This implies that deflecting torque is directly proportional to  $V^2$  if  $\frac{dM}{d\theta}$  is kept nearly constant.

This is possible if  $\theta$  varies from  $45^\circ$  to  $135^\circ$  over the range of instrument scale.

**Remarks:** Electro-dynamic meter’s use is much more common for ac voltmeters than for ac ammeters because of practical limitation on the current through the moving coil. Electro-dynamic ammeter needs to read r.m.s values of alternating current accurately irrespective of signal waveform or distortion of signal waveform.

**Wattmeter:** Perhaps the most important use of the electro-dynamometer is for the wattmeter. The mechanism of electro dynamic wattmeter closely resembles that of an electro-dynamic ammeter, but the moving coil of wattmeter is connected in series with a high non-inductive resistance. It provides with separate terminals to connect across the load terminals. The fixed coil is connected in series with the load to have the same load current. A typical connection of an electro-dynamometer for use as a wattmeter is shown in Fig. 43.6.

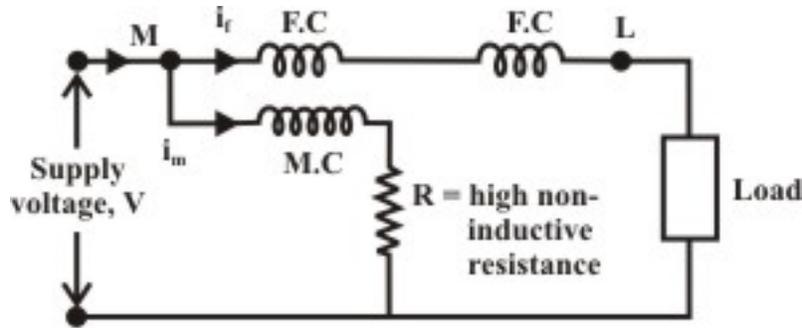


Fig. 43.6: Wattmeter Connection

For a d.c circuit, the fixed coil current  $i_f = I_L$  is the load current, and the moving-coil current  $i_m$  has a value  $V/R$ . The deflecting torque in a d.c circuit is proportional to the power and it is given by

$$\theta \propto i_f i_m \frac{dM}{d\theta}$$

or

$$\theta \propto I_L \frac{V}{R} \frac{dM}{d\theta}$$

$$\propto \text{power} \left( \text{if } \frac{dM}{d\theta} \text{ is nearly constant} \right).$$

**For a.c circuit:** Let the applied voltage  $v(t) = V_m \sin \omega t$ ; and the currents through the moving and fixed coils are given by

$$i_m(t) = \frac{V_m}{R} \sin \omega t \quad (\text{assuming inductance of moving coil is negligible})$$

$$i_f(t) = i_L(t) = I_m \sin(\omega t \pm \phi)$$

where  $\phi$  is the power factor angle of the load (  $+\phi$  leading p.f of the load and  $-\phi$  for lagging p.f of the load).

Instantaneous deflecting torque

$$T_d(t) \propto i_f(t) i_m(t) \frac{dM}{dt} \propto V_m I_m \sin \omega t \sin(\omega t \pm \phi) \quad (43.11)$$

The mean or average torque

$$T_{d,av} \propto \frac{1}{T} \int_0^T V_m I_m \sin \omega t \sin(\omega t \pm \phi) dt$$

$$\propto V I \cos \phi \quad (43.12)$$

where  $V$  and  $I$  are the r.m.s values of load voltage and current respectively. It may be noted that the developed torque must be equal to the controlling torque at steady state. In other words, the controlling torque  $T_c \propto \theta$  and this implies that  $\theta \propto \text{power (average)}$ . Thus an electro-dynamic instrument, connected as shown in Fig. 43.6, becomes a wattmeter which will give a direct deflection of the power in either dc or ac circuit.

**Remarks:**

- The moving coil is usually called the voltage coil (or pressure coil) and carries a small current proportional to voltage across the coil.
- The fixed coils are called the current coils and will carry load current.
- The terminal 'M' is connected to the source side where as the terminal 'L' is connected to the load side.

**Wattmeter Errors:**

A wattmeter is normally required to measure power in the load. Two modes of wattmeter connections to the load are shown in Fig. 43.7(a) and Fig.43.7(b). For the connection shown in Fig. 43.7(a), the power supplied by the source to load =  $VI \cos \phi$  where  $\phi$  is the load power factor.

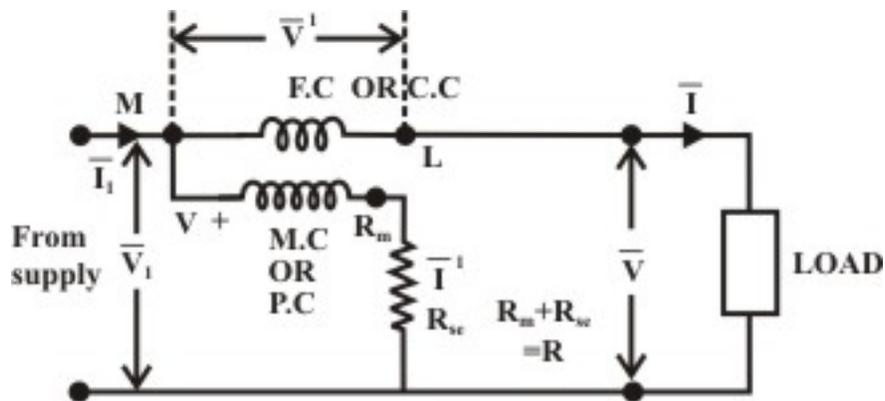


Fig. 43.7(a)

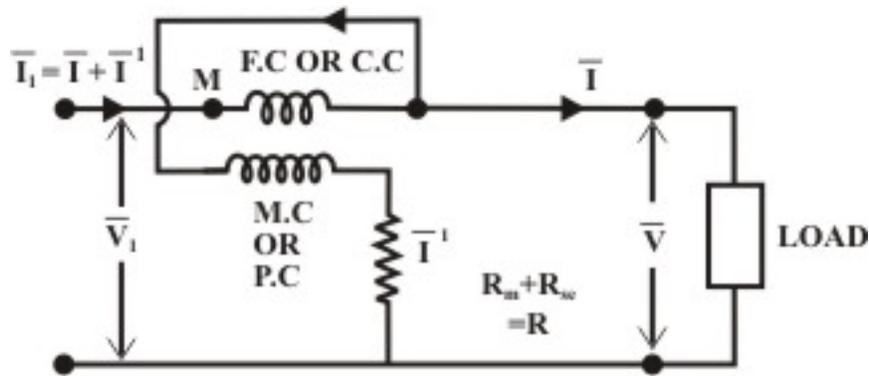
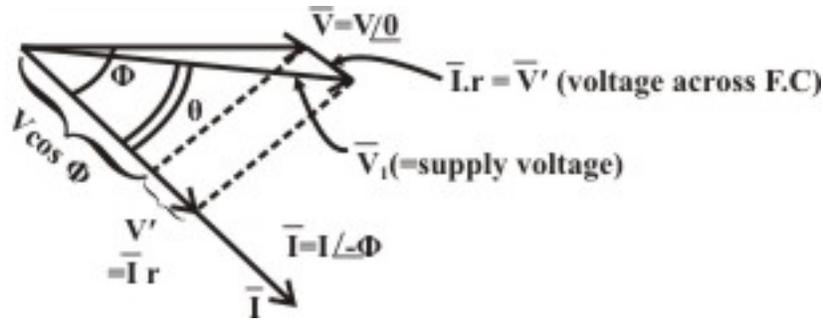


Fig. 43.7(b)

Refer to Fig.43.7(a), and let us study the reading of the wattmeter and its is expressed as

$$\begin{aligned}
 \text{Wattmeter reading} &= V_1 I \cos(\angle \bar{V}_1, \angle \bar{I}) = V_1 I \cos \theta = (V_1 \cos \theta) I \\
 &= (V \cos \phi + V') I \quad (\text{see phasor diagram}) \\
 &= VI \cos \phi + V'I = VI \cos \phi + I.r.I = VI \cos \phi + I^2 r \quad (43.13)
 \end{aligned}$$

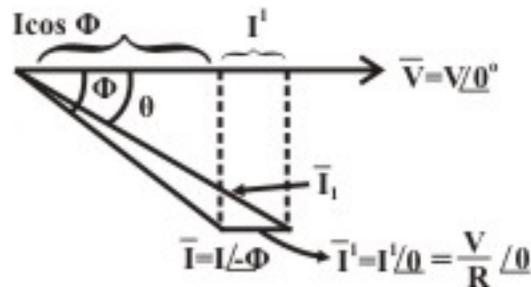


where  $V'$  is the magnitude voltage across the current coil or fixed coil.

Equation (43.13) shows that the wattmeter reading is equal to the sum of power consumed in the load ( $VI \cos \phi$ ) + power loss ( $I^2 r$ ) in the fixed coil of resistance ' $r$ '  $\Omega$ .

If the connections are those of Fig. 43.7(b) the total current  $\bar{I}_1$  through the current coil will be the vector sum of the load current  $\bar{I}$  and the voltage coil or pressure coil or moving coil  $\bar{I}'$  ( $I' = \frac{V}{R}$  where  $R$  is the resistance of the voltage coil). The wattmeter reading corresponding to the circuit configuration Fig. 43.7(b) is given by

$$\begin{aligned}
 \text{Wattmeter reading} &= V I_1 \cos(\angle \bar{V}, \angle \bar{I}_1) = V I_1 \cos \theta \\
 &= V(I \cos \phi + I') = VI \cos \phi + VI' \\
 &= VI \cos \phi + V \frac{V}{R} = VI \cos \phi + \frac{V^2}{R} \\
 &= \text{Power in load} + \text{Power in voltage coil.}
 \end{aligned} \tag{43.14}$$



These results can be also applied in d.c circuits; the verification of this is simple, as phase angles are not involved.

**Remarks:**

- Losses  $\left( I^2 r \text{ or } \frac{V^2}{R} \right)$  are normally small in instrument-circuit.
- Normally the connection Fig. 43.7(a) is better, but under some heavy load current conditions the wattmeter circuit configuration Fig 43.7(b) is preferable.

## How to Compensate Wattmeter Error?

The error involved in wattmeter reading with the circuit configuration Fig 43.7(b) can easily be eliminated by introducing double-wound current coil. One conductor being the current coil as usual. The additional conductor is an internal connection, corresponding to the lead from  $L$  to  $V^+$  of Fig. 43.8(a), which carries the voltage-coil current in a reverse direction through the winding.

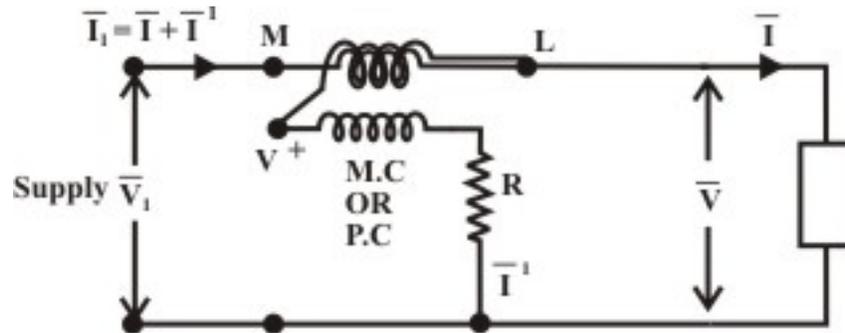


Fig. 43.8(a): Compensated wattmeter

Thus any extra torque due to the voltage-coil current in the current coil itself is neutralized by the torque due to the voltage-coil current in the additional winding.

**Note:** (i) There are watt-meters, that directly read the power consumed by the load  $P_L$ . In such a meter, the moving-coil (voltage coil) current goes through an additional fixed coil located so as to cancel the effect of  $I$  on the current in the fixed coil.

(ii) The input terminals of each coil (fixed and moving coils) is identified as  $\pm$  sign as shown in Fig.43.8(b). The marked  $\pm$  terminal of the current coil should be connected to the incoming line or to the source side and the voltage coil marked  $\pm$  terminal should be connected to the positive side of the load. With the wattmeter terminal connection specified, the meter will read up-scale when power is delivered to the load. If one of the coils is connected in reverse polarity (backwards), the wattmeter will drive downscale and may be damaged. To get, up-scale reading of watt-meters, the current coil connection should be reversed.

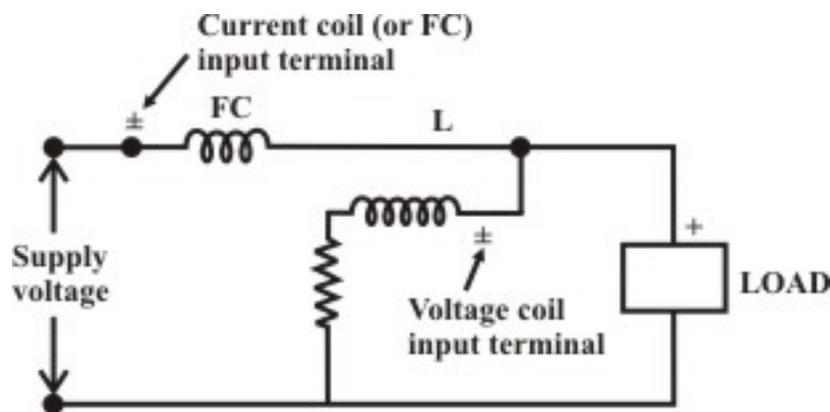
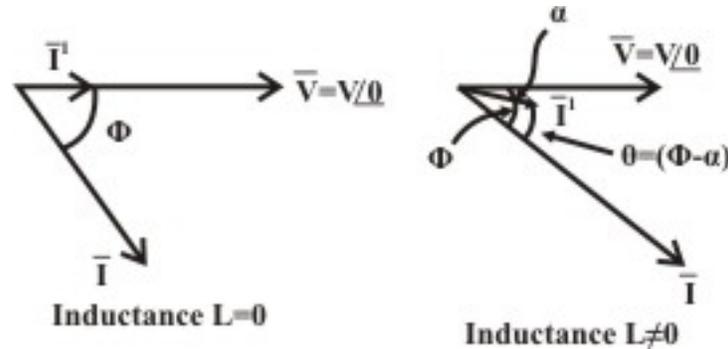


Fig. 43.8(b): Wattmeter terminal connection

## L.43.4 Inductance of Voltage–Coil Introduces an Error in Wattmeter

Let us consider the wattmeter connection as shown in Fig. 43.7(b). The mean torque of an electro-dynamic instrument is proportional to the mean value of the product  $i_1 i_2$ , where  $i_1$  and  $i_2$  being instantaneous values of the two currents. It may also be said that the mean torque is proportional to



$I_1 I_2 \cos \theta$ , where  $I_1$  and  $I_2$  are r.m.s values and  $\theta$  the phase angle between the two currents (pressure coil current and current coil current). The current through moving coil  $I = \frac{V}{R}$ , is in the same phase as that of load voltage, where  $R$  is the resistance of a non-inductive voltage-coil circuit. Then  $\theta = \phi$ , load phase angle.

Instantaneous Torque Expression (neglecting inductance of voltage coil or moving coil):-

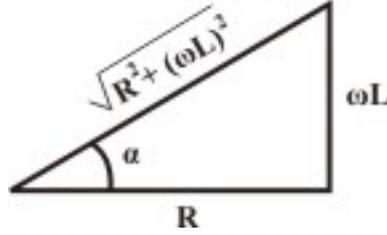
$$\begin{aligned}
 T_{\text{inst}} &= \frac{V_m \sin \omega t}{R} I_m \sin(\omega t - \phi) \frac{dM}{d\theta} \\
 T_{\text{av}} &= \frac{1}{RT} \left[ \int_0^T \frac{V_m I_m}{2} 2 \sin \omega t \sin(\omega t - \phi) dt \right] \frac{dM}{d\theta} \\
 &= \frac{1}{RT} VI \left[ \int_0^T [\cos(\phi) - \cos(2\omega t - \phi)] dt \right] \frac{dM}{d\theta} \\
 &= \frac{VI}{RT} [\cos \phi]_0^T = \frac{VI}{R} \cos \phi \frac{dM}{d\theta} \tag{43.15}
 \end{aligned}$$

In practice, the voltage-coil must possess some inductances; at a given frequency, let the resulting reactance be  $X_L = \omega L$ .

The instantaneous current through the voltage

$$i'(t) = \frac{V_m \sin(\omega t - \alpha)}{\sqrt{R^2 + (\omega L)^2}} \quad \text{where } \alpha = \tan^{-1} \frac{\omega L}{R}$$

where  $v(t) = V_m \sin \omega t$  = voltage across the load.



$$\begin{aligned}
 T_{\text{instantaneous}}(t) &= I_m \sin(\omega t - \phi) \frac{V_m \sin(\omega t - \alpha)}{\sqrt{R^2 + (\omega L)^2}} \frac{dM}{d\theta} \\
 &= \frac{V_m I_m}{\sqrt{R^2 + (\omega L)^2}} \sin(\omega t - \phi) \sin(\omega t - \alpha) \frac{dM}{d\theta} \\
 T_{\text{av}} &= \frac{1}{T} \int_0^T \frac{V_m I_m}{\sqrt{R^2 + (\omega L)^2}} \sin(\omega t - \phi) \sin(\omega t - \alpha) \frac{dM}{d\theta} dt \\
 &= \frac{VI}{\sqrt{R^2 + (\omega L)^2}} \cos(\phi - \alpha) \frac{dM}{d\theta} \\
 &= \frac{VI}{R} \cos \alpha \cos(\phi - \alpha) \frac{dM}{d\theta} \tag{43.16}
 \end{aligned}$$

Comparison of equations (43.15) and (43.16) shows that the correction factor by which the deflection must be multiplied is  $\frac{\cos \phi}{\cos \alpha \cos(\phi - \alpha)}$ .

**Remarks:**

- As  $\alpha$  is very small, it is usually sufficiently accurate to take the correction factor as (i)  $\frac{\cos \phi}{\cos(\phi - \alpha)}$  (43.17)  
for lagging power factor of the load. (ii)  $\frac{\cos \phi}{\cos(\phi + \alpha)}$  for leading power factor of the load.
- The effect of inductance in the moving coil circuit is to cause the wattmeter to read high on **lagging power factor** (see the equation (43.16)).
- For **leading power factor** the wattmeter will read low.
- Correction factor is zero at load of unity power factor.

## L43.5 Advantages and disadvantages of electro-dynamic instruments

### Advantages:

- i Free from hysteresis and eddy current errors.
- ii Applicable to both dc and ac circuits.
- iii Precision grade accuracy for 40 Hz to 500 Hz.
- iv Electro-dynamic voltmeters give accurate r.m.s values of voltage irrespective of waveforms.

### Disadvantages:

- i Low torque/weight ratio, hence more frictional errors.
- ii More expensive than PMMC or MI instruments.
- iii Power consumption higher than PMMC but less than MI instruments.

For these reasons, dynamometer ammeters and voltmeters are not in common use (except for calibration purpose) especially in dc circuits. The most important application of the dynamometer type instruments used as dynamometer wattmeter.

## L43.6 Test your Understanding

Marks: 50

T.1 Derive an expression for the torque on a dynamometer ammeter in terms of the currents in the coils and the rate of change of mutual inductance with deflection. [10]

T.2 A dynamometer wattmeter is connected with the voltage coil on the supply side of the current coil. Derive an expression for a correction factor to allow for the inductance of voltage coil. Calculate the correction factor of the wattmeter if the phase angle of the voltage coil is  $1^\circ$  and the power factor of the load is (i) 0.8 lagging(ii) 0.8 leading. [8]

[Answer: (i) 0.987, (ii) 1.013.]

T.3 An electro dynamic wattmeter is used to measure the power consumed by the load. The load voltage is 250 v and the load current is 10A at a lagging power factor of 0.5. The wattmeter voltage circuit has a resistance of  $2000 \Omega$  and inductance 40 mH. The voltage coil (moving coil) is connected directly across the load. Estimate the percentage error in the wattmeter reading. (Assume the supply frequency = 50 HZ) [8]

(Answer: 0.2% high)

T.4 A dynamometer ammeter is arranged so that  $1/100^{\text{th}}$  of the total current passes through the moving coil and the remainder through the fixed coil. The mutual inductances between the two coils varies with the angle of displacement of the moving coil from its zero position as indicated below:

$\theta$	0	$15^\circ$	$30^\circ$	$60^\circ$	$90^\circ$	$105^\circ$	$120^\circ$
$M(\mu H)$	-336	-275	-192	0	192	275	336

If a torque of  $1.05 \times 10^{-5} \text{ N-m}$  is required to give a full-scale deflection of  $120^\circ$ , calculate the current at half and full-scale deflection. [12]

(Answer: 1.187A, 2.46A)

T.5 A voltmeter has a range of 120 volts and a resistance of  $1550 \Omega$  at  $20^\circ\text{C}$ , of which  $77 \Omega$  is due to the resistances of fixed and moving coils and the remainder  $1473 \Omega$  for non-inductive resistance connected to the moving coil. The inductances of the instrument (measured at  $50 \text{ Hz}$ ) for number of position of the moving system are given in Table. When voltmeter used on a 100 volt dc circuit and  $50 \text{ Hz}$ , 100 volt a.c. circuit.

<b>Applied voltage</b>	0	40	60	80	100	120
<b>Angular deflection</b>	0	7	13.8	24	37.1	54
<b>Inductance (mH)</b>	70.1	72.5	74.8	78.3	82.8	88.6

The inductance of the fixed coil is  $74.5 \text{ mH}$  and moving coil is  $2.2 \text{ mH}$ .

Calculate (i) Mutual inductance between fixed and moving coils against applied voltage using above data

(ii) Plot mutual inductances against angular deflection and comments on  $\frac{dM}{d\theta}$ .

(iii) Power loss in the instrument.

[6+3+3]

Answer: (i)

<b>Applied voltage</b>	0	40	60	80	100	120
<b>Mutual inductance (mH)</b>	-3.3	-2.1	-0.95	0.8	3.05	5.95

(ii)  $\frac{dM}{d\theta}$  is constant.

(iii) 6.45 watts.