If $M_3$ is off & $M_4$ is on

Then $V_{out} = V_{DD} - V_{TN_{M3}} - V_{TN_{M4}} = V_{DD} - 2V_{TN}$

Then no current goes through $M_3$. But $M_4$ must provide $2I_{DS}$ current, as it must cater to $I_{DS2}$ & $I_{DS1}$ currents. $I_{DS1}$ comes through charging of Capacitor $C$. 

\[ I_{DS} \rightarrow I_{DS2} \rightarrow I_{DS1} \rightarrow C \]
Case 2: Now if M3 is ON and M4 is OFF
Then \( V_{out} \) will be similar to that of \( V_{out} \) as in case 1.

Hence \( V_1 \) and \( V_2 \) will alternate in time frame.

Since \( I_{DS} \) charges capacitor in time \( \Delta t \), hence delivered charge in capacitor \( C = I_{DS} \cdot \Delta t \)

However, capacitor sees change of \( V_{DD} - V_{TN} - V_{DD} - 3V_{TN} = 2V_{TN} \)

\[ \text{Charge in Capacitor} = C \cdot 2V_{TN} \]

\[ \Delta t = \frac{2CV_{TN}}{I_{DS}} \]

But frequency \( f = \frac{1}{2\Delta t} = \frac{I_{DS}}{4CV_{TN}} \)
Current Starved VCO

This is essentially a Ring Oscillator

\[ f_{osc} = \frac{I_{DS}}{N \cdot C_{Total} \cdot V_{DD}} \]
**LC Oscillators**

All LC oscillators use a Resonant Tank Circuit to sustain oscillation.

**Ideal Inductor** \( f = \frac{1}{2\pi\sqrt{LC}} \)

Series resistance of inductor coil.

In IC chips, Spiral Inductor Coils are printed to have desired inductance value.
For the Non-Ideal Tank Circuit

\[
Z_T(s) = \frac{R_s + L_1 s}{1 + L_1 C_1 s^2 + R_s C_1 s} = (R_s + j\omega L_1) \left(1 - \frac{1}{j\omega C_1}\right)
\]

\[
|Z_T(j\omega)|^2 = \left| \frac{R_s + j\omega L_1}{1 - \omega^2 L_1 C_1 + j\omega R_s C_1} \right|^2
\]

\[
= \frac{R_s + \omega_0^2}{(1 - \omega_0^2)^2 + R_s^2 \omega^2}
\]

In ideal LC Circuit at Oscillation frequency, the \( Z_T \) (\( Z \) for Tuned Circuit) is infinite.

But in non-ideal case as above \( Z_T(j\omega) \to 0 \) at any frequency. Which essentially means Circuit has finite Q.
Tank circuit representation in parallel form.

Let us keep $C_1 = C_p$, then

$$R_s + j\omega L_1 = \frac{R_p L_p S}{R_p + L_p S} = \frac{j\omega R_p L_p}{R_p + j\omega L_p} - (i)$$

$$= \frac{j\omega R_p L_p (L_p - j\omega L_p)}{R_p^2 + \omega^2 L_p^2}$$
From (i)
\[(R_s + j\omega L_1)(R_p + j\omega L_p) = j\omega R_p L_p\]

or
\[R_s R_p - \omega^2 L_1 L_p + j\omega (L_p R_s + L_1 R_p) = j\omega (R_p L_p - c_{ii})\]

Equating Real & Imaginary parts in (cii)
\[L_p R_s + L_1 R_p = R_p L_p - c_{iii}\]
and
\[R_s R_p - \omega^2 L_1 L_p = 0 - c_{iv}\]

From (iv)
\[\omega^2 L_1 L_p = R_s R_p\]

\[L_p = \frac{R_s R_p}{\omega^2 L_1} \rightarrow (v)\]
\[ R_p = \frac{\omega^2 L_1 L_p}{R_s} \]  

Substituting \( R_p \) in (iii):

\[ L_p \frac{R_s}{L_p} + L_1 \frac{\omega^2 L_1 L_p}{R_s} = \frac{\omega^2 L_1 L_p}{R_s} \cdot L_p \]

\[ L_p \frac{R_s}{L_p} + \frac{\omega^2 L_1^2 L_p^2}{R_s \cdot L_p} = \frac{\omega^2 L_1^2 L_p^2}{R_s \cdot L_1} \]

\[ L_p \frac{R_s}{L_p} + \frac{\omega^2 L_1^2 L_p^2}{R_s} \left( \frac{1}{L_p} - \frac{1}{L_1} \right) = 0 \]

\[ L_p \frac{R_s^2}{L_p} + \omega^2 L_1^2 L_p^2 = \frac{R_s L_p}{R_s L_1} \cdot \frac{\omega^2 L_1^2 L_p^2}{L_1} \]

\[ 1 + \frac{L_p \frac{R_s^2}{\omega^2 L_1^2 L_p^2}}{L_1} = \frac{L_p}{L_1} \frac{\omega^2 L_1^2 L_p^2}{\omega^2 L_1^2 L_p^2} \]
\[ 1 + \frac{R_s^2}{\omega^2 L_1^2} = \frac{L_p}{L_1} \]

\[ L_p = L_1 \left( 1 + \frac{1}{Q^2} \right) \]

Since \( Q \gg 1 \) (Typical \( Q \) of Spiral Inductor on Silicon is \( \geq 3 \))

\[ L_p \equiv L_1 \]

Substitute this in eq. (vi)

\[ R_p = \frac{\omega^2 L_1 L_p}{R_s} = \frac{\omega^2 L_1^2}{R_s} = \frac{\omega^2 L_1^2}{R_s^2} \cdot R_s = Q^2 R_s \]

\[ R_p = Q^2 R_s \]

And \( C_p = C_1 \)
The Magnitude & Phase of Input Impedance \( Z \) of Parallel Tank Circuit is like:

\[
\omega_1 = \frac{1}{\sqrt{LC}} \\
\omega_0 = \sqrt{\frac{L}{C'}} \\
f_{0.5} = \frac{1}{2\pi\sqrt{L/C'}} \\
= \frac{1}{2\pi\sqrt{L/C_p}}
\]
Cross Coupled Oscillator

\[ H(j\omega) = H_1(j\omega) \cdot H_2(j\omega) \]

\[ H_0 = g_{m1}R_p \cdot g_{m2}R_p \]

If \( H(0) \geq 1 \)

then oscillation will be sustained.
Colpitts and Hartley Oscillators are example for Cross Coupled LC Oscillators

Principle:

\[
\frac{V_{out}}{I_{in}} = \frac{L_p S}{R_p} \parallel \frac{1}{C_p S}
\]

Colpitts Oscillator
\[ V_1 = -\left( I_{in} - \frac{V_{out}}{L_P S} - \frac{V_{out}}{R_P} \right) \cdot \frac{1}{C_1 S} \]

\[ i_{C_2} = \{ V_{out} - (-V_1) \} C_2 S = (V_{out} + V_1) C_2 S \]

\[ i_{C_2} = \left[ V_{out} - \left( I_{in} - \frac{V_{out}}{L_P S} - \frac{V_{out}}{R_P} \right) \frac{1}{C_1 S} \right] C_2 S \]

Using Kirchhoff's law at node Vout-
we have

\[ g_m V_i + I_{c2} + \frac{V_{out}}{L_p} + \frac{V_{out}}{R_p} = 0 \]

Solving this equation we get:

\[ \frac{V_{out}}{I_{in}} = \frac{R_p L_p S (g_m + C_2 S)}{R_p C_1 C_2 L_p S^2 + (C_1 + C_2) L_p S^2 + [g_m L_p + R_p (C_1 + C_2)] S + g_m R_p} \]

For oscillation \( \frac{V_{out}}{I_{in}} \to \infty \) at oscillatory frequency \( \omega_R \)

We obtain

\[ \omega_R^2 = \frac{1}{L_p \frac{C_1 C_2}{C_1 + C_2}} \quad \& \quad g_m R_p = \frac{C_1}{C_2} \left( 1 + \frac{C_2}{C_1} \right)^2 \]

Minimum required DC gain gives \( g_m R_p \geq 4 \) when \( \frac{C_2}{C_1} = 1 \)

With \( C_p \) present \( \omega_R^2 = \frac{1}{\left[ C_p + \frac{C_1 C_2}{C_1 + C_2} \right] L_p} \)
Single Port Oscillators.

Oscillations die as $R_p$ dissipates power.

$$f_{osc} = \frac{1}{2\pi \sqrt{L_p C_p}}$$

$-ve$ Resistance
Negative Impedance Generator

\[ I_x = g_{m2} V_2 = -g_{m1} V_1 \]

But \( V_x = V_1 - V_2 \)

\[ V_x = \frac{I_x}{g_{m1}} - \frac{I_x}{g_{m2}} \]

or \( \frac{V_x}{I_x} = -\left( \frac{1}{g_{m1}} + \frac{1}{g_{m2}} \right) = -\frac{2}{g_m} \) if \( g_{m1} = g_{m2} = g_m \)

\[ R_{in} = -\frac{2}{g_m} \]
Differential version
LC Oscillator
with -ve Resistance Generator.