Analog Circuits and Systems
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Lecture 31: Waveform Generation
Review

- Phase Locked Loop (self tuned filter)
- 2\textsuperscript{nd} order High – Q low-pass output phase compared with the input
- 90\degree phase shift causes zero average using analog multiplier and filtering
- Negative feedback with loop-gain high maintains lock
Review (contd..)

- Capture must occur before lock
- Capture range less than lock range
- Lock range decided by limits of any one of the blocks in PLL
- Capture occurs always close to the quiescent state where the loop-gain can be high
Review (contd.,)

- Summary of filter design
  - Second order filter blocks cascaded, first order may be required
  - Poles of the second order filter peak around the bandwidth
  - Zeros are always located outside the band
Review (contd.,)

◦ First locate the zeros wherever narrow band noise is present, then locate the poles with peaking near the pass-band edge
◦ Higher-Q poles with higher frequencies get located closer to the bandwidth
◦ Effect of gain bandwidth product is to cause Q enhancement which can be compensated
Waveform Generation

- Testing and operation of analog and digital systems require
  - Sine wave
  - Clock
  - Square wave
  - Saw tooth waveform
  - Triangular waveform
  - Arbitrary waveform

- Requirements of signals generated
  - Constant amplitude as frequency varies
  - Precise adjustment of amplitude
  - Frequency stability
Waveforms

Sine

Clock

Triangle

Sawtooth

Square

Arbitrary
Sine Wave Oscillators

have their origin in harmonic oscillator

\[ X = A \sin(\sqrt{k}t + \phi) \]

is the solution of a second order linear differential equation

\[ \frac{\partial^2 X}{\partial t^2} + kX = 0 \]

\(\sqrt{k}\) is known as the frequency of oscillation in rad/sec

A and \(\phi\) are determined by initial conditions. In the case of electrical oscillators \(X\) is replaced by voltage \(v\) or current \(i\)

\[ \frac{\partial^2 v}{\partial t^2} + kv = 0 \]
LC network

- A network can be easily formed by connecting two energy storage elements L and C
- Inductor stores energy in electromagnetic form
- Capacitor stores energy in electrostatic form
LC network (contd.,)

Capacitor stores it in electrostatic form as voltage.

\[ v = -L \frac{di}{dt} = \frac{1}{C} \int idt \]

\[ \frac{d^2i}{dt^2} + \frac{i}{LC} = 0 \]

\[ i = I_p \sin \left( \left\{ \frac{t}{\sqrt{LC}} \right\} + \phi \right) = I_p \sin \left( \omega_n t + \phi \right) \]

\[ I_p \text{ and } \phi \text{ depend on initial conditions; } \omega_n = \frac{1}{\sqrt{LC}} \]
In practice

if capacitor is initially charged to say 1V at $t = 0$
and inductor current at $t=0$ is 0

$i = I_p \sin \omega_n t; \phi = 0$ because at $t = 0; i = 0$

inductor acts as an open circuit.

\[ \frac{di}{dt} = \frac{I_p}{\omega_n} \cos \omega_n t; \omega_n = \frac{1}{\sqrt{LC}} \]

natural frequency of the system

\[ L \left. \frac{di}{dt} \right|_{t=0} = \frac{I_p}{\omega_n} = 1 \text{ V}; \ I_p = \omega_n \text{ amps;} \]

\[ i = I_p \sin \omega_n t; \ v = I_p \cos \omega_n t \]
Simulation of the Oscillator

- Time period: 63.61 m sec; L=1 mH and C =0.1 mF
- Voltage across C at t = 0 is 1V and current through the inductor at t = 0 is zero
RLC Circuit

\[
\frac{v}{R_p} + \frac{1}{L} \int v \, dt + C \frac{dv}{dt} = 0
\]

\[
\frac{d^2v}{dt^2} + \frac{1}{R_p C} \frac{dv}{dt} + \frac{v}{LC} = 0
\]

\[
\frac{d^2v}{dt^2} + \frac{\omega_n}{Q} \frac{dv}{dt} + \omega_n^2 v = 0
\]

\[
Q = \omega_n R_p C = R_p \sqrt{\frac{C}{L}}
\]

\( \omega_n \) is natural frequency.

\( Q \) is known as quality factor of the resonant system.
RLC Circuit (contd.,)

- The circuit is popularly known as tank circuit (because its ability to store electrical energy).

\[
s^2 + \frac{s\omega_n}{Q} + \omega_n^2 = 0; \quad s = -\frac{\omega_n}{2Q} \pm \frac{1}{2} \sqrt{\frac{\omega_n^2}{Q^2} - 4\omega_n^2}
\]

\[
s = -\frac{\omega_n}{2Q} \pm \frac{\omega_n}{2Q} \sqrt{1 - 4Q^2} \quad \text{for } Q < \frac{1}{2}
\]

\[
s = -\frac{\omega_n}{2Q} \pm j\frac{\omega_n}{2Q} \sqrt{4Q^2 - 1} \quad \text{for } Q > \frac{1}{2}
\]

\[
v = Ae^{-(\alpha_n/2Q)t} \sin \left(\frac{\omega_n t}{2Q} \sqrt{4Q^2 - 1} + \phi\right) = Ae^{-(\alpha_n/2Q)t} \sin \left(\omega_n t \sqrt{1 - \frac{1}{4Q^2}} + \phi\right)
\]

A resonant system with ringing can be present only if \( Q > \frac{1}{2} \).
Decay of oscillations

- $R = 1 \text{ kW}$, $L = 1 \text{ mH}$ and $C = 0.1 \text{ mF}$; $Q = 10$
- $Q=$ no. of visible peaks (from first peak current up to $1/10$ of first peak)
For sustained oscillations a negative resistance, must be used across the lossy tank circuit.
Negative Resistance Oscillator

- Frequency of oscillation = \( \frac{1}{\sqrt{LC}} \)
- \( R_p = R_p' \)
LCR Circuit with negative resistance (contd.,)

The effective resistance \[- \frac{1}{R'_p} + \frac{1}{R_p} \] must be negative i.e., \( R'_p < R_p \).

Then oscillation amplitude will progressively increase

To make the oscillator produce sinusoidal waveform, start with \( R'_p < R_p \).
Need for Non-linear Resistance

- The resistance $R_p$ must be made non-linear such that it increases in magnitude.
- As voltage across it increases to a given amplitude $R(V_p)=R_p$ it stabilizes.
- This effectively means net admittance at a specific frequency $w_n$ if becomes zero it is a sine wave oscillator.
- Negative resistance can be obtained by using tunnel diodes or BJT$s$, FET$s$ or Op Amps.
- Tunnel diode BJT$s$ and FET$s$ are used for oscillators in the RF range or microwave range.
- Op Amps are used for lower frequencies.
Non-linear negative resistance

- A grounded negative resistance can be simulated by using an non-inverting voltage amplifier of gain greater than 1.
- Here a gain of 2 amplifier is used to simulate $-R_p$ across the non-ideal tank circuit. The negative resistance simulated in n-type negative resistance which is short circuit stable and open circuit unstable.
- Frequency of oscillation $= \frac{1}{\sqrt{LC}}$; amplitude $= \frac{V_s}{2}$. 
Non-linear negative resistance
Amplitude Stabilization

- $R_p' = 600\,\text{W}$; $R_p = 1\,\text{kW}$; $C = 1\,\text{mF}$; $L = 1\,\text{mH}$; $R = 10\,\text{kW}$; opamp = LM741
Zenor Diode for Amplitude Stabilization

- $R_p'=800\,\text{W}$; $R_p=1\,\text{kW}$; $C=1\,\mu\text{F}$; $L=1\,\text{mH}$; $R=10\,\text{kW}$; $R_z=600\,\text{W}$; $\text{opamp} = \text{LM741}; V_{Z1}=V_{Z2}=1\,\text{V}$
- $R_p' = 800\,\text{W}$; $R_p = 1\,\text{kW}$; $C = 1\,\text{mF}$; $L = 1\,\text{mH}$; $R = 10\,\text{kW}$; $R_z = 1\,\text{kW}$; $\text{opamp} = \text{LM741}$; $C' = 0.1\,\text{nF}$; $V_{\text{ref}} = 0.45\,\text{V}$
Simulation - Precision amplitude stabilization

- $V_p = 3V$ for $V_{ref} = 0.45V$
Automatic Gain Control (AGC/AVC)

- Used in almost all communication receivers at the front end (RF)
Simulation 1 - AGC/AVC

- Input voltage = 3V; $V_{ref} = 0.2V$; expected output voltage = 2V
Simulation 2 - AGC/AVC

- Input voltage = 10V; $V_{\text{ref}} = 0.2V$; expected output voltage = 2V
AGC/AVC

- Lock range – it is determined by the range of operation of the multiplier and saturation range of the opamp.
- Loop gain – evaluation
Conclusion