

Single Side Band (SSB) Modulation

- In DSB-SC it is observed that there is symmetry in the bandstructure. So, even if one half is transmitted, the other half can be recovered at the receiver. By doing so, the bandwidth and power of transmission is reduced by half.

Depending on which half of DSB-SC signal is transmitted, there are two types of SSB modulation

1. Lower Side Band (LSB) Modulation
2. Upper Side Band (USB) Modulation

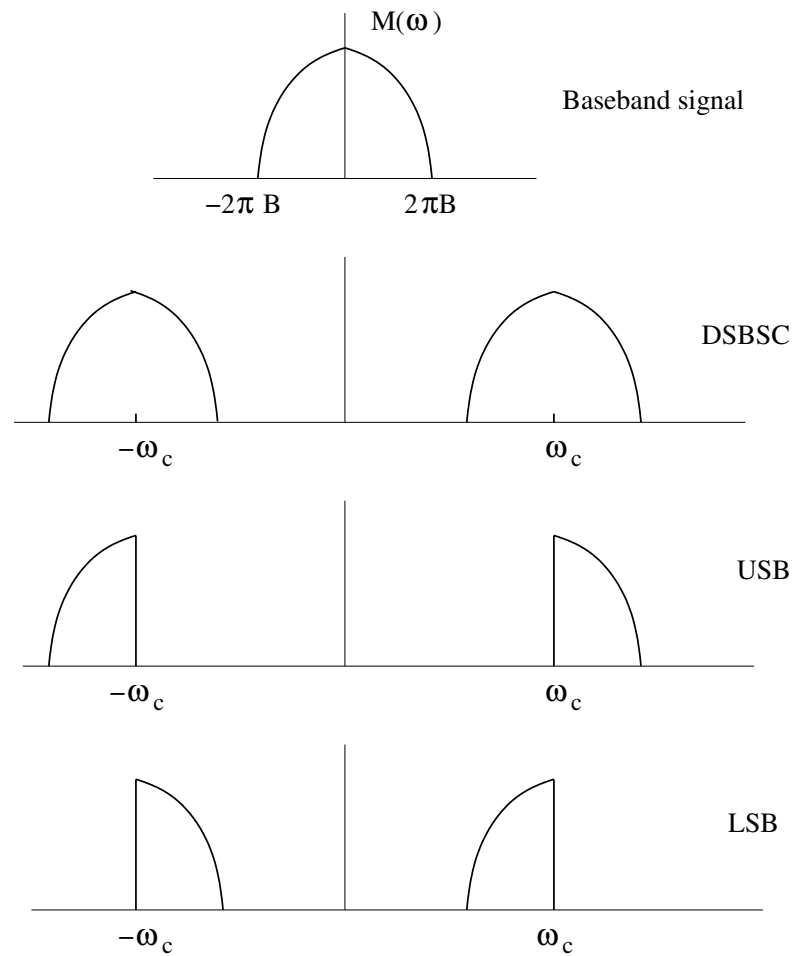


Figure 1: SSB signals from original signal

- Mathematical Analysis of SSB modulation

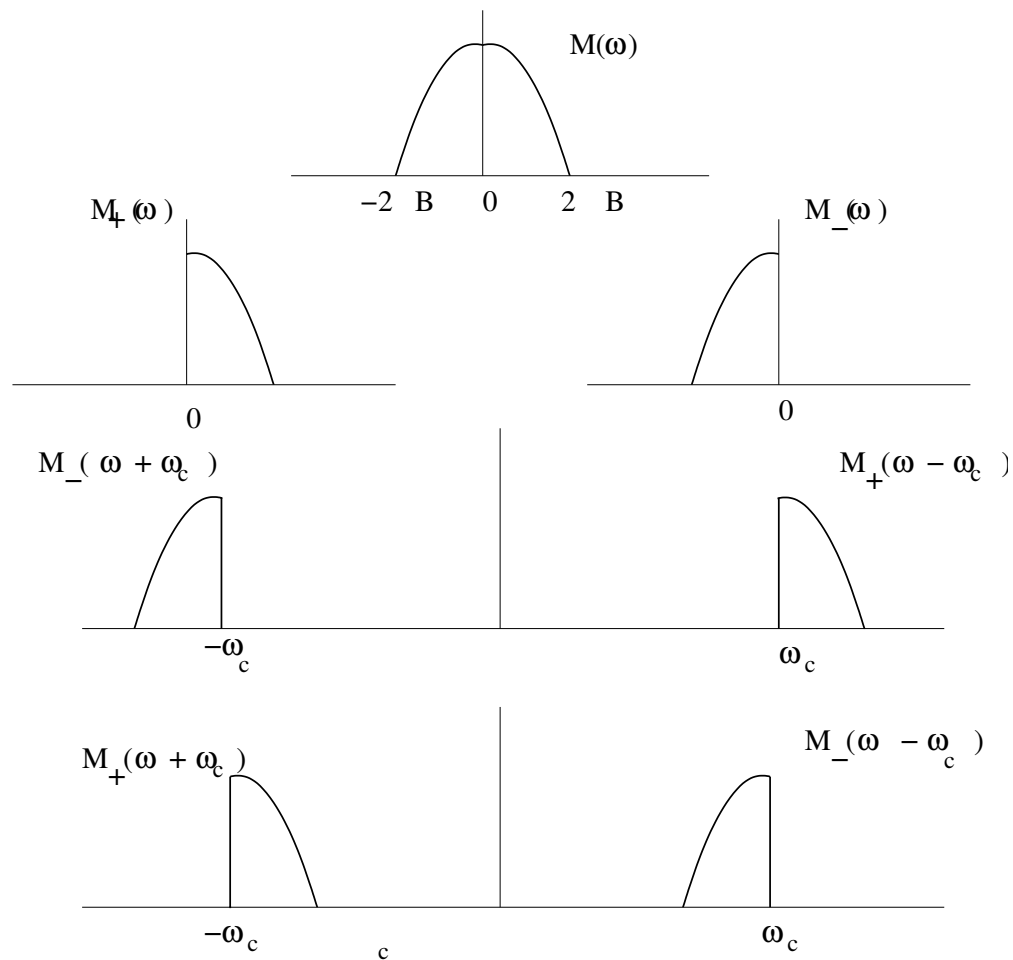


Figure 2: Frequency analysis of SSB signals

- From Figure 2 and the concept of the Hilbert Transform,

$$\begin{aligned}\Phi_{USB}(\omega) &= M_+(\omega - \omega_c) + M_-(\omega + \omega_c) \\ \phi_{USB}(t) &= m_+(t)e^{j\omega_c t} + m_-(t)e^{-j\omega_c t}\end{aligned}$$

But, from complex representation of signals,

$$\begin{aligned}m_+(t) &= m(t) + j\hat{m}(t) \\ m_-(t) &= m(t) - j\hat{m}(t)\end{aligned}$$

So,

$$\phi_{USB}(t) = m(t) \cos(\omega_c t) - \hat{m}(t) \sin(\omega_c t)$$

Similarly,

$$\phi_{LSB}(t) = m(t) \cos(\omega_c t) + \hat{m}(t) \sin(\omega_c t)$$

- Generation of SSB signals A SSB signal is represented by:

$$\phi_{SSB}(t) = m(t) \cos(\omega_c t) \pm \hat{m}(t) \sin(\omega_c t)$$

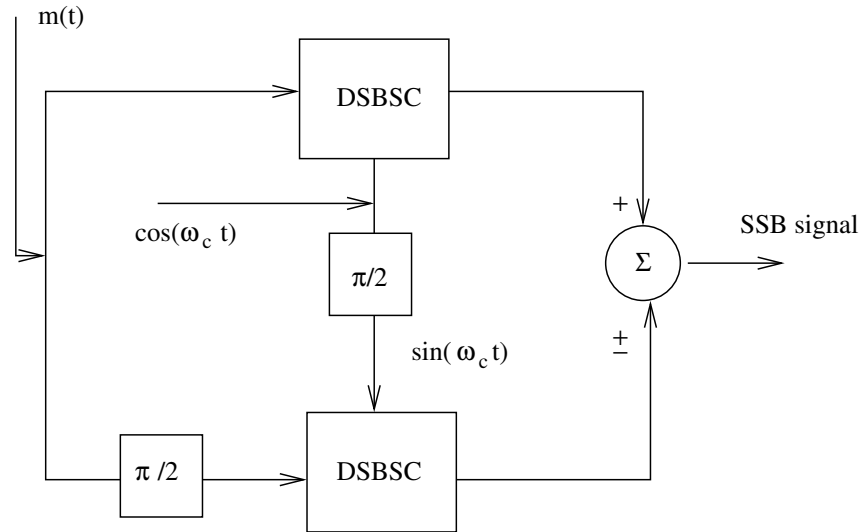


Figure 3: Generation of SSB signals

As shown in Figure 3, a DSB-SC modulator is used for SSB signal generation.

- **Coherent Demodulation of SSB signals** $\Phi_{SSB}(t)$ is multiplied with $\cos(\omega_c t)$ and passed through low pass filter to get back the original signal.

$$\begin{aligned}\phi_{SSB}(t) \cos(\omega_c t) &= \frac{1}{2}m(t) [1 + \cos(2\omega_c t)] \pm \frac{1}{2}\hat{m}(t) \sin(2\omega_c t) \\ &= \frac{1}{2}m(t) + \frac{1}{2} \cos(2\omega_c t) \pm \frac{1}{2}\hat{m}(t) \sin(2\omega_c t)\end{aligned}$$

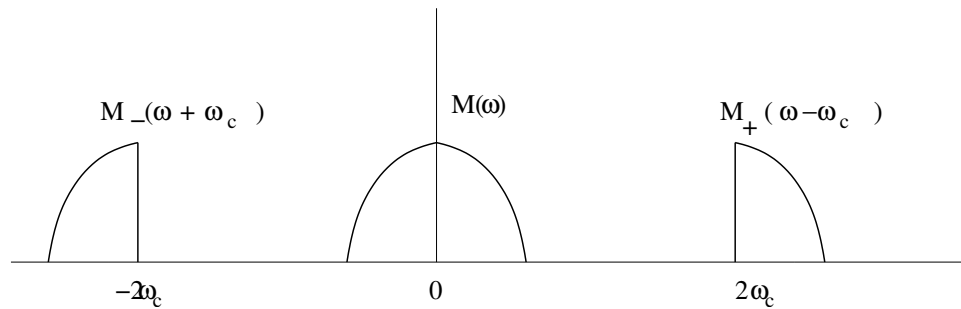


Figure 4: Demodulated SSB signal

The demodulated signal is passed through an LPF to remove unwanted SSB terms.

Vestigial Side Band (VSB) Modulation

- The following are the drawbacks of SSB signal generation:
 1. Generation of an SSB signal is difficult.
 2. Selective filtering is to be done to get the original signal back.
 3. Phase shifter should be exactly tuned to 90^0 .
- To overcome these drawbacks, VSB modulation is used. It can be viewed as a compromise between SSB and DSB-SC. Figure 5 shows all the three modulation schemes.

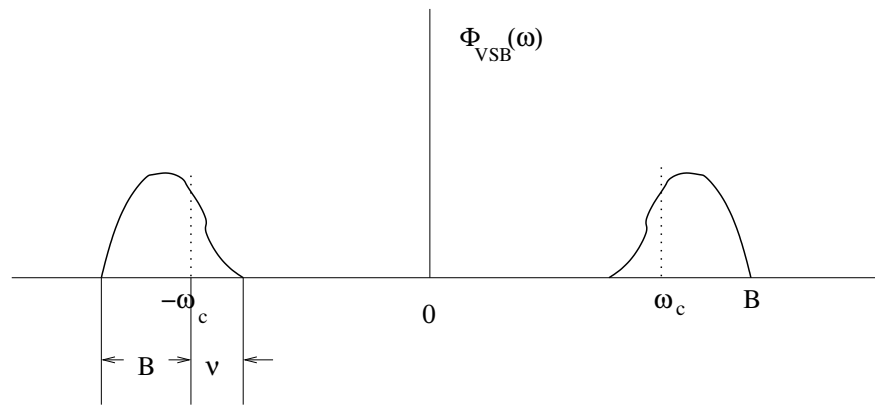


Figure 5: VSB Modulation

- In VSB
 1. One sideband is not rejected fully.
 2. One sideband is transmitted fully and a small part (vestige) of the other sideband is transmitted.
- The transmission BW is $BW_v = B + v$. where, v is the vestigial frequency band. The generation of VSB signal is shown in Figure 6

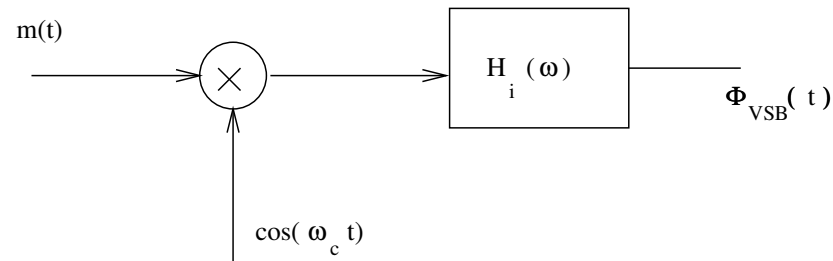


Figure 6: Block Diagram - Generation of VSB signal

Here, $H_i(\omega)$ is a filter which shapes the other sideband.

$$\Phi_{VSB}(\omega) = [M(\omega - \omega_c) + M(\omega + \omega_c)] \cdot H_i(\omega)$$

To recover the original signal from the VSB signal, the VSB signal is multiplied with $\cos(\omega_c t)$ and passed through an LPF such that original signal is recovered.

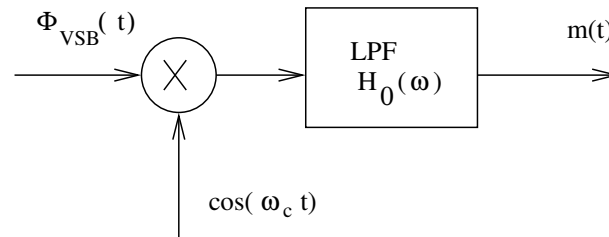


Figure 7: Block Diagram - Demodulation of VSB signal

From Figure 6 and Figure 7, the criterion to choose LPF is:

$$\begin{aligned}
 M(\omega) &= [\Phi_{VSB}(\omega + \omega_c) + \Phi_{VSB}(\omega - \omega_c)] \cdot H_0(\omega) \\
 &= [H_i(\omega + \omega_c) + H_i(\omega - \omega_c)] \cdot M(\omega) \cdot H_0(\omega) \\
 \implies H_0(\omega) &= \frac{1}{H_i(\omega + \omega_c) + H_i(\omega - \omega_c)}
 \end{aligned}$$

Appendix: The Hilbert Transform

The Hilbert Transform on a signal changes its phase by $\pm 90^\circ$. The Hilbert transform of a signal $g(t)$ is represented as $\hat{g}(t)$.

$$\hat{g}(t) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{g(\tau)}{t - \tau} d\tau$$

$$g(t) = -\frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{\hat{g}(\tau)}{t - \tau} d\tau$$

We, say $g(t)$ and $\hat{g}(t)$ constitute a Hilbert Transform pair. If we observe the above equations, it is evident that Hilbert transform is nothing but the convolution of $g(t)$ with $\frac{1}{\pi t}$.

The Fourier Transform of $\hat{g}(t)$ is computed from signum function $sgn(t)$.

$$\begin{aligned} \operatorname{sgn}(t) &\longleftrightarrow \frac{2}{j\omega} \\ \implies \frac{1}{\pi t} &\longleftrightarrow -j\operatorname{sgn}(\omega) \end{aligned}$$

Where,

$$\operatorname{sgn}(\omega) = \begin{cases} 1, & \omega > 0 \\ 0, & \omega = 0 \\ -1, & \omega < 0 \end{cases}$$

Since, $\hat{g}(t) = g(t) * \frac{1}{\pi t}$,

$$\hat{G}(\omega) = G(\omega) \times -j\operatorname{sgn}(\omega)$$

Properties of Hilbert Transform

1. $g(t)$ and $\hat{g}(t)$ have the same magnitude spectrum.
2. If $\hat{g}(t)$ is HT of $g(t)$ then HT of $\hat{g}(t)$ is $-g(t)$.
3. $g(t)$ and $\hat{g}(t)$ are orthogonal over the entire interval $-\infty$ to $+\infty$.

$$\int_{-\infty}^{+\infty} g(t)\hat{g}(t) dt = 0$$

Complex representation of signals

If $g(t)$ is a real valued signal, then its complex representation $g_+(t)$ is given by

$$\begin{aligned}g_+(t) &= g(t) + j\hat{g}(t) \\G_+(\omega) &= G(\omega) + \text{sgn}(\omega)G(\omega)\end{aligned}$$

Therefore,

$$G_+(\omega) = \begin{cases} 2G(\omega), & \omega > 0 \\ G(0), & \omega = 0 \\ 0, & \omega < 0 \end{cases}$$

$g_+(t)$ is called *pre-envelope* and exists only for positive frequencies.
For negative frequencies $g_-(t)$ is defined as follows:

$$\begin{aligned}g_-(t) &= g(t) - j\hat{g}(t) \\G_-(\omega) &= G(\omega) - \text{sgn}(\omega)G(\omega)\end{aligned}$$

Therefore,

$$G_-(\omega) = \begin{cases} 2G(\omega), & \omega < 0 \\ G(0), & \omega = 0 \\ 0, & \omega > 0 \end{cases}$$

- Essentially the *pre-envelope* of a signal enables the suppression of one of the sidebands in signal transmission.
- The *pre-envelope* is used in the generation of the SSB-signal.