A simple ac amplifier noise can be now evaluated.

If $R_{out}$ is output resistance of $M1$ ($\gamma_{01}$) and current source has $R_{source}$ then noise current $\sqrt{\gamma_n}$ flows through $R_{out}$ output resistance.

\[ \sqrt{\gamma_{out}} = \sqrt{\gamma_n} \cdot \gamma_{01} \]

\[ = 4kT \left( \frac{2}{3}g_m \right) \gamma_{01} \]

\[ 4kT \frac{2}{3}g_m \gamma_{01} \sqrt{\gamma_{out}} = \sqrt{\frac{8kTg_m}{3} \gamma_{01} \gamma_{source}} \]

\[ \therefore \text{Low Noise} \Rightarrow \text{low } g_m \text{ (Lower Gains)} \]
$1/f$ Noise

- Famously known as Flicker Noise.
- Most believe that this Noise is due to random carrier trapping at Interface States (Insulator/Semiconductor Interface) in MOS device.
- It is difficult to accurately predict the Noise Power due to $1/f$ Noise, as 'Interface' property is strongly dependent on Process Technology.
- This noise power is seen to follow $1/f$ relation with frequency $f$. 
In general one models this noise by Noise Voltage Source in series to Gate terminal.

Typically it can be represented as

\[ V_{nVf}^2 = \frac{K}{C_{ox}W_L} \cdot \frac{1}{f} \]

\[ I_{nVf}^2 = \frac{K}{C_{ox}W_L} \cdot g_{m}^2 \cdot \frac{1}{f} \]

The spectral density of Flicker Noise Current source is given by

\[ \overline{I_{nVf}^2} = \int_{f_1}^{f_2} \frac{K}{L^2} \cdot I_{DS} \cdot \frac{df}{C_{ox}f} \]
\[ \bar{I}_{n_{1/f}}^2 = \frac{K_1}{L^2} \frac{I_{DS}}{C_{ox}} \ln \left( \frac{f_2}{f_1} \right) \]

\[ \bar{I}_{n_{1/f}}^2 = \frac{K_2}{L^2} \frac{I_{DS}}{C_{ox}} \log \left( \frac{f_2}{f_1} \right) \]

We also know that Thermal Noise Voltage in a frequency band can be written as

\[ \bar{V}_{n_{th}}^2 = 4kT \left( \frac{g_m}{3} \right) \]

At a frequency \( f_c \) called 1/f Noise Corner Frequency, the two noise voltages are equal.

\[ 4kT \left( \frac{2g_m}{3} \right) = \frac{K}{C_{ox}W\cdot L} \frac{1}{f_c} g_m^2 \]

or

\[ f_c = \frac{Kg_m}{C_{ox}W\cdot L} \frac{3}{8kT} \]
20 \log \bar{V}_n^2

(\log \text{Scale})

\frac{1}{f}

\text{Noise}

\text{Thermal Noise}

f (\log)

\text{Constants:}

K_{1\text{NMOs}} = 2 \times 10^{-29} \text{ A} \cdot \text{F}

K_{1\text{PMOS}} = 3.5 \times 10^{30} \text{ A} \cdot \text{F}

K = 1.06 \times 10^{-25} \text{ V}^2 \text{F} \quad (90 \text{ nm Tech.})
(i) As \( K_1 \) values are larger for NMOS devices compared to PMOS devices, \( f_c \) for NMOS devices is larger than that for PMOS devices.

(ii) Further \( f_c \propto \frac{1}{L^2} \) i.e. \( f_c \) increases with Technology Scaling.

(iii) Finally \( f_c \propto \frac{1}{g_m/|I_{DS}|} \), gives understanding that larger \( g_m \) devices reduces \( f_c \), meaning 1/f noise dominates over thermal noise.
Noise in CS Amplifier

\[ \frac{V_{n_{\text{out}}}^2}{R_L} = -\frac{4kT}{R_L} \]

\[ V_{n_{\text{out}}}^2 = \left[ 4kT \frac{2}{3} g_m + \frac{K}{C_{ox W L}} \cdot \frac{1}{f} g_m^2 + \frac{4kT}{R_L} \right] R_L^2 \]

Input Referenced Noise

\[ \frac{V_{n_{\text{in}}}^2}{A_{vo}^2} = V_{n_{\text{out}}}^2 / A_{vo}^2 \]

\[ A_{vo} = -g_m R_L \]

\[ = \left[ 4kT \left( \frac{2}{3} g_m \right) + \frac{K}{C_{ox W L}} \cdot \frac{g_m^2}{f} + \frac{4kT}{R_L} \right] \frac{R_L^2}{g_m^2 R_L^2} \]
\[ \bar{V}_{n_{\text{min}}}^2 = \frac{8}{3} \frac{kT}{g_m} + \frac{K}{C_{\text{ox}}W L} \cdot \frac{1}{f} + \frac{4kT}{g_m^2 R_L} \]

\[ \bar{V}_{n_{\text{out}}}^2 = 4kT \left( \frac{2}{3g_m} + \frac{1}{g_m^2 R_L} \right) + \frac{K}{C_{\text{ox}}W L} \cdot \frac{1}{f} \]

gives Input Referred Noise.

\[ \bar{V}_{n_{\text{out}}}^2 = \left[ \frac{8}{3} \frac{kT g_m}{R_L} + 4kT \right] \left[ \frac{R_L}{1 + j\omega R_L C_L} \right]^2 \text{df} \]

(\(\omega > \omega_c\) is assumed)

\[ \bar{V}_{n_{\text{out}}}^2 = \int_0^\infty 4kT \left( \frac{1}{R_L} + \frac{2}{3} g_m \right) \left| \frac{R_L}{1 + j\omega R_L C_L} \right|^2 \text{df} \]

\[ = \frac{kT}{C_L} \left( 1 + g_m \cdot \frac{2}{3} R_L \right) \leq \frac{kT}{C} (1 + 1 \Delta v_{\text{off}}) \]
Noise in DIFFAMP

\[ \overline{i_{n}^2} = 4kT \left( \frac{2}{3} g_m \right) + \frac{K}{C_{ox} W L} \frac{g_m^2}{f} \]

\[ i_{\text{inout}}^2 = \overline{i_{n1}^2} + \overline{i_{n2}^2} + \overline{i_{n3}^2} + \overline{i_{n4}^2} \]
\[ \sqrt{V_{\text{Total\_out}}} = \frac{i_{\text{out}}^2}{g_{m_1}} \cdot (\gamma_{o2\|o4})^2 \sqrt{V^2/1+e} \]

\[ \therefore \ \sqrt{V_{\text{TR\_RM}}} = \frac{V_{\text{Total\_out}}}{A_{V_0}} = \frac{1}{g_{m_1}^2} \cdot \frac{V_{\text{Total\_out}}}{(\gamma_{o2\|o4})^2} \]

\[ \therefore \ \sqrt{V_{\text{TR\_RM}}} = \frac{i_{\text{out}}^2}{g_{m_1}^2} \cdot (\gamma_{o2\|o4})^2 \]

\[ \therefore \ \sqrt{V_{\text{TR\_RM}}} = \frac{i_{\text{out}}^2}{g_{m_1}^2} \]

\[ i_{\text{out}}^2 = 4kT \left( \frac{2}{3} g_{m_1} \right) + \frac{K}{C_{ox} W_{1} L_1} \cdot \frac{g_{m_1}^2}{f} + 4kT \left( \frac{2}{3} g_{m_2} \right) + \frac{K}{C_{ox} W_{2} L_2} \cdot \frac{g_{m_2}^2}{f} + 4kT \left( \frac{2}{3} g_{m_3} \right) + \frac{K}{C_{ox} W_{3} L_3} \cdot \frac{g_{m_3}^2}{f} + 4kT \left( \frac{2}{3} g_{m_4} \right) + \frac{K}{C_{ox} W_{4} L_4} \cdot \frac{g_{m_4}^2}{f} \]
\[ g_{m_1} = g_{m_2}, \quad \frac{w_1}{l_1} = \frac{w_2}{l_2} \]

\[ g_{m_3} = g_{m_4}, \quad \frac{w_3}{l_3} = \frac{w_4}{l_4} \]

\[ \therefore \quad t_{\text{out}}^2 = 8kT \left( \frac{2}{3} g_{m_1} \right) + 2K g_{m_1}^2 \cdot \frac{1}{f} \cdot \frac{1}{C_{ox} w_1 l_1} \]

\[ + 8kT \left( \frac{2}{3} g_{m_3} \right) + 2K g_{m_3}^2 \cdot \frac{1}{f} \cdot \frac{1}{C_{ox} w_3 l_3} \]

\[ \therefore \quad V_{TN,\text{INA}}^2 = \frac{16kT}{3g_{m_1}} + \frac{2K}{C_{ox} w_1 l_1} \cdot \frac{1}{f} \]

\[ + \frac{16kT}{3} \cdot \frac{g_{m_3}}{g_{m_1}} + 2K \left( \frac{g_{m_3}}{g_{m_1}} \right)^2 \cdot \frac{1}{C_{ox} w_3 l_3} \cdot \frac{1}{f} \]
In Sampling Mode, the MOS Transistor is used as Switch. Switch is characterised as

\[ \text{Clock} \quad R \quad M \quad X \quad \text{out} \]

\[ \text{In} \quad \text{out} \quad \Rightarrow \quad \text{In} \quad \text{out} \]

In sampling we have circuit like shown. When Switch is ‘ON’, C charges to \( V_{in} = V_{our} \)

and when Switch is ‘OFF’, last \( V_{out} \) is retained or 'hold'.

This RC circuit gives \( \frac{kT}{C} \) noise.

For such a circuit

\[ \text{SNR} = \frac{P_{av}}{P_{noise}} = \frac{\frac{1}{2} V_{out}^2}{kT} \times C \frac{V_{out}^2}{kT} \text{ at a Temp.} \]

SNR obtained is around 80 dB.