M2 & M4 form Cascode Stage leading to Higher Rout. However, we must ensure that M2 & M4 are in Saturation.

In current source case, not only do we need High Rout, but very small $V_{min}$

For a $5 \mu $ ($V_{DD} = 5V$) process

$V_{Th} = 0.83V$ & $V_{OV} = 0.37V$

i.e. $V_{GS} = 1.2V$

$V_{min}$ is drop across current source.
If $V_{gs} = 1.2 \text{ V}$
then $V_{gs1} = V_{gs2} = 1.2$
and $V_{gs3} = V_{gs4} = 2.4 \text{ V}$

For $M4$ in Saturation

$V_{ds4} > V_{gs4} - V_T$

Now $V_{D2} = V_{S4}$

For $M2$ to saturate $V_{ds2} > V_{gs2} - V_T$

If we keep $V_{ds2} = V_{gs2}$, then $M2$ is always in Saturation

$V_{ds2} = V_{gs1} = V_{gsL} = 1.2 \text{ V}$

For $M4$ to be in saturation $V_{D4} - V_{D2} > V_{QA4} - V_{D2} - V_T$
or $V_{D4} > 2.4 - 0.83 = 1.57 \text{ V} = (2V_{OV} + V_T)$
i.e. $V_{min} > 2V_{OV} + V_T$
To reduce $V_{min}$, we use an additional battery of $V_T$ between gates of $M_3$ & $M_4$.

This gives $V_{Ga4} = 2V_{ov} + V_T$

$\therefore V_{min} = 2V_{ov} + V_T - (V_{ov} + V_T) = V_{ov}$

$\text{Rout} = r_{o4}(1 + g_{m4}r_{o2}) + r_{o2}$

$\approx r_{o4}(1 + g_{m4}r_{o2})$

$= g_{m4}r_{o2}r_{o4}$ (Cascode Effect)
Sensitivity Analysis

\[ S_x = \lim_{\Delta x \to 0} \frac{\Delta y}{y} \frac{\Delta x}{x} \]  
Definition

(i) Sensitivity of Current Source wrt \( V_{DD} \)

In a Simple Current Mirror (\( W_1/L \) are equal for \( M_1 \& M_2 \))

We have \( I_0 = I_{DS1} = 10\mu A \)

However \( I_{DS1} = \frac{V_{DD} - V_{SS} - V_{GS}}{R} \)

We need to find \( S_{V_{DD}} = \lim_{AV_{DD} \to 0} \frac{\Delta I_{0}}{I_{0}} \)

\[ S_{V_{DD}} = \frac{V_{DD}}{I_{0}} \frac{\Delta I_{0}}{\Delta V_{DD}} \]

\[ \frac{\Delta I_{0}}{I_{0}} = 0.66 \times 0.2 \] (if \( V_{DD} \) becomes \( V_{DD} \pm 0.1V \) ) \( \approx 5\% \)
\[ R_c = R_s \frac{L}{W} \]

\[ A = w \cdot 2s \]

\[ t = xj \]

\[ \delta = 9 \mu n^t \]

\[ s = \frac{1}{9 \mu n^t} \]

\[ R = \frac{X \cdot r \cdot t}{A} \]

\[ R_s = \frac{\rho}{t} \]
1. $I_{Cf}(R) = \frac{1}{R} \frac{dR}{dt} = +2000 \text{ ppm} / ^{\circ}C$

   Where $R$ is created from diffused $n^+$ region

2. $I_{Cf}(V_T) = -3000 \text{ ppm} / ^{\circ}C$ for $V_T = 0.8V$

   Typically $\frac{\partial V_T}{\partial T} = -2.4 \text{ mV} / ^{\circ}C$

3. For a MOSFET $\beta' = \mu C_{ox}$, so $\beta'(T) = \beta'(0) \left( \frac{T}{T_0} \right)^{-3/2}$

   This gives $\frac{1}{\beta'} \frac{\partial \beta'}{\partial T} = -\frac{1.5}{T}$, $T$ in $^\circ K$

   Or $I_{Cf}(\beta') = -\frac{1.5}{T} \Rightarrow \frac{1.5}{300} = \frac{1}{200} = \frac{106}{200} \text{ ppm} / ^{\circ}C$

   $= 5000 \text{ ppm} / ^{\circ}C$
(1) Temperature Sensitivity

\[ T_{C_f}(I_0) = \frac{1}{I_0} \frac{\partial I_0}{\partial T} \]

\[ S_T = \lim_{\Delta T \to 0} \frac{\Delta I_0/I_0}{\Delta T/T} = \frac{T}{I_0} \frac{\partial I_0}{\partial T} = T \cdot T_{C_f}(I_0) \]

\[ T_{C_f}(I_0) \Rightarrow \text{evaluation} \]

\[ I_0 = I_{DS1} = \frac{V_{DD} - V_{DS} - V_{SS}}{R} \quad \text{for Simple Minor} \]

\[ \frac{\partial I_0}{\partial T} = \frac{\partial I_{DS1}}{\partial T} = -\frac{1}{R} \frac{\partial V_{DS}}{\partial T} + \frac{1}{R^2} V_{DS} \frac{\partial R}{\partial T} \]

\[ \therefore T_{C_f}(I_0) = \frac{1}{I_0} \left[ -\frac{1}{R} \frac{\partial V_I}{\partial T} - \frac{1}{R} \frac{2}{\partial T} \sqrt{2 I_0 R} + \frac{1}{R} \frac{\partial R}{\partial T} \right] \]

Typical Value \( T_{C_f}(I_0) = 0.17 \% /^\circ C = 1700 \text{ ppm/}^\circ C \)
Using negative feedback, Simple Current Mirror can further be improved. Two such circuits are:

1. Wilson Mirror
2. Regulated Cascode

Wilson Current Mirror:

By using P-device with proper bias, we can create stable & reference current \( I_{ds1} \). \( V_{bias} \) is normally taken from a 'Stable Band Gap reference.'

This Current Mirror has:

1. \( I_o \) more stable than Simple Case.
2. Output impedance is further improved.
Vo increases

- \( I_{DS4} \) increases
- \( I_{DS4} = I_{DS3} \) & hence increase of \( I_{DS4} \) increases \( I_{DS2} \).
- \( V_A \) decreases
- \( V_{gs4} \) decreases \( \Rightarrow I_{DS4} \).

\[ V_{SB4} = V_{gs2} = V_{gs3} = I_X \left( \frac{1}{g_{m3}} + \frac{1}{r_{03}} \right) \]

\[ V_{gs4} = -g_{m2} V_{gs2} \left( \frac{1}{r_{01} \left( r_{02} \right)} \right) - V_{gs2} \]
or \( V_{gs4} = -\left[1 + g_m (\frac{1}{\beta_m})\right] V_{gs2} \)

\[ = -\left(1 + g_m (\frac{1}{\beta_m})\right) V_{sb4} \]

\[ : \quad V_{gs4} = -\left[1 + g_m (\frac{1}{\beta_m})\right] I_x (\frac{1}{\frac{1}{\beta_m}}) \]

Further

\[ I_x = g_m v_{gs4} - g_m b v_{sb4} + \frac{V_x - V_{gs2}}{\gamma_0} \]

From (1) & (2)

\[ R_{out} = \frac{V_x}{I_x} = \gamma_0 \left[1 + g_m (\frac{1}{\beta_m}) (1 + g_m (\frac{1}{\beta_m}) + \frac{1}{\gamma_0} (\frac{1}{\beta_m}) \right] \]
Assume $M_3$ & $M_4$ identical, then

$$f_{m_3} = f_{m_4}, \quad r_{0_3} \parallel \frac{1}{f_{m_3}} = \frac{1}{f_{m_4}}$$

Further we see $r_0 = r_{0_1} = r_{0_2} = r_{0_4}$ can be assumed

$$\therefore \text{ } R_{out} = r_0 \left[ \frac{1}{1 + \frac{g_{m_3}r_0}{2}} \right] + \frac{g_{m_4}r_0}{f_{m_3}} \frac{1}{r_0} + \frac{1}{r_0 f_{m_3}}$$

$$R_{out} \leq r_0 + g_{m_2}r_0^2 = r_0 \left( 1 + g_{m_2}r_0 \right)$$

$$\therefore \text{ } R_{out} \text{ is Boosted due to cascode configuration.}$$

Next thing we wish to know $V_{\text{min}} = V_{0_{\text{min}}}$

$$\therefore \text{ } V_{0_{\text{min}}} = V_{a_{\text{3}}} + V_{D_{\text{4}}_{\text{sat}}}$$

$$= V_{a_{\text{3}}} + V_{a_{\text{4}}} - V_{T_{\text{4}}}$$. 
\[ V_{\text{omin}} = \sqrt{\frac{2I_0}{\beta_3}} + V_T + \sqrt{\frac{2I_0}{\beta_4}} \]

If \( \beta_3 = \beta_4 \), then

\[ V_{\text{omin}} = 2 \sqrt{\frac{2I_0}{\rho'(W/L)}} + V_T \]

\( \propto V_{\text{omin}} \propto \sqrt{2I_0} \)
This is a good stable source.

If $I_0 \uparrow$, $I_{DS_2} = IR_2$ also increases. Drop across $R_2$ increases ($V_A = I_0 \cdot R_2$) increases $V_{GS_1}$ of $M_1$. Hence $I_{DS_1}$ increases with increase of $I_0$.

Since $I_n$ is a good current source, hence increase in $I_{DS_1}$ causes $V_B$ to reduce. But $V_B = V_{G_2}$. Thus reduction in $V_{G_2}$ reduces $V_{GS_2}$ which in turn reduces $I_0$. Thus negative feedback leads to stability.
Clearly \( I_0 = I_{DS2} = I_{R2} \)

\[
\therefore I_0 = \frac{V_A}{R_2} = \frac{V_{ass}}{R_2}
\]

\[
\therefore I_0 = \frac{V_{OV} + V_T}{R_2}
\]

\[
I_0 = V_T + \sqrt{\frac{2 |U_{IN}|}{\beta'(W/L)}} \frac{1}{R_2}
\]

Thus \( I_0 \) is function of \( V_T \), which is relatively fixed value.
A Better version of $V_T$ reference CS, is Regulated Cascade CS.

If we have large $(W/L)$ for $M_2$, then $V_{ov} = V_{gs2} - V_T$ & $s < V_T$

$\therefore I_0 = \frac{V_T}{R_2}$

However, if we do Small Signal Analysis

$R_{out} \approx \frac{1}{g_m \gamma_0} \{1 + \frac{1}{2} g_m \gamma_0 \} \approx \frac{1}{2} g_m^2 \gamma_0$