Module 2: Transmission Lines
Lecture 6: Loss Less Transmission Line

Objectives

In this course you will learn the following

- What is a loss-less transmission line?
- Variation of voltage and current on a loss less line.
- Standing waves on a loss-less line.
- Voltage standing wave ratio (VSWR) and its relation to the voltage reflection co-efficient.
- Importance of VSWR and its values for various impedances.
- Concept of return-loss (RL). Return loss a measure of reflection on the line.
Module 2: Transmission Lines
Lecture 6: Loss Less Transmission Line

Analysis of Loss Less Transmission Line

In any electrical circuit the power loss is due to ohmic elements. A loss less transmission line therefore implies $R = 0$ and $G = 0$. For a loss less transmission line hence we get

Propagation constant:

$$\gamma = \sqrt{j\omega L - j\alpha C} = j\omega\sqrt{LC} = \text{Purely imaginary}$$

That is, $\alpha = 0$ and $\beta = \omega\sqrt{LC}$.

The characteristic impedance

$$Z_0 = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}} = \text{Purely real}$$

The reflection coefficient at any point on the line is

$$\Gamma(l) = \Gamma e^{-j2\beta l} = \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) e^{-j2\beta l}$$

The voltage and current expressions become

$$V(l) = V e^{j\beta l} \left\{1 + Z_L e^{-j2\beta l}\right\}$$
$$I(l) = V e^{j\beta l} \left\{1 - Z_L e^{-j2\beta l}\right\}$$

Let the reflection coefficient at the load end be written in the amplitude and phase form as

$$\Gamma_L = |\Gamma_L| e^{i\phi}$$

then we have

$$V(l) = V e^{j\beta l} \left\{1 + |\Gamma_L| e^{i(\phi - 2\beta l)}\right\}$$
$$I(l) = V e^{j\beta l} \left\{1 - |\Gamma_L| e^{i(\phi - 2\beta l)}\right\}$$

As we move towards the generator the phase $(\phi - 2\beta l)$ becomes more negative and point P rotates clockwise on the dotted circle. The radius of the circle is $|\Gamma_L|$. Length of the vector OP gives the magnitude of the quantity $(1 + |\Gamma_L| e^{i(\phi - 2\beta l)})$.
Spatial Variation of Current & Voltage

The previous equations indicate that the amplitudes of the voltage and current vary as a function of distance on the line.

Wherever \( \theta - 2\beta \ell = 0 \) or even multiple of \( \pi \), the quantity in the brackets is maximum \( (1 + |\Gamma_L|) \) in the voltage expression, and minimum \( (1 - |\Gamma_L|) \) in the current expression. That is wherever the voltage amplitude is maximum, the current amplitude is minimum.

Similarly wherever \( \theta - 2\beta \ell = \text{odd multiple of} \pi \), the voltage is minimum and the current is maximum.

Note

The voltage and current variation at every point on the line is \( \theta \text{ just} \) only.

The distance between two adjacent voltage maxima (or minima) or two adjacent current maxima (or minima) corresponds to

\[
2\beta \ell = 2\pi \\
\Rightarrow 2 \cdot \frac{2\pi}{\lambda} \ell = 2\pi \\
\ell = \frac{\lambda}{2}
\]

The distance between adjacent voltage and current maxima or minima corresponds to

\[
2\beta \ell = \pi \\
\Rightarrow \ell = \frac{\lambda}{4}
\]

We then say that the voltage and current are in space quadrature, i.e, when voltage is maximum the current is minimum and vice versa.
Voltage Standing Wave Ratio

- The maximum and minimum peak voltages measured on the line are

\[
|V|_{\text{max}} = |V|^+ \left(1 + |\Gamma_L|\right)
\]
\[
|V|_{\text{min}} = |V|^+ \left(1 - |\Gamma_L|\right)
\]

- Let us define a quantity called 'Voltage Standing Wave Ratio (VSWR)' as

\[
\rho = \frac{|V|_{\text{max}}}{|V|_{\text{min}}}
\]

- Substituting for \(|V|_{\text{max}}\) and \(|V|_{\text{min}}\) we get

\[
\rho = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}
\]

or \(|\Gamma_L| = \frac{\rho - 1}{\rho + 1}\)

- The VSWR is a measure of the reflection on the line. Higher the value of VSWR, higher is \(|\Gamma_L|\) i.e., higher is the reflection and is lesser the power transfer to the load.

- Since \(0 \leq |\Gamma_L| \leq 1\), we get

\[
1 \leq \rho \leq \infty
\]

VSWR of 1 corresponds to the \(|\Gamma_L| = 0\). That is the best situation.

- Ideally for a perfect match VSWR = 1. However, generally a VSWR \(\leq 2\) is considered acceptable in all experimental works.
Module 2: Transmission Lines

Lecture 6: Loss Less Transmission Line

**Return Loss & Reflection Co-efficient**

- The return loss is defined as

  \[ \text{Return loss (RL)} = -20 \log |\Gamma_L| \text{ dB} \]

- The return loss indicates the factor by which the reflected signal is down compared to the incident signal.

- For perfect match \( |\Gamma_L| = 0 \) and the return loss is \( \infty \), whereas for the worst case of \( |\Gamma_L| = 1 \) the return loss is 0 dB.

- Higher the return loss better is the match.

For acceptable value of VSWR = 2,

\[
|\Gamma_L| = \frac{\text{VSWR} - 1}{\text{VSWR} + 1} = \frac{2 - 1}{2 + 1} = \frac{1}{3}
\]

\[
\Rightarrow \text{Return Loss RL} = -20 \log \left( \frac{1}{3} \right) = 9.54
\]

The return loss should be higher than 9.54.
Recap

In this course you have learnt the following

- What is a loss-less transmission line?
- Variation of voltage and current on a loss less line.
- Standing waves on a loss-less line.
- Voltage standing wave ratio (VSWR) and its relation to the voltage reflection co-efficient.
- Importance of VSWR and its values for various impedances.
- Concept of return-loss (RL). Return loss a measure of reflection on the line.