Frequency Response of Amplifiers

Concepts of Poles & Zeros.

A Transfer Func $A_v(s) = A_v \cdot \frac{s/\omega_2}{(s/\omega_1 + 1)(s/\omega_2 + 1)}$

$\omega_2$ is called a 'zero' frequency at which $A_v(s) \to 0$

$\omega_1, \omega_2$ are poles (Pole frequencies) at which $A_v(s) \to \infty$

In a typical MOS Amplifier we have around two 'poles' and one 'zero'.

Two poles occur from Input Side & Output Side.

These can be termed as $\omega_{in}$ and $\omega_{out}$.

And $\omega_2$ is a 'zero' frequency occurring due to feedback.
**SINGLE POLE TRANSFER FUNCTION**

**FIGURE 1.22** Two examples of STC networks: (a) a low-pass network and (b) a high-pass network.

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<td>( \frac{K}{1 + (s/\omega_0)} )</td>
<td>( \frac{Ks}{s + \omega_0} )</td>
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<td>3-dB Frequency</td>
<td>( \omega_0 = 1/\tau ); ( \tau = ) time constant</td>
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Bode Plots of STC

**FIGURE 1.23** (a) Magnitude and (b) phase response of STC networks of the low-pass type.

**FIGURE 1.24** (a) Magnitude and (b) phase response of STC networks of the high-pass type.
Input pole & Output Pole

\[ \omega_{in} = \frac{1}{R_s C_{gs}} \]

\[ \omega_{out} = \frac{1}{R_D C_{db}} \]

However in equivalent cut & Amplifier we also have capacitance \( C_{gd} \).
Then circuit looks like

\[ V_{gs} = \frac{1}{C_{gs} \cdot s} \quad V_{in} = \frac{V_{in}}{1 + s \cdot R_s \cdot C_{gs}} \]

\[ C_{gs} \text{ may include } C_{gb} \text{ as it may occur in saturation.} \]

Then

\[ A_v(s) = \frac{V_o(s)}{V_{in}(s)} = -\left( \frac{g_{m} R_D}{g_{m}} \right) \left\{ \frac{1 - \frac{C_{gd} \cdot s}{g_{m}}}{1 + s \cdot C_{gd} \cdot R_D} \right\} \]

\[ A_v(s) = \frac{V_o(s)}{V_{in}(s)} = -\left( g_{m} R_D \right) \left( \frac{1 - s \cdot C_{gd}/g_{m}}{1 + s \cdot R_s \cdot C_{gs} \cdot (1 + s \cdot R_D \cdot C_{gd})} \right) \]
Approximately looking at denominator we see

\[ D = \gamma_m (1 + s R_s C_{qs}) (1 + s R_D C_{gd}) \]

\[ = (1 + s R_s C_{qs}) (\gamma_m + s \gamma_m R_D C_{gd}) \]

\[ = (1 + s R_s C_{qs}) (\gamma_m + (-A_v o) s C_{gd}) \]

\[ = 1 + s \left\{ C_{qs} + (1 + A_v o) C_{gd} \right\} R_s \]

\[ = 1 + R_s (C_{qs} + (1 + \gamma_m R_D) C_{gd}) \cdot s \]

\[ = 1 + R_s C_{in} \cdot s \quad C_{in} = C_{qs} + (1 + \gamma_m R_D) C_{gd} \]

\[ \therefore \omega_m = \frac{1}{R_s C_{in}} \]
Similar result we can attain by using Miller's Theorem.

Given

\[ V_x \rightarrow 2 \rightarrow V_y \]

we can convert this to

where

\[ A = \frac{V_y}{V_x} \quad \text{(Gain Function)} \]

\[ z_1 = \frac{z}{1-A} \]

\[ z_2 = \frac{A}{1-A} \cdot z \]

If \( z \) is capacitive \( C_{xy} \)
Limitations of Miller's Theorem

We must have two paths from input to output for validity of Miller's theorem.
Continuing similar argument, we can see that a 'Zero' of Transfer Function is only possible if we have two paths from input to output and at a frequency the value of current in two paths are equal in magnitude and 180° out of phase.

This is due to effect of "Feed Forward"

\[ I_{cs} = -I_{ds} \]

\[ \therefore I = 0 \quad \therefore \text{Vout} = 0 \rightarrow \text{'Zero' Occurrence} \]
Returning to our problem of frequency response of a CS amplifier, whose HF model can be shown with $e_\text{q}$ omitted as:

Using DC gain expression $A_{V0} = -g_m R_D$, we see that $e_\text{q}$, Ckt could be shown as using Miller's theorem as:
Then
\[ \omega_{in} = \frac{1}{R_s \left[ C_{gs} + (1+\beta_m R_D) C_{gd} \right]} \]

\[ \omega_{out} = \frac{1}{R_D \left( C_{gd} + C_{db} \right)} \]

And Transfer \( F_n \) can be written as
\[ A_{v(s)} = \frac{A_{vo}}{(1+\frac{s}{\omega_{in}}) \left(1+\frac{s}{\omega_{out}}\right)} \]
\[ A_{vo} = -\beta_m R_D \]
If we solve complete eq. circuit with node analysis, we get

\[
Av(s) = -\frac{g_m R_D - C_{gd} \cdot s}{1 + R_s \cdot R_D \cdot C_{eq} \cdot s^2 + \left[ R_s \left( 1 + g_m R_D \right) C_{gd} \right. \\
\left. + R_s C_{gs} + R_D \left( \frac{C_{gd} + C_{ab}}{2} \right) \right] s}
\]

This leads to

\[
\omega_{p1} = \frac{1}{R_s \left( 1 + g_m R_D \right) C_{gd} + R_s C_{gs} + R_D \left( \frac{C_{gd} + C_{ab}}{2} \right)}
\]

\[
\omega_{p2} = \frac{1}{\omega_{p1}} \cdot \frac{1}{R_s R_D \left( C_{gs} C_{gd} + C_{gs} C_{db} + C_{gd} C_{ab} \right)}
\]

\[
C_{eq} = \left( C_{gs} C_{gd} + C_{gs} C_{db} + C_{gd} C_{ab} \right)
\]
If $R_D \left(C_{gd} + C_{db}\right) \ll R_s C_{gs} + R_s \left(1 + g_m R_D\right) C_{gd}$

and $C_{gs} \gg \left(1 + g_m R_D\right) C_{gd} + \frac{R_D}{R_s} \left(C_{gd} + C_{db}\right)$

then

$$\omega_{p1} = \frac{1}{R_s \left[C_{gs} + \left(1 + g_m R_D\right) C_{gd}\right]}$$

$$\omega_{p2} = \frac{1}{R_D \left(C_{gd} + C_{db}\right)}$$

If we see values of $\omega_{in}$ & $\omega_{out}$ as obtained by Miller's theorem use, we observe that

$$\omega_m = \omega_{p1} \quad & \quad \omega_{out} = \omega_{p2}$$
Hence in most cases we can find Dominant Pole, Non Dominant poles and even zeros using simple evaluations of \( \omega_n \) and \( \zeta \), which essentially are inversely related to input time constant \( R_i C_{in} \) and output time constant \( R_{out} C_{out} \), being output Time Constant.

This suggests a simple Technique of evaluation of poles using a technique called "Zero-Value Time Constant" analysis. This is also at times referred as "Open Circuit Time Constant analysis."
Step 2:

\[ R_{cgd} = \frac{V_{gd}}{i_{test}} = \frac{i_{test} \cdot R_s + R_D (g_m V_{gs} + i_{test})}{i_{test}} \]

\[ R_{cgd} = R_s + g_m R_D \cdot i_{test} \cdot R_s + R_D \]

\[ R_{cgd} = R_s + R_D + g_m R_s R_D i_{test} \]
ZVTC Technique:

1. Remove all but one Capacitor.
   Short all independent Voltage Sources,
   open all independent Current Sources.

2. Calculate resistance seen by capacitor $C_1$
   and then evaluate Time constant $\tau_1 = R_1 C_1$.

3. Repeat this for all Capacitors and obtain
   $\tau_j = R_j C_j$.

4. Sum all the Time Constants and then we get
   $\omega_{-3db} = \frac{1}{\sum_{j=0}^{N} \tau_j}$
   $N$ is no. of Capacitors.
ZVTC and SCTC Techniques

1. These do not lead to finding 'Zeros' of the systems.

2. It is accurate enough if there exists a dominant pole separated by larger frequency to next non-dominant pole. In most amplifiers which we use in analog systems, this condition is mostly met.

3. In these techniques, remove coupling capacitors from evaluation as they provide 'shorts' at higher frequency.
For the Ckt. of CS Amplifier, we obtain $\omega_{-3db}$ now using ZVTC technique.

Step 1: Time constant for $C_{gs}$. (Open $C_{gd}$ & $C_{db}$ arm for $C_{gd} = C_{db} = 0$)

Clearly $R_{egs} = R_S \quad \therefore \tau_1 = R_S C_{gs}$
\[
\omega_{-3\text{dB}} = 2\pi f_{-3\text{dB}} = \frac{1}{R_s C_g + \frac{R_o C_d}{2} + (R_s + R_D)C_g d + \frac{g_m R_s R_D}{C_d}}
\]

Please pay attention to our earlier calculations of \(\omega_{p1}\). It seems that now we get
\[
\omega_{-3\text{dB}} = \omega_{p1}
\]
This is a dominant pole.

Hence ZVT technique is good enough approximation to get Dominant Pole. For non-dominant pole too, we use technique called `Short Circuit Time Constant` technique.
Example**: We continue with same amplifier design and neglect $C_{ab}$ for this example.

\[
\begin{align*}
\text{Solve for:} \\
R_s &= 10K = R_L \\
C_{gs} &= 1 \text{ pf} \\
C_{gd} &= 20 \text{ pf} \\
q_m &= 3mA/V
\end{align*}
\]

We have two capacitors in the circuit, and hence we have two poles, one of them will be dominant pole and other non-dominant pole.

(i) We use $2VTC$ technique to obtain dominant pole

(ii) We use 'short circuit time constant' technique to get non-dominant pole.

*Ref: Gray, Myer et al.*
Data: \( R_s = 1 \, k \), \( R_D = 5 \, k \)
\( I_D = 1 \, mA \), \( B' (W/L) = 100 \times mA/V^2 \)
\( C_{gd} = 0.5 \, pf \), \( C_{gs} = 0 \)
\( C_{gs} = 5 \, D \, pf \)

Now, \( g_m = \sqrt{2 \times 100 \times 10^3 \times 10^3} = 14.1 \, mA/V \)

Using Miller Technique, the dominant pole is
\[
\omega_p = -\frac{1}{(1 + g_m R_D) R_s C_{gd} + R_s C_{gs} + R_D C_{gd}}
\]

\( 1 + g_m R_D = 1 + 10^{-3} \times 14.1 \times 5 \times 10^3 = 71.5 \)

\( (R_s + R_D) = 6 \times 10^3 \times 0.5 \times 10^{-12} \)
\( R_s \cdot C_{gs} = 10^3 \times 5 \times 10^{-12} = 5 \times 10^{-9} \)
\( = 3 \times 10^{-9} \)
\[ g_m R_D = 70.5 \quad ; \quad R_s R_D = 5 \times 10^6 \]

\[ R_s R_D C_{gd} = 5 \times 10^6 \times 5 \times 10^{-13} = 2.5 \times 10^{-6} \]

\[ g_m R_s R_D C_{gd} = 14.1 \times 10^{-3} \times 2.5 \times 10^{-6} \]

\[ = 3.525 \times 10^{-9} \]

\[ \therefore \omega_{p1} = \frac{1}{3 \times 10^{-9} + 5 \times 10^{-9} + 35.25 \times 10^{-9}} \]

\[ = \frac{10^{-9}}{43.25 \times 10^{-9}} = 23.12 \times 10^6 \]

\[ \therefore f_{p1} = \frac{1}{2\pi} \times 23.12 \times 10^6 = 3.68 \text{ MHz} \]

\[ \omega_{p2} = \frac{1}{R_D C_{gd}} = \frac{1}{5 \times 10^3 \times 5 \times 10^{-13}} = \frac{10^{10}}{25} = 4 \times 10^8 \]

\[ f_{p2} = \frac{1}{2\pi} \times 4 \times 10^8 = 63.6 \text{ MHz} \]
We compare these values with 2VTC & ASCTC ('Short Ckt Time constant') technique based evaluations of $f_{p1}$ & $f_{p2}$

[1] 2VTC technique gives Dominant Pole:

Here $\tau_1 = R_S C_{gs} = 5 \times 10^{-9}$

$\tau_2 = (R_S + R_D) C_{gd} + \Im R_S R_D C_{gd}$

$= 3 \times 10^{-9} + 35.25 \times 10^{-9}$

$\therefore \omega_{p1} = \frac{1}{\sqrt{\tau_j}} = \frac{1}{\tau_1 + \tau_2} = \frac{1}{43.25 \times 10^{-9}}$

$\therefore f_{p1} = 3.68 \text{ MHz} \quad \text{Dominant Pole}$
[2] Short Circuit Time Const. Technique:

This gives Non dominant Pole.

(a) \( C_{gd} \) seen Resistance when \( C_{gs} \) is shorted

\[
\begin{align*}
R_s & \quad C_{gd} \\
\text{Vin} & = 0 \\
\text{shorted} & \quad g_m \\
R_D & \quad C_{gd} \\
\end{align*}
\]

\[
R_{Cgd} = R_D
\]

\[
\therefore \quad \tau_{Cgd} = R_D C_{gd} = 5 \times 10^{-9} \times 0.5 \times 10^{-12} = 2.5 \times 10^{-9}
\]

(b) \( C_{gs} \) seen Resistance when \( C_{gd} \) is shorted

\[
\begin{align*}
R_s & \quad C_{gs} \\
g_m V_{gs} & \quad R_D \\
\end{align*}
\]

\[
\therefore \quad R_{Cgs} = (R_s \parallel R_D) \parallel \frac{1}{g_m}
\]
\[ Z_{cs} = \text{Re} Z_{cs} \cdot C_{gs} \]
\[ = 65 \times 0.5 \times 10^{-12} \times 10 = 0.325 \times 10^{-9} \]

\[ Z_{sc} = 2.5 \times 10^{-9} + 0.325 \times 10^{-9} \]
\[ = 2.825 \times 10^{-9} \]

\[ \omega_{non-dc} = \omega_{p2} = \frac{1}{2.825 \times 10^{-9}} \]

\[ f_{p2} = \frac{10^9}{6.28 \times 2.825} = 56.3 \text{ MHz} \]

\[ f_{p2} \text{ we obtained earlier with nodal analysis} = 63.6 \text{ MHz} \]
\[ \therefore \tau_2 = R_{c_{gd}} \cdot C_{gd} \]

\[ \therefore \tau_2 = (R_s + R_D + G_m R_s R_D) C_{gd} \quad - (2) \]

**Step 3:** We get Time constant as seen by $C_{db}$

\[ \tau_3 = R_D \cdot C_{db} \quad - (3) \]

\[ \therefore \frac{1}{\tau_{\text{cdd}}} = \frac{1}{\tau_2 + \tau_3} = \frac{1}{\frac{1}{\tau_2} + \frac{1}{\tau_3}} \]