36.4 Superelevation

Superelevation is defined as the difference in elevation of water surface between inside and outside wall of the bend at the same section.

\[ \Delta y = y_0 - y_1 \]  

This is similar to the road banking in curves. The centrifugal force acting on the fluid particles, will throw the particle away from the centre in radial direction, creating centripetal lift.

Superelevation in other words means the greater depth near the concave bank than near convex bank of a bend. This phenomenon was first observed by Ripley in 1872, while he was surveying Red River in Loussiana for the removal of the great raft obstructing the stream.

36.4.1 Transverse water surface slope in bends

Gockinga was first to derive the following formula for determining the difference in elevation of the water surface on opposite sides of channel bends.

\[ y = 0.235V^2 \log \left(1 - \frac{x}{r}\right) \]

in which \( V \) is the velocity in \( \text{ms}^{-1} \), \( x \) is the distance at which \( y \) is to be determined, \( r \) is the radius of the river bend. But above equation is found to fit a particular stream to which it was designed. Also he found that the transverse slope was twice greater than the longitudinal slope. He showed that the increased depth in bend is caused by the helicoidal flow induced by centrifugal force. Fargue while conducting studies on scour in meandering devised the formula and called "Fargue's law of greatest depth" in 1908. But unfortunately it was found to be applicable to the stream Garonne at Barsac only.

\[ C_1 = 0.03H^3 - 0.23H^2 + 0.78H - 0.76 \]

in which \( C_1 \) is the reciprocal of radius of curvature in kilometer and \( H \) is the lowest water depth at the deepest point of the channel in meter.

Mitchell also derived another equation applicable to Delaware river, Philadelphia.
Ripley in 1926 arrived at the formulae based on the field observations, having their own limitations.

\[ y = D_1 \left( 1 - \frac{4x^2}{T^2} \right) + D_1 \frac{17.52}{r} \left( 1 - \frac{4x^2}{T^2} \right)x \]

\[ y = P_1 \left( 1 - \frac{4x^2}{T^2} \right) + 26.28P_1 \frac{1}{r} \left( 1 - \frac{4x^2}{T^2} \right)x \]

In the above equations, parabolic sections are assumed whose focal distances are \( \frac{T^2}{D_1} \) and \( \frac{T^2}{P_1} \) respectively and with the origin at a point on the axis at a distance of \( D_1 \) and \( P_1 \) from apex respectively.

The above two equations when combined yield a simplified form in FPS units. Figure represents the general profile for equation given below.

\[ y = 6.35D_1 \left( \sqrt{0.437 - \frac{x^2}{T^2}} - 0.433 \right) \left( 1 + \frac{17.52x}{r_0} \right) \]
Grashof was the first to try an analytical solution for superelevation. He obtained equation by applying Newton’s second law of motion to every streamline and integrating the equation of motion. Equation gives a logarithmic profile. Referring to figure given below one may write

\[
\text{Centrifugal force} = \frac{W_1}{g} \frac{V_{\text{max}}^2}{r_c}
\]

\[
\frac{dy}{dr} = \frac{g}{W_1} \frac{r_c}{V_{\text{max}}} = \frac{V_{\text{max}}^2}{g r_c}
\]

Assuming the boundary conditions near inside wall of the bend and integrating above equation reduces to the form

\[
\Delta y = 2.3 \frac{V_{\text{max}}^2}{g} \log \frac{r_0}{r_i}
\]

in which \( r_i \) and \( r_0 \) are the inner and outer radii of the bend respectively.

Woodward in 1920 assumed the velocity to be zero at banks and to have a maximum value at the centre of the bend and the velocity distribution varying in between
according to parabolic curve. Using Newton's second law of motion he obtained the following equation for superelevation.

\[
\Delta y = v_{\text{max}}^2 \left[ \frac{20}{3} \frac{r_c}{b} - 16 \left( \frac{r_c}{b} \right)^3 + 4 \left( \frac{r_c}{b} \right)^2 - 1 \right] \ln \left( \frac{2r_c + b}{2r_c - b} \right)
\]

Shukry obtained the following equation for maximum superelevation based on free-vortex flow and principle of specific energy.

\[
\Delta y_{\text{max}} = \frac{C^2}{2g r_0^2 r_i^2} \left( r_0^2 - r_i^2 \right)
\]

The Euler equation of motion.

\[
\frac{\partial}{\partial s} \left( p + \gamma z \right) + \rho a_s = 0
\]

\[
(p + \gamma z) = \gamma h
\]

Since

\[
\frac{\partial}{\partial s} (p + \gamma z) = -\gamma \frac{\partial h}{\partial n}
\]

\[
H_0 = y + \frac{v^2}{2g} + z
\]

\[
= h + \frac{v^2}{2g} \quad (\because h = y + z)
\]
differentiating above equation
\[
\frac{dh}{dn} + \frac{v}{g} \frac{\partial v}{\partial n} = 0
\]
\[
\frac{v^2}{gr} + \frac{v}{g} \frac{\partial v}{\partial n} = 0
\]
\[
\frac{v}{r} + \frac{\partial v}{\partial n} = 0
\]
Thus \( v \) decreases and \( h \) increases from inner boundary to outer boundary.
The equation can be rewritten as \( \frac{\partial v}{v} + \frac{\partial r}{r} = 0 \).
Therefore it can be shown as \( v \frac{r}{r} = \text{constant} \) which is in the free vortex condition.

Consider a rectangular channel bend,
Discharge per unit width \( q = v y \).
\[
q = \frac{c}{r} y \quad \text{or} \quad \frac{y}{r} = \frac{q}{c} \quad \text{a constant.}
\]

Modification to the bed profile to obtain the horizontal water surface in a bend
\[
\frac{dy}{dr} = \frac{y}{r}
\]
\[
\frac{dy}{dr} = - \frac{dz}{dr} \frac{1}{1-F^2}
\]
z decreases from inner wall to outer wall for subcritical flow (as shown in the above figure).
\[
\frac{dz}{dr} = - \frac{dy}{dr} \left(1-F^2\right) = - \frac{y}{r} \left(1-F^2\right)
\]
and
\[
\frac{dh}{dr} = \frac{dz}{dr} + \frac{dy}{dr} = - \frac{dy}{dr} \left(1-F^2\right) + \frac{dy}{dr}
\]
\[
\frac{dy}{dr} \left[ -1+F^2 + 1 \right] = \frac{y}{r} F^2 = \frac{v^2}{gr}
\]
Transverse bed profile.

\[ H_0 = z + y + \frac{v^2}{2g} = y_0 + \frac{v_0^2}{2g} \]

\[ H_0 = z + \frac{rq}{C} + \frac{C^2}{2gr^2} \]

The above equation gives reasonably good result as long as the angle of the circular bend in plan is greater than 90°. A correction factor was suggested by Shukry for circulation constant C, assuming it to vary linearly from 0° to 90°.

\[ \nu_r = C U_f = C \left[ \frac{\theta}{90} + \left( 1 - \frac{\theta}{90} \right) \frac{r V_{mA}}{C} \right] \]

\[ \frac{v}{r} = w_1 U_x = w_1 \left[ \frac{\theta}{90} + \left( 1 - \frac{\theta}{90} \right) \frac{V_{mA}}{r w_1} \right] \]

However, by applying Newton's 2nd law of motion based on one dimensional analysis i.e., all the filamental velocities in the bend are equal to the mean velocity \( V_{mb} \) and that of all the streamlines having the same radii of curvature \( r_c \), an equation can be obtained for a rectangular channel namely

\[ \Delta y_{\text{max}} = \frac{V_{mb}^2}{2g} \left( \frac{2b}{r_c} \right) \]

For the channels other than the rectangular channel, the bed width \((b)\) can be replaced by the water surface width \((T)\) then

\[ \Delta y_{\text{max}} = \frac{V_{mb}^2}{2g} \left( \frac{2T}{r_c} \right) \]

The above equation is only a first approximation and gives transverse profile as the straightline. This assumes that the rise and the drop of the water surface level from the normal level is equal on either side of center line of bend.

As a better approximation, Lpren and Drinker obtained an equation for superelevation. The derivation of equation is based on the assumptions of free vortex or irrotational flow with the uniform specific head over the cross section, and the mean depth in bend being equal to the mean approaching flow depth.
The bends in nature will not have the symmetry due to entrance conditions, length of curvature and boundary resistance. Hence above equation will not give accurate result.

If the forced vortex condition exists with constant stream cross section and constant average specific energy, then equation for superelevation assumes the form

\[ \Delta y = \frac{V^2}{2g} \frac{2T}{r_c} \left( \frac{1}{1 - \frac{T^2}{4r_c^2}} \right) \]

The above equation is applicable to a smooth rectangular boundary with circular bend with the flowing fluid being ideal.

Better results can be obtained by combining the effects of the free and forced vortex conditions simultaneously. The minimum angle of bend is to be 90° for applying the above equation in combination. For the smaller angles the difference in computed values from the above equation becomes larger than the actual ones.

However, for a rectangular channel, circular bend with , applying the free vortex formula, velocity at inside wall of the bend becomes as thus a depth of should exist at the boundary (i.e. at ), which is physically impossible.

Muramoto obtained an equation for superelevation based on equations of motion.

\[ v_r = C_1 \]
\[ U = \frac{g S_0 r^2}{3C_1} + \frac{C_2}{r} \]

Then

\[ \Delta y = \left[ \frac{g S_0^2 r^4}{36 C_1^2} \left( \frac{C_1^2 + C_2^2}{2gr^2} \right) + \frac{2S_0 C_2 r}{3C_1} \right]_{t_0}^{t_i} \]

in which \( C_1 \) and \( C_2 \) are circulation constants obtained, after integration. The special feature of above equation lies in including the effect of bed slope on superelevation.
Thus it can be observed from the above discussion that superelevation in a bend is a function of shape of the cross section, Reynolds number, approach flow, slope of the bed, Froude number, \( \frac{\theta}{180} \cdot \frac{r_c}{b} \) and boundary resistance. The superelevation is also affected by the presence of secondary currents and separation. However, before applying to field situations, the validity of the transverse flow profile equation has to be justified. The observations in the field have shown the occurrence of troughs near the concave profile in the bend. Further, these troughs have been observed during rising and as well as falling stages of the channels. The channels of smaller width have exhibited accumulation of debris instead of troughs.

### 36.4.2 Superelevation and transverse profile

Normalised super elevation \( \frac{\Delta y}{\Delta y_{\text{max}}} \) was correlated with normalised bend angle \( \frac{\theta}{\theta_0} \) for all the three bends, for two different Reynolds numbers. From Figure, it may be observed that two peaks of superelevation occur at \( \frac{\theta}{\theta_0} = 0.17 \) and \( \frac{\theta}{\theta_0} = 0.67 \) for all the cases except for bend B1 for \( R_e = 42,280 \). The influence of Reynolds number on the trend of the variation of superelevation at various section of the bend is insignificant in all the three bends.
Variation of normalised Super elevation with normalised angle for Reynolds number 42280
Variation of normalised Super elevation with normalised angle for Reynolds number 101,700

The maximum value of superelevation was normalised with the approaching velocity head $\frac{V^2}{2g}$ and correlated with the Froude number. The equations of these lines is in the form

$$\frac{\Delta y_{max}}{\frac{V^2}{2g}} = m \log F + C$$

in which 'm' is the slope of the line and 'C' a constant.

Bend 1:  \[ \frac{\Delta y_{max}}{\frac{V^2}{2g}} = -1.19 \log F + 0.20 \]

Bend 2:  \[ \frac{\Delta y_{max}}{\frac{V^2}{2g}} = 5.33 \log F - 0.49 \]
Bend 3: \[ \frac{\Delta y_{\text{max}}}{V^2/2g} = 1.96 \log F - 0.06 \]
Flow Conditions D/S of B1

The greatest difference in elevation between the longitudinal profiles at outer and inner walls is maximum at the 30° section of the 180° bend.
Wheel volve
2. Inlet pipe
3. Adjustment valve
4. Entrance chamber
5. Transition
6. Leading Channel
7. Rectangular notch
8. Stilling chamber
9. Masonry honey comb
10. Transition
11. Main channel
12. Tail control

Experimental Set - Up
Scale = 1: 100 (all dimensions in cm)
Section and points of measurement

b) points of measurement

All dimensions are in cm

Station A

Station B

Station C

Station D

Section AA

Inner

Outer

G.L

2 m

1 m

r_c = 300

r_c = 200

0°

15°

30°

45°

60°

75°

90°

1 m

1 m

2 m

-22.5 -12.5 0 12.5 22.5

-22.5 -12.5 0 12.5 22.5

0.4

0.4

150°

180°

120°

90°

60°
Variation of $\frac{\Delta y}{V^2/2g}$ with $\log F$

Bend 1

Bend 2

Bend 3
Comparison of Observed and Theoretical Superelevation

Bed slope 1:1000

STN A STN B STN C STN D

Q = 26.1 lps Re 42280 F = 0.49

STN A STN B STN C STN D

Q = 71.9 lps Re 101760 F = 0.55

Longitudinal Water Surface Profile

Scale
x axis 1:100
y axis 5:1
Outer wall
Inner wall
ISOVELS in open channel bend [Normalised with $V_{max}$] $Q = 26.1 \text{ lps, } F = 0.18, R_e = 36050$
ISOVELS in open channel bend [Normalised with $V_{max}$]

**Station A**
- $Q = 71.9$ lps, $F = 0.44$, $R_e = 95420$

**Station B**
- $Q = 83.5$ lps, $F = 0.41$, $R_e = 103460$

**Station C**
- $Q = 71.9$ lps, $F = 0.44$, $R_e = 95420$

**Station D**
- $Q = 83.5$ lps, $F = 0.41$, $R_e = 103460$
Reference

