i. Multiple choices of each questions are marked as A to D. One of the choices is unambiguously correct. Choose the most appropriate one amongst the given choices.

ii. Assume spherical earth with average radius of 6378 Km unless specified.

1. Determine the rise in antenna noise temperature $\Delta T$ during sun conjunction for 3 GHz receiver with antenna efficiency of 60% and beam width of $4^\circ$.
   A. 2.274K
   B. 22.74K
   C. 227.4K
   D. 22274K

   **Solution:**
   The generic equation for rise in antenna noise temperature is $\Delta T = P\eta T_s D^2$. Here $P = 0.5$ as the sun generates random polarization, $\eta = 0.6$, antenna efficiency.
   The temperature of the sun, $T_s = 0.12 \times 10^6 \times f^{-0.75}$K, where $f$ is measured in GHz.
   So, $T_s = 0.12 \times 10^6 \times 3^{-0.75} = 52642.97$K.
   The value of $D_s = \frac{0.48}{9} = 0.12$. Hence, by utilizing the formula given above, we get $\Delta T = 227.4K$. Hence the appropriate option is C.

2. A Satellite is orbiting in an elliptical orbit with apogee height at 20000 Km and perigee height at 400 Km. The ratio of velocity at perigee to that at apogee will be
   A. 3.89
   B. 7.07
   C. 15.15
   D. 2.56

   **Solution:**
   It is mentioned that the orbit has apogee height at 20000 Km and perigee height at 400 Km. So, the distance of apogee from the center of the earth can be calculated as $r_a = 20000 + 6378 = 26378$ Km. Similarly, the distance of perigee will be $400 + 6378 = 6778$ Km. The semi major axis can be calculated as $r = \frac{r_a + r_p}{2} = 16578$Km. So, the ratio of velocity at the perigee to that of at apogee is $\frac{v_p}{v_a} = \frac{\sqrt{\frac{r_a}{r_p} \cdot f}}{\sqrt{\frac{r_p}{r_a} \cdot f}} = \frac{0.01532}{3.93 \times 10^{-3}} = 3.89$.
   So, the appropriate option is A.

3. A satellite is orbiting in a circular orbit which is 1000 Km away from the surface of the earth. Then how many times in a day, the satellite will be overhead from a particular location on the earth.
   A. 16 times
   B. 15 times
   C. 14 times
D. 13 times

**Solution:**
The satellite is orbiting in a circular orbit with height 1000 Km from the surface of the earth. So, the distance from the center of the earth is 1000 + 6378 = 7378 Km. According to the Kepler’s Law, orbital period \( T = \sqrt{\frac{4\pi^2r^3}{3.986 \times 10^5}} \) = 6306.9472 seconds. Hence, the number of times the satellite will be overhead is \( \frac{86400}{6306.9472} \approx 13 \) times. As, the fractional times we cannot see a satellite overhead. So, the appropriate option is **D**.

4. A satellite is orbiting in an elliptical orbit. Thus a generic conclusion can be drawn as

A. the velocity at apogee is greater than that of perigee
B. the velocity at apogee is less than that of perigee
C. velocity at perigee is minimum
D. velocity at apogee is maximum

**Solution:**
The apogee is the furthest point and perigee is the closest point from a focus of an ellipse. According to Kepler’s Law any elliptical body covers same area in a specific time. Mathematically, the velocity at a particular point of an elliptical orbit can be written as

\[
v = \sqrt{\frac{\mu}{r} - \frac{1}{a}}.
\]

Where, \( a = \frac{r_a + r_p}{2} \). As \( r_a > r_p \), therefore, \( v_p > v_a \). So, the appropriate option is **B**.

5. Determine the orbital height in Km of a satellite orbiting in a circular orbit with orbital period of a sidereal day.

A. 42164 Km
B. 36712 Km
C. 42379 Km
D. 35786 Km

**Solution:**
Applying the Kepler’s Law, \( r = \left(\frac{T^2 \mu}{4\pi^2}\right)^{\frac{1}{3}}. \) Where, \( T = \)Orbital period, \( \mu = 3.986 \times 10^5 \text{km}^3/\text{s}^2 \) and \( r = \)Orbital height from the center of the earth. Now, it is mentioned in the question that, \( T = 86164 \)s. So, \( r = \left(\frac{86164^2 \times 3.986 \times 10^5}{4\pi^2}\right)^{\frac{1}{3}} = 42164.12 \text{ Km}. \) So, the orbital height is 42164 – 6378 = 35786 Km. So, the appropriate option is **D**.

6. The difference between the farthest and the closest point in a satellite’s elliptical orbit from the surface of the earth is 30000 Km, and the sum of the distances is 50000 Km, if the mean radius of the earth is considered to be 6400 Km, determine the eccentricity and length of semi-major axis of the orbit.

A. 0.32 & 31500 Km
B. 0.48 & 31400 Km
C. 0.61 & 31500 Km
D. 0.27 & 31400 Km
Solution:
The apogee and perigee distances are measured from the center of the earth. Now in the question, it is given that, Apogee-perigee=30000 Km (due to subtraction the radius of the earth is cancelled). Now apogee+perigee =50000 + (2 × 6400) = 62800Km. Now the orbit eccentricity can be calculated as \( e = \frac{\text{Apogee-Perigee}}{\text{Apogee+Perigee}} = 0.478 \). Now solving the previous two equations we get apogee distance as 46400 Km. Hence, the semi-major axis of the satellite orbit is \( \frac{\text{Apogee Distance} + \text{Perigee Distance}}{2} = \frac{62800}{2} = 31400 \)Km. So, the appropriate choice is B.

7. A satellite is in a circular equatorial orbit moving in the same direction as of earth rotation with a period 24 hours exactly. Determine the rate of drift of sub-satellite point around the equator in degrees per solar day.

A. 0.5 degree towards east  
B. 0.5 degree towards west  
C. 0.98 degree towards west  
D. 0.98 degree towards east

Solution:
The orbital period is greater than the one sidereal day. The earth is rotating around its axis from west to east. As the orbital period is more than a sidereal day, satellite will be rotating slower than that of the earth. Therefore, the sub satellite point would move towards west. Hence, the amount of shift can be determined as 360 × \( \frac{86400 - 86164}{86400} \) = 0.983°. Hence, the appropriate option is C.

8. Determine the visibility arc on earth equator from the satellite located at 87°E in the geostationary orbit

A. 5.7°E to 168.3°E  
B. 87°E to 168.3°E  
C. 163.3°W to 8.7°W  
D. Cannot be determined

Solution:
For the maximum visibility arc, the line joining the satellite and the visibility point on the earth will create tangent on the earth’s surface. Therefore, the angle between the line joining the satellite and center of the earth and that joining the center of the earth and the location on the earth’s surface, \( \alpha \) can be determined as,

\[
\alpha = \cos^{-1} \left( \frac{r_e}{r_e + r_h} \right) \\
= \cos^{-1} \left( \frac{6378}{6378 + 36000} \right) \\
= 81.34°
\]

Hence, the arc will be in between 87 – 81.3 = 5.7°E to 87 + 81.3 = 168.3°E. So the correct option is A.

9. A satellite is moving in an elliptical orbit with the semi major axis equals to 24571 Km. If the perigee distance is 6978 Km, find the apogee height and orbit eccentricity
A. 30000 Km, 0.62 
B. 42164 Km, 0.9 
C. 35786 Km, 0.72 
D. 42164 Km, 0.72 

**Solution:**

It is mentioned that the satellite orbit has semi major axis 24571 Km. The semi major axis, $a$ is half of the sum of distance of apogee($r_a$) and that of perigee($r_p$). Hence,

$$a = \frac{r_a + r_p}{2}$$

$$r_a = 2a - r_p$$

$$= 49142 - 6978$$

$$= 42164 Km$$

Hence, the apogee height would be $42164 - 6378 = 35786$ Km and the eccentricity, $e = \frac{r_a - r_p}{2a} = \frac{42164 - 6978}{49142} = 0.72$. So, the correct one is **C**.

10. An earth station at IIT Kharagpur campus $(22^0N, 87^0E)$ is pointing towards INSAT located at $93^0E$. Select the correct option for the Earth station’s elevation, azimuth and distance to the satellite. Assume the radius of the spherical earth=6378 Km and orbit height of INSAT is 35786 Km.

A. Elevation=63.340, Azimuth=164.330 and Distance=36367km 
B. Elevation=31.340, Azimuth=195.670 and Distance=42475km 
C. Elevation=63.340, Azimuth=165.670 and Distance=37164km 
D. none of these 

**Solution:**

Given, Latitude of IITKGP=$LB=22^0N$, Longitude of IITKGP=$lB=87^0E$, Latitude of INSAT, $LA=0^0N$ and Longitude of INSAT=$lA=93^0E$. Let the central angle be $\gamma$. So, we know that,

$$\cos(\gamma) = \cos(LB) \cos(LA) \cos(lA-lB) + \sin(LB) \sin(LA)$$

$$= 0.9221$$

So, $\gamma = 22.7643^0$. Now applying the triangular law,

$$\cos \gamma = \frac{r_s^2 + r_e^2 - d^2}{2r_sr_e}$$

and hence, $d = \sqrt{r_s^2 + r_e^2 - 2r_s r_e \cos \gamma} = 36366.85$ Km. Now, applying the sin rule, we can easily calculate the elevation angle $EL = 63.345^0$.

we know that, $\tan \alpha = \tan \frac{\tan(Ls-Le)}{\sin(Le)}$, where, $L_s=LA$ and $L_e=LB$. So, $\alpha = 15.672^0$. as the satellite us in the east of earth station, $Az=(180^0 - \alpha) = 164.32^0$. So, the appropriate choice is **A**.

11. A satellite in circular orbit with 1000 Km orbital height transmits at 2.65 GHz. A station in the plane of the satellite orbit receives the signal from the satellite when it is rising from horizon. The Doppler shift of the received signal will be in the range of
A. +50 KHz to +55 KHz  
B. −50 KHz to −60 KHz  
C. +55 KHz to +60 KHz  
D. −55 KHz to −65 KHz  

Solution:

It is given that the satellite is orbiting at 1000 Km from the surface of the earth. The velocity of the satellite will be

\[ v = \sqrt{\frac{\mu}{a}} = \sqrt{\frac{3.986 \times 10^5}{1000 + 6378}} = 7.35 \text{ Km/s}. \]

The component of the velocity towards an observer, located in the orbital plane is \( v_r = v \cos \theta \). Where \( \cos \theta = \frac{r_r}{r_h} = \frac{6378}{6378} = 0.6844 \). Hence, the radial velocity is \( v_r = 7.35 \times 0.6844 = 6.36 \text{ Km/s} \). As the satellite rising from the horizon, the Doppler shift will be positive. The amount of Doppler shift will be

\[ f_d = \frac{v_r \times f_c}{c} = \frac{6.36 \times 10^3 \times 2.65 \times 10^9}{3 \times 10^8} = 56180 \text{ Hz}. \]

Hence, the appropriate option is C.

12. A satellite was launched from a satellite launching pad located at \((13.7^0 N, 80.2^0 E)\) with azimuth 102\(^0\) and was launched into a Geosynchronous Transfer orbit with perigee at 250 Km and apogee at Geosynchronous height. Determine the minimum incremental velocity required to place the satellite in Geostationary orbit.

A. 1.63 Km/s  
B. 2.64 Km/s  
C. 4.56 Km/s  
D. none of these

Solution:

In GEO orbit the orbital period is equal to one sidereal day, i.e \( T = 86164 \) seconds. Therefore, from the orbital period and using Kepler's Law we can determine the height of the GEO orbit. Hence,

\[ r_a = \sqrt[3]{\frac{T^2 \mu}{4\pi^2}} = \sqrt[3]{\frac{3.986 \times 10^5 \times 86164}{4\pi^2}} = 42164.12 \text{ Km}. \]

Now, semi major axis of the elliptical orbit, \( a \) can be calculated as \( a = \frac{r_a + r_p}{2} = \frac{42164.12 + 6378 + 250}{2} = 24396.08 \text{ Km}. \) So, the velocity at the apogee of the orbit can be calculated as

\[ v_a = \sqrt{\mu \left[ \frac{2}{r_a} - \frac{1}{a} \right]} = \sqrt{3.986 \times 10^5 \left[ \frac{2}{42164.12} - \frac{1}{24396.08} \right]} = 1.602 \text{ Km/s}. \]

Launching is made from non zero latitude with 102\(^0\) azimuth angle. GTO plane will have inclination with earth equatorial plane. Since at GEO orbit inclination with equatorial plane is zero, launch inclination needs to be corrected. It is given in the question that the launching pad is located at \( L = 13.7^0 \) and have azimuth of \( A = 102^0 \). Hence, the orbit inclination can be determined by using the following formula,

\[ \cos i = \sin A \times \cos L \]
\[ i = \cos^{-1}[\sin A \times \cos L] \]
\[ = 18.14^0 \]

The GEO orbit, being circular one, the velocity of the satellite at the orbit can be calculated as

\[ v = \sqrt{\frac{\mu}{a}} = \sqrt{\frac{3.986 \times 10^5}{42164.12}} = 3.074 \text{ Km/s} \]
Now, the incremental velocity can be calculated as
\[
\Delta v_i = \left[ v^2 + v_a^2 - 2vv_a \cos(i) \right]^{0.5}
\]
\[
= [3.07^2 + 1.602^2 - (2 \times 3.07 \times 1.602 \times \cos(18.14))]^{0.5}
\]
\[
= 1.63
\]

Hence, the appropriate option is A.