Lecture: 4

Modal Propagation of Light in an Optical Fiber
By now, the reader may be well aware of the fact that any ray of light which is launched within the Numerical Aperture Cone (Acceptance cone) of an optical fiber propagates through the optical fiber core if and only if its launching angle is such that the angle of refraction ($\theta$) of its refracted ray, in the core, satisfies the following phase condition given by equation (3.12). It should be also obvious that, whenever we talk of a single launched ray, there exist an annular ring of similarly launched rays making the same launching angle with the fiber axis. This situation will be further obvious from the figure 4.1 below.

![Figure 4.1: Annular rings of different modes.](image)

The number of different values of ‘$m$’ signifies the number of different possible launching angles which can successfully propagate in the optical fiber core. The ray that is launched along the axis of the fiber propagates without any phase condition requirement to be satisfied and corresponds to the first mode of propagation, also called as the zero order mode of propagation. This is shown by $m=0$ in the figure above and few other modes are shown by their respective annular rings represented by different colours. There may be $N$ possible modes of propagation for which the rays successfully travel along the fiber creating unique light intensity patterns around the axis of the core. However, one should carefully note here that, the number allowable values of ‘$m$’ depend on the acceptance angle of the optical fiber. This is because although there are infinite integral values of ‘$m$’ (according to equation 3.12) only those modes would propagate along the fiber whose launching angles lie within the N.A. cone of the fiber. Any ray that is launched outside this cone does not propagate along the fiber although it might correspond to a particular mode. This is shown in the figure above by the ray AO. This ray simply refracts out of the core because its angle of incidence at the core-cladding interface is smaller than the critical angle of the core with respect to the cladding. Thus the N.A. cone can no longer be assumed as a solid cone of rays, but has to be viewed as composed of annular rings of rays which correspond to particular modes of propagation that satisfy basic phase conditions. In other words, the optical fiber too is selective in accepting only those rays which satisfy the basic phase conditions and the other...
rays are rejected by the fiber although they may lie within the acceptance cone of the fiber. Thus there are only a finite number of modes that are allowed in an optical fiber and the other modes are rejected. This leads to a further decrease in the light accepting efficiency of fiber.

Let us reconsider the propagation of the rays in the optical fiber in relevance to wave theory of light. Treating light a transverse electromagnetic wave, we find that when meridional rays propagate along the fiber, their electric and magnetic fields of all the rays superimpose to result in electric and magnetic field distribution which may be either transverse electric (TE\(_x\)) or transverse magnetic (TM\(_x\)) in nature. The subscript ‘x’ denotes the definite number of maxima and minima in the resultant light intensity pattern. The propagation of skew rays, on the other hand, results in a particularly special form of modes which are neither TE nor TM in nature and are called as Hybrid modes. When we refer to modal propagation in dielectric waveguides, we find that unlike metallic waveguides, there is a special set of modes that exists in a dielectric waveguide in addition to TE and TM modes. This set of modes is called as hybrid mode. The optical fiber is actually a cylindrical dielectric waveguide and so it can exhibit hybrid modes as well. Rigorous analysis shows that hybrid mode is in fact the lowest order mode that can propagate in an optical fiber. Since hybrid mode is the lowest order mode, it can be analytically shown that the mode of the ray that propagates in the fiber along the axis is hybrid in nature.

Let us now have a glimpse of the different types of modes that propagate inside an optical fiber which may be TE\(_x\), TM\(_x\), or hybrid in nature. Figure 4.2 below shows different intensity patterns created by superposition of the wave-fronts of all the light rays for Transverse Electric modes that propagate in an optical fiber.

![Figure 4.2: Different TE modes in an optical fiber](image)

The fields that are shown in the cladding region are actually the evanescent fields that exist in the cladding owing to the boundary condition requirement at the core-cladding interface. For very low launching angles with respect to the axis of the fiber, the intensity pattern created is the one which is shown by TE\(_0\) in the above figure. There exists a maximum intensity region around the axis of the core and as we move towards the periphery of the core the fields start to decay. These fields
eventually decay down to negligibly low value in the cladding as shown in the figure. If the launching angle is increased further, we get the intensity patterns as that shown for TE1 and TE2 in the above figure. The subscript of TE in fact indicates the number of destructive interferences in the pattern where the field intensity crosses the zero level, or in other words, creates an optically dark area. So, for TE0 we have no dark area, for TE1 we have one dark area at the axis, for TE2 we have two dark areas and so on. This situation is well obvious from the above figure which shows the number of times the field intensity pattern crosses the zero level corresponding to the subscript of TE. This subscript is also termed as the index of the mode. As we further increase the launching angle with respect to the axis, more zeros are crossed and we get the higher indices of the mode. The above discussion is also true for TM mode as well and hence these modes are not referred to separately.

Let us now go back to our discussion about launching light into an optical fiber. The ray-model of light showed us that launching angle of the light ray must be smaller than the acceptance angle of the optical fiber core. But the consideration of the wave-fronts showed us that this condition of the launching angle is not enough to ensure a successful propagation of light in an optical fiber. The launching angle must be such that the angle of refraction of the launched ray into the fiber must satisfy the phase condition of equation 3.12 for sustained propagation inside the optical fiber core. Let us rewrite the equation 3.12 with the terms having their usual meanings. (The reader is assumed to be familiar with the equation 3.12)

\[ \frac{2 \pi n_1 d \sin \theta}{\lambda} + \delta = \pi m \quad (m=0,1,2,3,\ldots) \]

The different discrete values of the angle \( \theta \) indirectly signify the different allowable launching angles of the light rays into the optical fiber. If we substitute the first value of \( m \) (i.e. \( m=0 \)) in the above equation we get \( \theta=0^\circ \). This refers to the ray that propagates along the axis. This ray will inevitably propagate inside the fiber because it does not require any phase condition to be satisfied. Let us now substitute the next integral value of \( m \) to obtain the first order mode as in figure 4.1.

We get:

\[ \theta_1 = \sin^{-1} \left( \frac{\lambda(\pi-\delta)}{2\pi dn_1} \right) \quad (4.1) \]

This value of \( \theta_1 \) signifies the first annular ring of rays that propagates inside the fiber. Similarly we may obtain the other modes that propagate in the fiber by subsequent substitution of the corresponding values of \( m \) until the condition \( \theta \leq \alpha \) is reached, where \( \alpha \) is the N.A. of the fiber core.

When a pulse of light is aligned onto the tip of the optical fiber core, the light energy in the pulse divides into numerous rays which become incident on the tip of the optical fiber core. But only those rays propagate which satisfy both the requirements for a successful propagation of light in the core. Yet there numerous
rays that enter the optical fiber core at all the allowed launching angles. This causes different rays to travel by different paths which indeed lead to pulse broadening of light in the core. (For Pulse-Broadening, the reader is advised to go through the transcript of Lecture-3). Pulse broadening is also referred to as dispersion and is greatly an undesirable phenomenon because it reduces the bandwidth of the fiber. Thus a basic and obvious question that comes to the mind is that, how can pulse broadening be reduced? The answer to this question lies in the very cause which is responsible for the effect. The pulse broadening is caused by the time delay in between the axially launched ray and the ray corresponding to the largest order mode possible in the optical fiber because it is the largest order mode that travels the longest path inside the fiber. Let us now ask a very conceptual question that, what if we do not allow any mode to get launched into the fiber except the axial ray? This would ideally lead to a zero pulse broadening. But how is this possible? The answer to this question lies in the equation (4.1). One very interesting situation to note in equation (4.1) is that, though the propagation of the ray along the axis is inevitable, the propagation of next mode and the subsequent modes depends on many parameters. These parameters are quite obvious in the RHS of equation (4.1). If by any means of variation in these parameters we could make θ>α, the ray corresponding to this θ will not get launched into the fiber. The different parameters that we can possibly vary are the refractive index (n₁) of core and the diameter of the core (d) for a given wavelength of light (λ). But the refractive index n₁ cannot be varied because we have already chosen the material glass for the core which has a fixed refractive index of about 1.5. This leaves us with only one option and that is to vary the diameter of the core. If we reduce the diameter of the core to a very low value such that θ₁ exceeds the numerical aperture of the fiber core, then the rays corresponding to this θ cannot be launched into the fiber. Thus only the axial ray would be launched and any higher mode would not be launched into the fiber thereby reducing the pulse broadening effect to a negligibly low value. These types of fibers which allow only a single mode of light to propagate inside them are called as Single Mode Optical Fibers (SMOF). And the optical fibers which allow the propagation of multiple modes are called as Multimode Optical Fibers (MMOF). Thus it is obvious that SMOF have very low pulse broadening in comparison to MMOF and thus have higher bandwidths. But MMOF have higher N.A. than SMOF. SMOF and MMOF are also called as step-index type optical fibers because the transition from cladding refractive index n₂ to core refractive index n₁ or vice versa is in the form of a step function.

The above discussion suggests that by reducing the diameter of the core of the optical fiber, the pulse broadening can be decreased and thus its bandwidth can be increased. That is why almost for all practical data communication purposes single mode optical fibers are used. One significant observation to note here is that though SMOF have high bandwidths, they have very low N.A. values, which makes it very difficult to launch light into a single mode optical fiber. First of all, the source of light has to have a highly directional beam and secondly, the fiber core has to be
carefully aligned to the source. The slightest disturbance to this arrangement would prevent any available light to enter the fiber even with a highly directional optical source. Hence LASER like sources are used in case of single mode optical fibers. LASERs have highly directional beams which are apt for SMOF. The only trouble is now to align the fiber to the LASER source and prevent any external disturbance to the arrangement. On the other hand, MMOF, on account of their high N.A., accept large percentage of the incident light. Even LEDs could serve as a source in case of MMOF because they do not require highly directional sources.

The prominent thing to note here is that, single mode optical fibers attain their high bandwidth at cost of light gathering efficiency. The obvious question that may come to the reader’s mind is that, is it possible to make a multimode fiber to have both high N.A. and low pulse broadening (or high bandwidth)? The answer to this query again can be derived from the very cause of the pulse broadening effect. All the rays of light for a given wavelength propagate with the same velocity inside the core of the optical fiber. This causes different rays to take different time intervals to propagate a particular length of the fiber because they travel along different paths. The axially launched ray thus takes the lowest time to travel and the rays corresponding to the largest allowed mode take the largest amount of time because they travel the longest distance inside the fiber and suffer the most number of total internal reflections. This difference in the time intervals is in fact the pulse broadening $\Delta T$. If by some means all the rays could be made to travel with different velocities so that they all take the same time to travel a given length of the optical fiber, we could achieve our goal of having a multimode optical fiber with high N.A. and low pulse broadening. This means that we have to make the rays which travel the longest distance travel with the fastest velocity and the other rays to travel with correspondingly lower velocities with the axial ray having the lowest velocity. To achieve this we can refer back to the basic definition of refractive index of a material which says:

$$\text{Refractive Index of a medium}(\mu) = \frac{\text{Velocity of light in Vacuum} (c)}{\text{Velocity of light in the medium}(v)}$$

The above definition signifies that light travels faster in materials with lower refractive index. That is, if we make the axial ray to travel through a region of highest refractive index so that it travels with the lowest velocity and make the other rays to travel through regions of decreasing refractive indices whose refractive indices decrease in the same proportion as the increase in their distance of travel, then all the rays would travel with almost equal velocity along the axis and thus would take the same time to travel a given length of a fiber. We actually are suggesting of creating some sort of refractive index gradient that is symmetrical around the axis such that the refractive index is maximum at the axis and it gradually decreases as we move towards the periphery of the core and again constant in the cladding. This type of index grading is shown in figure 4.3 below. The way in which the launched rays would travel in such a fiber is also shown in the figure.
The maximum refractive index of the core is at the axis of the optical fiber and it decreases gradually towards the periphery of the core and then in the cladding it is constant at $n_2$. These types of fibers are called Graded Index Optical Fibers (GIOF). The axial ray travels through a region of highest refractive index compared to the rest of the core and hence travels with the lowest velocity. The velocities of the rays increase as their lateral displacement from the axis increase because they encounter regions of lower refractive index. This causes them to travel together without any delay between themselves and thus reduce the pulse broadening to a considerably low value. The perfect profiling of the refractive index has not yet been possible practically. Hence the delay between the rays is never practically zero though it may be very small. So, GIOFs are not as better in bandwidth as SMOF but do have higher N.A. than SMOFs. This is why, where light gathering is more a concern over bandwidth, GIOFs becomes the appropriate choice. GIOFs are obviously better than MMOFs in terms of bandwidth. Let us now have a comparative glimpse into the three types of fiber in quantitative terms as given in the table 4.1 below.

<table>
<thead>
<tr>
<th>Index Profile and Cross-sectional View of different types of fibers</th>
<th>Core Diameter (2a) (µm)</th>
<th>Cladding Diameter (d) (µm)</th>
<th>Bandwidth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Index Profile $n_1 &lt; n_2$ Fiber Cross-section and Ray Paths</td>
<td>8-12</td>
<td>125</td>
<td>High</td>
</tr>
</tbody>
</table>

Figure 4.3: Graded Index Optical Fiber
Table 4.1: Comparative Study of the three types of Optical Fibers

From the above table it can be very well concluded that single mode fibers are the best choice when distance of communication is very large and also the bandwidth requirement is the primary concern (for example in long distance high-speed communications like WAN etc.). It has the best dispersion performance out of the three and hence has the highest bandwidth out of the three. Next to single mode fibers is the multimode graded index optical fiber which has N.A. higher than single mode fiber but its dispersion performance is about 10 times poorer than that of a single mode fiber. Applications where the distance of communication is short and the designer does not want to sacrifice much on the light gathering efficiency, this type of optical fibers appropriately serve the purpose (for example in local area communications like LANs, Intranet etc.). Multimode step index fibers are left with only academic importance and for use in laboratory demonstrations because though they have high N.A., their dispersion performance too poor to be of any use in communication. They may be used in optical sensors for their high N.A., but have very limited range of applications.

Thus the simplest model of light, “The Ray-Model” helped us to understand the propagation of light inside the optical fiber core in a detailed manner beginning from the very basics of refraction to the modal propagation of light in the optical fiber. The reader may now have a basic question rising in his mind and that is whether to stop the discussion on propagation of light here because apparently all concepts of propagation of light in an optical fiber seem to have been made clear by the Ray-Model. The answer to this query is a straight no. The reason behind this negativity is due to the fact that though the Ray-model has been successful in explaining many issues about light propagation in an optical fiber, but they are not all. There are other
underlying concepts that are equally important as the ones already discussed above and those which the Ray-model fails to explain. One such concept is the dependence of the propagation characteristics of light on the wavelength of the launched light. If we look into this through the ray model, it suggests that all wavelengths travel with the same characteristics. But it is not so. To understand the relationship between the propagation characteristics and the wavelength of the launched light and also a host of other related phenomena, we have to refer to the next higher model of light, “The Wave-Model”, which shall be discussed in the subsequent section. In this model, light is treated as an electromagnetic wave. Hence, for these discussions, the reader will be assumed to have been familiar with electromagnetic wave theory and the propagation of electromagnetic waves in a bound medium.