Module 4 : Uniform Plane Wave

Lecture 26 : Polarization of a transverse Electro Magnetic Wave

Objectives

In this course you will learn the following

- Definition of Wave Polarization.

- Ellipse of Polarization for time harmonic fields.

- Different types of Polarization.

- Conditions for different types of Polarization.
Wave Polarization

The wave polarization is defined by the time behaviour of the electric field of a TEM wave at a given point in space. In other words, the state of polarization of a wave is described by the geometrical shape which the tip of the electric field vector draws as a function of time at a given point in space. Polarization is a fundamental characteristic of a wave, and every wave has a definite state of polarization.

Let us consider two waves with their electric fields oriented in $x$ and $y$ directions respectively

\[ E_x = E_1 = \text{Re} \left[ E_{x0} e^{i(\omega t - \beta z)} \right] = E_{x0} \cos(\omega t - \beta z) \]
\[ E_y = E_2 = \text{Re} \left[ E_{y0} e^{i(\omega t + \phi - \beta z)} \right] = E_{y0} \cos(\omega t - \beta z + \phi) \]

The two waves have same frequency but the $y$-oriented field has phase $\phi$ with respect to the $x$-oriented field.

Without losing generality let us take $z = 0$, giving

\[ E_x = E_{x0} \cos \omega t \]
\[ E_y = E_{y0} \cos (\omega t + \phi) \]

Eliminating $\omega t$, we get the equation of the locus of the tip of the electric field vector traced over time as

\[ \frac{E_x^2}{E_{x0}^2} - \frac{2E_x E_y \cos \phi}{E_{x0} E_{y0}} - \frac{E_y^2}{E_{y0}^2} = \sin^2 \phi \]

This is the equation of an ellipse

The wave therefore is called to have 'Elliptical Polarization'
Wave Polarization

Case I: Linear Polarization ($\phi = 0$)

The two components $E_x$ and $E_y$ may or may not have the same amplitude but let us assume that the phase difference between them is zero. The equation of ellipse then reduces to

$$\left(\frac{E_x}{E_{x0}} - \frac{E_y}{E_{y0}}\right)^2 = 0$$

$$\Rightarrow E_y = \left(\frac{E_{y0}}{E_{x0}}\right)E_x$$

This is the equation of a straight line with slope ($\frac{E_{y0}}{E_{x0}}$). The tip of electric field vector therefore draws a straight line if $\phi = 0$, irrespective of the amplitudes of the two field components. This polarization is hence called the 'Linear Polarization' or the wave is said to be linearly polarized.

Depending upon the ratio of $E_{y0}$ and $E_{x0}$, the slope of the line changes.
Circular Polarization

Case II: Circular Polarization \( \phi = \pm \pi/2 \) and \( E_x = E_y = E_0 \).

\[
E_x^2 + E_y^2 = E_0^2
\]

- The tip of the electric field vector therefore draws a circle and the wave is said to be circularly polarized.
- If \( \phi = +\pi/2 \) then the vector rotates anti-clockwise and if \( \phi = -\pi/2 \) the vector rotates clockwise as shown in figure. The sense of rotation is governed by the phase \( \phi \).

Note:
- If \( E_y \) leads \( E_x \), the rotation is anticlockwise and if \( E_y \) lags \( E_x \), the rotation is clockwise.
- First, we orient ourselves in the direction of wave (wave going away). Then if the \( E \) vector rotates to our left hand (anti-clockwise) it is called 'LEFT HANDED ROTATION' and if the \( E \) vector rotates to our right hand (clockwise) it is called the 'RIGHT HANDED ROTATION'.

The circular polarization can be divided into Left-circular and Right-circular polarization.
Elliptical Polarization

Case III: Elliptical Polarization

• Here, the two field components $E_x$ and $E_y$ neither have the same amplitude nor they have the phase difference of zero or $\pm \pi / 2$ (i.e., $\phi \neq 0$ or $\pm \pi / 2$ and $E_x0 \neq E_y0$). We then obviously get general elliptically polarized wave.

• In this case again for positive $\phi$ the sense of rotation is Left handed and for negative $\phi$, the sense of rotation is Right handed (see figs.)

• The elliptical polarization is divided into two, the Right handed Elliptical (RHE) and the Left handed Elliptical (LHE).
Generation of arbitrary state of polarization using two linear polarization
Recap

In this course you have learnt the following

- Definition of Wave Polarization.
- Ellipse of Polarization for time harmonic fields.
- Different types of Polarization.
- Conditions for different types of Polarization.