Integrated Circuit Operational Amplifiers
Analog Integrated Circuit Design
A video course under the NPTEL

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National Programme on Technology Enhanced Learning
Cascode output resistance

\[ R_{\text{out}} = g_{mc}/g_{dsc} G_s + 1/G_s + 1/g_{dsc} \]

\[ R_{\text{out}} = g_{mc}/g_{dsc} g_{ds1} + 1/g_{dsc} + 1/g_{ds1} \]

Output resistance looking into one side of the differential pair is \(2/g_{ds1}\) \((g_{m1} = g_{mc}\) in the figure)
Opamp: dc small signal analysis

- Bias values in black
- Incremental values in red
- Impedances in blue

Total quantity = Bias + increment
Differential pair: Quiescent condition

\[ V_{dd} - V_{SG3} \text{ (by symmetry)} \]

\[ I_0/2 \quad I_0/2 \]

\[ V_{cm} \text{ (by symmetry)} \]

\[ V_{bias0} \]

M1 \quad M2

M3 \quad M4

zero current

\[ + \]

\[ V_{dd} - V_{SG3} \]

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Integrated Circuit Operational Amplifiers
Differential pair: Transconductance

\[ I_0 / 2 \]

\[ V_{bias0} \]

\[ V_{cm} \]

\[ -v_d/2 \]

\[ g_m v_d/2 \]

\[ I_0/2 \]

\[ V_{dd} \]

\[ M_1 \]

\[ M_2 \]

\[ M_3 \]

\[ M_4 \]

\[ V_{cm} \]

\[ +v_d/2 \]

\[ g_m v_d \]

\[ v_x \sim 0 \]

\[ V_{dd} - V_{SG3} \]
Differential pair: Output conductance

\[ V_{cm} \]

\[ I_0/2 \]

\[ I_0/2 \]

\[ V_{bias0} \]

\[ V_{dd} \]

\[ M_1 \]

\[ M_2 \]

\[ M_3 \]

\[ M_4 \]

\[ v_T g_{ds1}/2 \]

\[ v_T g_{ds1}/2 + v_T g_{ds3} \]

\[ v_T g_{ds1}/2 \]

\[ v_T g_{ds1}/2 \]

\[ v_T (g_{ds1} + g_{ds3}) \]

\[ v_T \]

\[ V_{dd} - V_{SG3} \]
Differential pair: Noise

- Carry out small signal linear analysis with one noise source at a time
- Add up the results at the output (current in this case)
- Add up corresponding spectral densities
- Divide by gain squared to get input referred noise
### Differential pair opamp

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_m$</td>
<td>$g_m$</td>
</tr>
<tr>
<td>$G_{out}$</td>
<td>$g_{ds1} + g_{ds3}$</td>
</tr>
<tr>
<td>$A_o$</td>
<td>$g_m/(g_{ds1} + g_{ds3})$</td>
</tr>
<tr>
<td>$A_{cm}$</td>
<td>$g_{ds0}/2g_m$</td>
</tr>
<tr>
<td>$C_i$</td>
<td>$C_{gs1}/2$</td>
</tr>
<tr>
<td>$\omega_u$</td>
<td>$g_m/C_L$</td>
</tr>
<tr>
<td>$p_k, z_k$</td>
<td>$p_2 = -g_m/(C_{db1} + C_{db3} + 2C_{gs3}); z_1 = 2p_2$</td>
</tr>
<tr>
<td>$S_{vi}$</td>
<td>$16kT/3g_m(1 + g_m/g_m)$</td>
</tr>
<tr>
<td>$\sigma_{Vos}^2$</td>
<td>$\sigma_{VT1}^2 + (g_m/g_m)^2\sigma_{VT3}^2$</td>
</tr>
<tr>
<td>$V_{cm}$</td>
<td>$\geq V_{T1} + V_{DSAT1} + V_{DSAT0}$</td>
</tr>
<tr>
<td></td>
<td>$\leq V_{dd} - V_{DSAT3} - V_{T3} + V_{T1}$</td>
</tr>
<tr>
<td>$V_{out}$</td>
<td>$\geq V_{cm} - V_{T1}$</td>
</tr>
<tr>
<td></td>
<td>$\leq V_{dd} - V_{DSAT3}$</td>
</tr>
<tr>
<td>$SR$</td>
<td>$\pm I_0/C_L$</td>
</tr>
<tr>
<td>$I_{supply}$</td>
<td>$I_0 + I_{ref}$</td>
</tr>
</tbody>
</table>
Telescopic cascode: Quiescent condition

\[ V_{bias0} \]

\[ V_{biasn2} \]

\[ V_{biasp2} \]

\[ V_{cm} \]

\[ I_0/2 \]

\[ V_{dd} \]

\[ V_{dd} - V_{SG3} \]

\[ 0 \text{ current} \]
Telescopic cascode: Transconductance

\[ M_1 \]

\[ V_{biasn2} \]

\[ V_{biasp2} \]

\[ V_{dd} \]

\[ V_{dd} - V_{SG3} \]

\[ \frac{g_m v_d}{2} \]

\[ \frac{I_0}{2} \]

\[ \frac{g_m v_d}{2} \]

\[ M_2 \]

\[ M_5 \]

\[ M_6 \]

\[ M_7 \]

\[ M_8 \]

\[ V_{cm} \]

\[ +\frac{v_d}{2} \]

\[ -\frac{v_d}{2} \]

\[ v_x \sim 0 \]

\[ V_{bias0} \]

\[ V_{bias0} \]

\[ I_0/2 \]
Telescopic cascode: Output conductance

\[ V_{bias0} \]

\[ V_{biasn2} \]

\[ V_{biasp2} \]

\[ V_{cm} \]

\[ I_0/2 \]

\[ I_0/2 \]

\[ V_{dd} \]

\[ V_{dd} - V_{SG3} \]

\[ +V_T \]

\[ v_T g_{ds5} g_{ds1}/2g_m5 \]

\[ v_T g_{ds5} g_{ds1}/2g_m5 + v_T g_{ds7} g_{ds3}/g_m7 \]

\[ g_{ds5} g_{ds1}/2g_m5 \]

\[ g_{ds1}/2 \]

\[ v_T (g_{ds5} g_{ds1}/g_m5 + g_{ds7} g_{ds3}/g_m7) \]

\[ v_T g_{ds5} g_{ds1}/2g_m5 \]

\[ v_T g_{ds5} g_{ds1}/2g_m5 \]
Telescopic cascode opamp

\[ \text{Vbias0} \quad \text{Vdd} \quad \text{M1} \quad \text{M2} \quad \text{M3} \quad \text{M4} \quad \text{Vbiasp2} \quad \text{M5} \quad \text{M6} \quad \text{M7} \quad \text{M8} \quad \text{out} \quad \text{inp} \quad \text{inn} \quad \text{Vbiasn2} \quad \text{Vbias0} \]
### Telescopic cascode opamp

<table>
<thead>
<tr>
<th>Parameter</th>
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<tbody>
<tr>
<td>$G_m$</td>
<td>$g_{m1}$</td>
</tr>
<tr>
<td>$G_{out}$</td>
<td>$g_{ds1} g_{ds5}/g_{m5} + g_{ds3} g_{ds7}/g_{m7}$</td>
</tr>
<tr>
<td>$A_o$</td>
<td>$g_{m1}/(g_{ds1} g_{ds5}/g_{m5} + g_{ds3} g_{ds7}/g_{m7})$</td>
</tr>
<tr>
<td>$A_{cm}$</td>
<td>$g_{ds0}/2g_{m3}$</td>
</tr>
<tr>
<td>$C_i$</td>
<td>$C_{gs1}/2$</td>
</tr>
<tr>
<td>$\omega_u$</td>
<td>$g_{m1}/C_L$</td>
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<tr>
<td>$p_k, z_k$</td>
<td>$p_2 = -g_{m3}/(C_{db1} + C_{db3} + 2C_{gs3})$</td>
</tr>
<tr>
<td></td>
<td>$p_3 = -g_{m5}/C_{p5}$</td>
</tr>
<tr>
<td></td>
<td>$p_4 = -g_{m7}/C_{p7}$</td>
</tr>
<tr>
<td></td>
<td>$p_{2,4}$ appear for one half and cause mirror zeros</td>
</tr>
<tr>
<td>$S_{vi}$</td>
<td>$16kT/3g_{m1}(1 + g_{m3}/g_{m1})$</td>
</tr>
<tr>
<td>$\sigma_{V_0s}^2$</td>
<td>$\sigma_{VT1}^2 + (g_{m3}/g_{m1})^2 \sigma_{VT3}^2$</td>
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<tr>
<td>$V_{out}$</td>
<td>$\geq V_{biasn1} - V_T5$</td>
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<td>$I_0 + I_{ref}$</td>
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Folded cascode: Quiescent condition

\[ V_{cm} \]

\[ M_1 \quad M_2 \]

\[ V_{bias0} \]

\[ I_0/2 \]

\[ I_0/2 \]

\[ V_{cm} \]

\[ M_3 \quad M_4 \]

\[ V_{biasn2} \]

\[ V_{GS3} \]

\[ + \]

\[ - \]

\[ M_5 \quad M_6 \]

\[ V_{biasp2} \]

\[ I_1 \]

\[ I_1 \]

\[ V_{biasp1} \]

\[ I_0/2 + I_1 \]

\[ I_0/2 + I_1 \]
Folded cascode: Transconductance

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Folded cascode: Output conductance

\[ V_{cm} \]

\[ I_0/2 \]

\[ I_0/2 \]

\[ V_{cm} \]

\[ M_1 \]

\[ M_2 \]

\[ V_{bias0} \]

\[ V_{bias} \]

\[ V_T g_{ds5} g_{ds1}/2g_{m5} \]

\[ V_T g_{ds5} g_{ds1}/2g_{m5} \]

\[ V_T (g_{ds5}(g_{ds1}+g_{ds9})/g_{m5} + g_{ds7}g_{ds3}/g_{m7}) \]

\[ +v_T \]

\[ V_{biasp2} \]

\[ V_{biasn2} \]

\[ V_{biasp1} \]

\[ V_{biasn1} \]

\[ M_3 \]

\[ M_4 \]

\[ M_5 \]

\[ M_6 \]

\[ M_7 \]

\[ M_8 \]

\[ M_9 \]

\[ M_{10} \]

\[ V_{dd} \]

\[ I_0/2 + I_1 \]

\[ I_0/2 + I_1 \]

\[ V_{gs3} \]

\[ 0 \]
Folded cascode opamp

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### Folded cascode opamp

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<td>$G_m$</td>
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<tr>
<td>$G_{out}$</td>
<td>$(g_{ds1} + g_{ds9}) g_{ds5} / g_{m5} + g_{ds3} g_{ds7} / g_{m7}$</td>
</tr>
<tr>
<td>$A_o$</td>
<td>$g_{m1} / ((g_{ds1} + g_{ds9}) g_{ds5} / g_{m5} + g_{ds3} g_{ds7} / g_{m7})$</td>
</tr>
<tr>
<td>$A_{cm}$</td>
<td>$g_{ds0} / 2g_{m3}$</td>
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<tr>
<td>$C_i$</td>
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<td>$16kT / 3g_{m1} (1 + g_{m3} / g_{m1} + g_{m9} / g_{m1})$</td>
</tr>
<tr>
<td>$\sigma^2_{V_{os}}$</td>
<td>$\sigma^2_{VT1} + (g_{m3} / g_{m1})^2 \sigma^2_{VT3} + (g_{m9} / g_{m1})^2 \sigma^2_{VT9}$</td>
</tr>
<tr>
<td>$V_{out}$</td>
<td>$\geq V_{biasn1} - V_{T5}$</td>
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</tr>
<tr>
<td>$SR$</td>
<td>$\pm \min{I_0, I_1} / C_L$</td>
</tr>
<tr>
<td>$I_{supply}$</td>
<td>$I_0 + I_1 + I_{ref}$</td>
</tr>
</tbody>
</table>
Body effect

- All nMOS bulk terminals to ground
- All pMOS bulk terminals to $V_{dd}$
- $A_{cm}$ has an additional factor $g_{m1}/(g_{m1} + g_{mb1})$
- $g_{m5} + g_{mb5}$ instead of $g_{m5}$ in cascode opamp results
- $g_{m7} + g_{mb7}$ instead of $g_{m7}$ in cascode opamp results
Two stage opamp

bias

stage 1

M3 → M4

M1 → M2

I₀

Iref

stage 2

Vdd

Rc

Cc

R_L

C_L

V_outbias

M₀ → M12

M11
First stage can be Differential pair, Telescopic cascode, or Folded cascode; Ideal $g_{m1}$ assumed in the analysis

Second stage: Common source amplifier

Frequency response is the product of frequency responses of the first stage $g_m$ and a common source amplifier driven from a current source
Common source amplifier: Frequency response

\[
\frac{V_o(s)}{V_d(s)} = \left( \frac{g_{m1} g_{m11}}{G_1 G_L} \right) \frac{s C_c (R_c - 1/g_{m11}) + 1}{a_3 s^3 + a_2 s^2 + a_1 s + 1}
\]

\[
a_3 = \frac{R_c C_1 C_L C_c}{G_1 G_L}
\]

\[
a_2 = \frac{C_1 C_c + C_c C_L + C_L C_1 + R_c C_c (G_1 C_L + C_1 G_L)}{G_1 G_L}
\]

\[
a_1 = \frac{C_c (g_{m11} + G_1 + G_L + G_1 G_L R_c) + C_1 G_L + G_1 C_L}{G_1 G_L}
\]

- \(G_1\): Total conductive load at the input
- \(G_L\): Total conductive load at the output
- \(C_1\): Total capacitive load at the input
- \(C_L\): Total capacitive load at the output
Common source amplifier: Poles and zeros

\[ p_1 \approx -\frac{G_1}{C_c\left(\frac{g_{m11}}{G_L} + 1 + \frac{G_1}{G_L} + G_1 R_c\right) + C_1\left(1 + \frac{G_1}{G_L}\right)} \]

\[ p_2 \approx -\frac{g_{m11}\frac{C_c}{C_1 + C_c} + G_L + G_1\frac{C_c + C_L}{C_1 + C_c} + G_1 G_L R_c\frac{C_c}{C_1 + C_c}}{C_1\frac{C_c}{C_1 + C_c} + C_L + \frac{R_c C_c (G_1 C_L + G_L C_1)}{C_c + C_L}} \]

\[ p_3 \approx -\left(\frac{1}{R_c}\left(\frac{1}{C_L} + \frac{1}{C_c} + \frac{1}{C_1}\right) + \frac{G_1}{C_1} + \frac{G_L}{C_L}\right) \]

\[ z_1 = \frac{1}{\left(1/g_{m11} - R_c\right)C_c} \]

Unity gain frequency

\[ \omega_u \approx \frac{g_{m1}}{C_c\left(1 + \frac{G_L}{g_{m11}} + \frac{G_1}{g_{m11}} + \frac{G_1 G_L R_c}{g_{m11}}\right) + C_1\left(\frac{G_L}{g_{m11}} + \frac{G_1}{g_{m11}}\right)} \]
Common source amplifier: Frequency response

- Pole splitting using compensation capacitor $C_c$
  - $p_1$ moves to a lower frequency
  - $p_2$ moves to a higher frequency (For large $C_c$, $p_2 = g_{m11}/C_L$)
- Zero cancelling resistor $R_c$ moves $z_1$ towards the left half $s$ plane and results in a third pole $p_3$
  - $z_1$ can be moved to $\infty$ with $R_c = 1/g_{m11}$
  - $z_1$ can be moved to cancel $p_2$ with $R_c > 1/g_{m11}$ (needs to be verified against process variations)
  - Third pole $p_3$ at a high frequency
- Poles and zeros from the first stage will appear in the frequency response—$Y_{m1}(s)$ instead of $g_{m1}$ in $V_o/V_i$ above
  - Mirror pole and zero
  - Poles due to cascode amplifiers
Compensation cap sizing

\[ p_2 \approx - \frac{g_{m11} C_c}{C_1 + C_c} \frac{1}{C_1 + C_c} + C_L \]

\[ \omega_u \approx \frac{g_{m1}}{C_c} \]

Phase margin (Ignoring \( p_3, z_1, \ldots \))

\[ \phi_M = \tan^{-1} \left( \frac{|p_2|}{\omega_u} \right) \]

\[ \frac{|p_2|}{\omega_u} = \tan \phi_M \]

\[ \frac{g_{m11}}{g_{m1}} \left( \frac{C_c}{C_L} \right)^2 = \frac{C_c}{C_L} \left( 1 + \frac{C_1}{C_L} \right) \tan \phi_M + \frac{C_1}{C_L} \tan \phi_M \]

- For a given \( \phi_M \), solve the quadratic to obtain \( C_c / C_L \)
- If \( C_1 \) is very small, \( p_2 \approx -g_{m2}/C_L \); further simplifies calculations
## Two stage opamp

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_o$</td>
<td>$g_{m1} g_{m11} / (g_{ds1} + g_{ds3}) (g_{ds11} + g_{ds12})$</td>
</tr>
<tr>
<td>$A_{cm}$</td>
<td>$g_{ds0} g_{m11} / 2 g_{m3} (g_{ds11} + g_{ds12})$</td>
</tr>
<tr>
<td>$C_i$</td>
<td>$C_{gs1} / 2$</td>
</tr>
<tr>
<td>$\omega_u$</td>
<td>$g_{m1} / C_c$</td>
</tr>
<tr>
<td>$p_{k}, z_{k}$</td>
<td>See previous pages</td>
</tr>
<tr>
<td>$S_{vi}$</td>
<td>$\approx 16kT / 3g_{m1} (1 + g_{m3} / g_{m1})$</td>
</tr>
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<td>$\sigma^2_{V_{os}}$</td>
<td>$\approx \sigma^2_{VT1} + (g_{m3} / g_{m1})^2 \sigma^2_{VT3}$</td>
</tr>
<tr>
<td>$V_{cm}$</td>
<td>$\geq V_{T1} + V_{DSAT1} + V_{DSAT0}$</td>
</tr>
<tr>
<td></td>
<td>$\leq V_{dd} - V_{DSAT3} - V_{T3} + V_{T1}$</td>
</tr>
<tr>
<td>$V_{out}$</td>
<td>$\geq V_{DSAT12}$</td>
</tr>
<tr>
<td></td>
<td>$\leq V_{dd} - V_{DSAT11}$</td>
</tr>
<tr>
<td>$SR+$</td>
<td>$I_0 / C_c$</td>
</tr>
<tr>
<td>$SR-$</td>
<td>$\min { I_0 / C_c, I_1 / (C_L + C_c) }$</td>
</tr>
<tr>
<td>$I_{supply}$</td>
<td>$I_0 + I_1 + I_{ref}$</td>
</tr>
<tr>
<td></td>
<td>Differential</td>
</tr>
<tr>
<td>----------------</td>
<td>--------------</td>
</tr>
<tr>
<td></td>
<td>pair</td>
</tr>
<tr>
<td>Gain</td>
<td>−</td>
</tr>
<tr>
<td>Noise</td>
<td>=</td>
</tr>
<tr>
<td>Offset</td>
<td>=</td>
</tr>
<tr>
<td>Swing</td>
<td>−</td>
</tr>
<tr>
<td>Speed</td>
<td>++</td>
</tr>
</tbody>
</table>
Differential pair

- Low accuracy (low gain) applications
- Voltage follower (capacitive load)
- Voltage follower with source follower (resistive load)
- In bias stabilization loops (effectively two stages in feedback)
Telescopic cascode

- Low swing circuits
- Switched capacitor circuits
  - Capacitive load
  - Different input and output common mode voltages
- First stage of a two stage opamp
  - Only way to get high gain in fine line processes
Folded cascode

- Higher swing circuits
- Higher noise and offset
- Lower speed than telescopic cascode
  - Low frequency pole at the drain of the input pair
- Switched capacitor circuits (Capacitive load)
- First stage of a two stage class AB opamp
Two stage opamp

- Highest possible swing
- Resistive loads
- Capacitive loads at high speed
- “Standard” opamp: Miller compensated two stage opamp
- Class AB opamp: Always two (or more) stages
Opamps: pMOS versus nMOS input stage

- nMOS input stage
  - Higher $g_m$ for the same current
  - Suitable for large bandwidths
  - Higher flicker noise (usually)

- pMOS input stage
  - Lower $g_m$ for the same current
  - Lower flicker noise (usually)
  - Suitable for low noise low frequency applications
Fully differential circuits

Two identical half circuits with some common nodes
Two arms of the differential input applied to each half
Two arms of the differential output taken from each half
Symmetrical linear (or small signal linear) circuit under fully differential (antisymmetric) excitation

- Nodes along the line of symmetry at 0 V (symmetry, linearity)
- Analyze only the half circuit to find the transfer function
Symmetrical circuit (maybe nonlinear) under common mode (symmetric) excitation

- Nodes in each half at identical voltages (symmetry)
- Fold over the circuit and analyze the half circuit
Common mode feedback

- Common mode feedback circuit for setting the bias
- Detect the output common mode and force it to be $V_{o,cm}$ via feedback
Common mode feedback loop has to be stable

- Analyze it by breaking the loop and computing the loop gain with appropriate loading at the broken point
- Apply a common mode step/pulse in closed loop and ensure stability
Calculate noise spectral density of the half circuit

Multiply by $2 \times$
Fully differential circuits: Offset

\[ v_{\text{off,full}} = 2v_{\text{off,half}} + \Delta V_T \]

\[ v_{\text{off,full}} = 2v_{\text{off,half}} + \Delta V_T \]

Calculate mean squared offset of the half circuit

- Multiply by 2 if mismatch (e.g. \( \Delta V_T \)) wrt ideal device is used

\[ v^2_{\text{off,full}} = 2v^2_{\text{off,half}} \]
Fully differential circuits: Offset

\[ V_{\text{off,full}} = V_{\text{off,half}} + \Delta V_{T12} - \Delta V_{T34} \]

\[ V_{\text{off,full}} = V_{\text{off,half}} + \Delta V_{T12} + \Delta V_{T34} \]

- Calculate mean squared offset of the half circuit
- Multiply by $1 \times$ if mismatch between two real devices is used

\[ V_{\text{off,full}}^2 = V_{\text{off,half}}^2 \]

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