Module 7 : Antenna

Lecture 48 : Hertz Dipole Antenna

Objectives

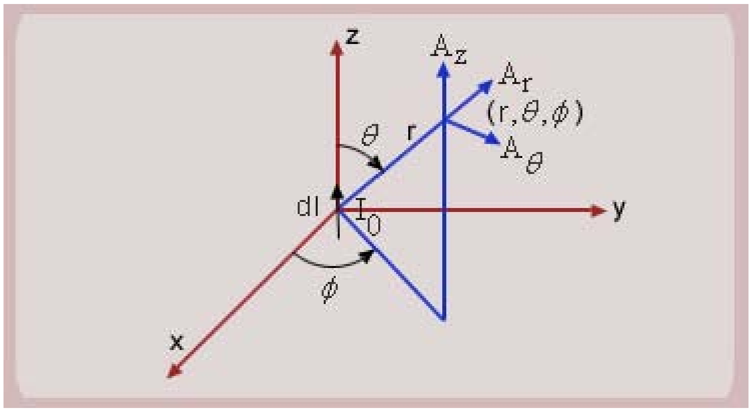
In this course you will learn the following

- Hertz Dipole
- Magnetic vector potential due to Hertz dipole
- Fields due to Hertz Dipole
- Electrostatic Induction and Radiation fields
- Near and Far-fields
- Power radiated by the Hertz Dipole
- Radiation resistance of the Hertz Dipole

Hertz Diople Antenna

- An infinitesimal element excited with an alternating current is called the **Hertz dipole**. In practice a linear antenna can be approximated by a Hertz dipole if the linear antenna is of size much smaller the wavelength.

- The analysis of Hertz dipole is important as any complicated radiating structure can be decomposed into Hertz dipoles.A Hertz dipole is shown in Fig.



- The Hertz dipole is oriented along the z-axis, has length dl and current $I_0 e^{j\omega t}$.

- Since the Hertz dipole is small the vector potential at point P is almost same as the Green's function multiplied by the volume integral of the current density.

Vector potential due to the Hertz dipole

- The vector potential due to the Hertz dipole is therefore given as

$$\mathbf{A} = A_z\hat{\mathbf{z}} = \frac{\mu}{4\pi} I_0 dl \frac{e^{-j\beta r}}{r} e^{j\omega t}\hat{\mathbf{z}}$$

- Note that the magnetic vector potential is in the z-direction and it has same direction every where in the space.

- Since the coordinate system used for the antenna analysis is spherical, the components of the magnetic vector potential in spherical coordinates are

$$A_r = A_z \cos\theta$$
$$A_\theta = -A_z \sin\theta$$
$$A_\phi = 0$$

Fields due to the Hertz Dipole

- The vector magnetic field is

$$\bar{\mathrm{H}} = \frac{1}{\mu}\frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{\mathbf{r}} & r\hat{\theta} & r\sin\theta\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ A_r & rA_\theta & r\sin A_\phi \end{vmatrix}$$

- The magnetic field components can be obtained as

$$H_r = 0$$
$$H_\theta = 0$$
$$H_\phi = -\frac{I_0 dl e^{j\omega t}}{4\pi r}\left\{\frac{\partial}{\partial r}\left(e^{-j\beta r}\sin\theta\right) + \frac{\partial}{d\theta}\left(\frac{e^{-j\beta r}\cos\theta}{r}\right)\right\}$$
$$= \frac{I_0 dl e^{j\omega t}}{4\pi r}\left\{ j\beta e^{-j\beta r}\sin\theta + \frac{e^{-j\beta r}}{r}\sin\theta\right\}$$

- The Hertz component has only ϕ-component. That is, the magnetic field loops around the z-axis.

- Substituting for H in the source-free Maxwell's curl equation the electric field can be obtained as

$$\mathbf{E} = \frac{1}{j\omega\epsilon}\nabla \times \mathbf{H}$$

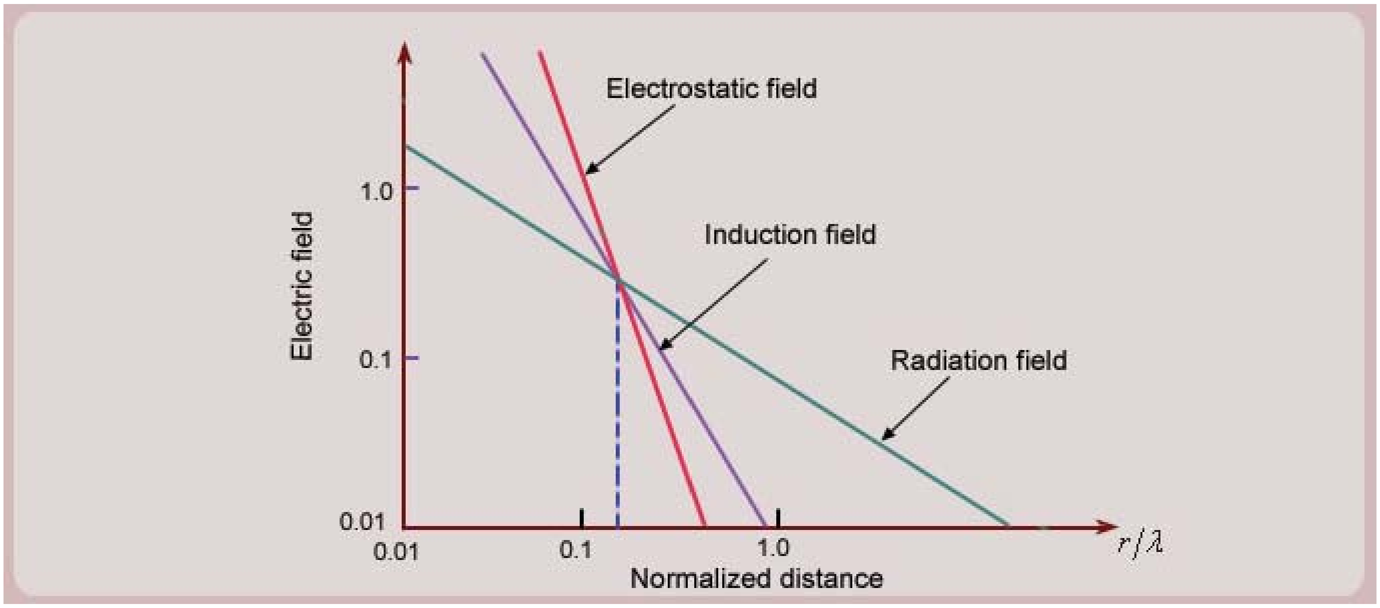
$$E_r = \frac{I_0 dl e^{j\omega t}\cos\theta e^{-j\beta r}}{4\pi\omega\epsilon}\left\{\frac{\beta}{r^2} - \frac{j}{r^3}\right\}$$

and $$E_\theta = \frac{I_0 dl \sin\theta e^{j\omega t} e^{-j\beta r}}{4\pi\in}\left\{\frac{j\beta^2}{\omega r} + \frac{\beta}{\omega r^2} - \frac{j}{\omega r^3}\right\}$$

- The ϕ-component of the electric field is zero.

Types of Fields

- For the Hertz dipole, the magnetic field has only ϕ-component and the electric field does not have the ϕ -component. The electric field lies in the (r, θ) plane.
- The fields can be divided into three categories depending upon their variation as a function of distance.
- **The field which varies as $1/r^3$, is called the electrostatic field.** This field is dominant in the close vicinity of the dipole since its amplitude decreases rapidly as function of distance.
- **The field which varies as $1/r^2$, is called the induction field .** This field extends little further than the electrostatic field but still decays rapidly as a function of distance.
- **The field which varies a $1/r$ is called the radiation field .** This is the field which extends over farthest distance from the antenna and is responsible for the radiation of power from the antenna.
- The electrostatic field is inversely proportional to the frequency. As the frequency of the current approaches zero, this field diverges to infinity. This field is essentially due to the accumulation of charges on the tip of the antenna. When the current flows in the dipole, the opposite charges get accumulated on the tips of the antenna giving a dipole. With the reversal of the current (every half cycle) dipole reverses its polarity giving an oscillating dipole. The electrostatic field is due to this oscillating dipole. As the frequency decreases, the accumulated charge for a given current increases and therefore the electrostatic field increases.
- The Induction field is independent of frequency. This field has same behavior as the magnetic field obtained from the Biot-Savart law, and hence the name given to the field.
- The radiation field is proportional to the frequency. This field is therefore practically absent at low frequencies. This field is essentially a high frequency phenomenon.
- The electrostatic and the induction fields together are called the **Near Fields**, and the radiation fields are called the **Far Fields**.
- The three field become equal in magnitude at a distance of $1/\beta = \lambda/2\pi \approx \lambda/6$ as shown in Fig. The distance within $\lambda/6$ is called the near-field zone and the distance $\gg \lambda/6$ is called the far-field zone.


Electrostatic field
1.0
Electric field
Induction field
0.1
Radiation field
0.01
0.01
0.1
1.0
r/λ
Normalized distance

Near Field

- The electric field for the dipole is given as (only $1/r^3$ terms)

$$E_r \approx \frac{-j2I_0 dl\cos\theta}{4\pi\in\omega r^3}e^{j\omega t}$$

$$E_\theta \approx \frac{-jI_0 dl\sin\theta}{4\pi\omega\in r^3}e^{j\omega t}$$

- The total near field is given as

$$|E| = \sqrt{\left|E_r^2\right| + \left|E_\phi^2\right|} = \frac{I_0 dl}{4\pi\in\omega r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$$

$$= \frac{I_0 dl}{4\pi\omega\in r^3}\sqrt{1+3\cos^2\theta}$$

- The Near field is minimum in the direction $\theta = \pi/2$ and maximum in the directions $\theta = 0, \theta = \pi$. However, along no direction the near field is zero.

- The near field essentially stores the electromagnetic energy around the dipole but does not contribute to the power flow from the antenna.

Far Field

- In the far field region only fields are the radiation fields.

- The far field components of the electric and magnetic fields are given as

$$E_\theta = \frac{jI_0 dl\,\beta^2\sin\theta e^{-j\beta r}e^{j\omega t}}{4\pi\in\omega r}$$

$$\text{and } H_\phi = \frac{jI_0 dl\sin\theta\beta e^{-j\beta r}e^{j\omega t}}{4\pi r}$$

- Important things to note about the far field (radiation field) are,

(1) The electric and magnetic fields are in time phase and they are in phase quadrature with the current. That is, these fields are proportional to the rate of change of current or acceleration of charges.

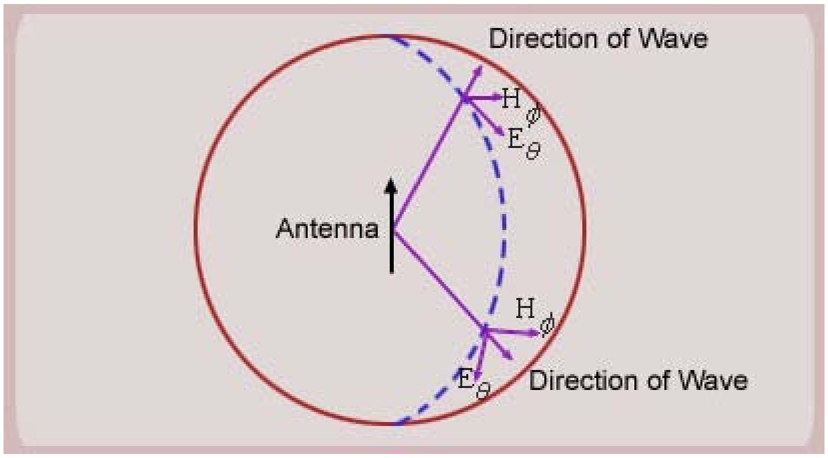
(2) The ratio of the electric and magnetic field at every point in space is equal to the intrinsic impedance of the medium.

$$\frac{E_\theta}{H_\phi} = \frac{\beta}{\omega\in} = \frac{\omega\sqrt{\mu\in}}{\omega\in} = \sqrt{\frac{\mu}{\in}}$$

$$= \eta(\text{intrinsic impedance of the medium})$$

(3) The wave travels in the r-direction, the electric field is in the θ-direction and the magnetic field is the ϕ-direction. That is, they are perpendicular to each other. These fields therefore represent transverse electromagnetic wave albeit spherical in

nature. (See Fig.)



(4) The fields are not uniform in all directions. The field strength is maximum along $\theta = \pi / 2$, and zero along $\theta = 0, \theta = \pi$.
The Hertz dipole hence does not have any radiation along its axis.

- Power Radiated by the Hertz dipole
- The power flow density in the space can be obtained by the Poynting vector.
- The average Poynting vector is given by

$$j60I_m \frac{e^{-j\beta R}}{R} F(\theta)$$

- The filed which contribute to the power flow are essentially the radiation fields. The average Poynting vector therefore is

$$P_{av} = \frac{1}{2}\left(\frac{I_0 dl \sin\theta}{4\pi r}\right)^2 \frac{\beta^3}{\omega \in}\hat{\mathbf{r}}$$

- The total power radiated by the antenna can be calculated by integrating the Poynting vector over a sphere of any radius enclosing the antenna. The total power radiated by the antenna is

$$W = \iint P_{av} r^2 \sin\theta d\theta d\phi = \int_{\phi=0}^{2\pi} \int_{\phi=0}^{2\pi} \frac{1}{2}\left(\frac{I_0 dl \sin\theta}{4\pi r}\right)^2 \frac{\beta^3}{\omega \in} r^2 \sin\theta d\theta d\phi$$

$$W = \pi \frac{I_0^2 dl^2}{16\pi^2} \frac{\beta^3}{\omega \in} \cdot \frac{4}{3}$$

- After substituting for β and doing some manipulations, and noting that the intrinsic impedance of the medium = $\sqrt{\mu_0 / \varepsilon_0} = 120\pi$, we get the total radiated power as

$$W = 40\pi^2 I_0^2 \left(\frac{dl}{\lambda}\right)^2$$

- The total radiated power of the Hertz dipole is proportional to the square of the normalized length (normalized with respect to the wavelength) of the dipole.

- Longer the length more will be the radiated power for a given excitation current. Note however, that for increasing the radiated power, the length can not be increased arbitrarily. For the Hertz dipole we should have $dl \ll \lambda$.

Radiation Resistance

- If the Hertz dipole is seen from its terminal, it appears like a resistance which consumes power. This resistance is directly related to the power radiated by the dipole.

- A hypothetical resistance which will absorb same power as that radiated by the Hertz dipole when excited with the same peak current I_0, is called the **Radiation Resistance** of the antenna.

- The radiation resistance of the Hertz dipole is

$$R_{rad} = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2$$

- For a Hertz dipole of length 0.1λ, the radiation resistance is about 8 ohms.

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Recap

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