Module 2: Nonlinear Frequency Mixing

Problems

1. Potassium Dihydrogen Phosphate (KDP) is a commonly used crystal for second harmonic generation of a laser with wavelength 1µm. At what angle should this laser propagate so that the phase velocity of the second harmonic matches that of the fundamental wave. Refractive indices of KDP are given below.

Sellmeier Equations of KDP: (\( \lambda \) is vacuum wavelength in µm)
\[
n_o^2 = 2.259276 + 0.01008956/(\lambda^2 - 0.012942625) + 13.005522 \lambda^2/(\lambda^2 - 400)
\]
\[
n_e^2 = 2.132668 + 0.008637494/(\lambda^2 - 0.012281043) + 3.2279924 \lambda^2/(\lambda^2 - 400)
\]

2. Beta Barium Borate (BBO) and LiNbO\(_3\) (LN) are two well known nonlinear optical crystals. Find out their refractive indices and other parameters from literature to answer the following questions:
   a. For SHG of a Nd:YAG laser are they phase matchable using birefringence phase matching.
   b. Find the value of \( \chi^{(2)} \) for the two cases.
   c. Which of these would you prefer and why?
   d. Compare the two crystals for quasi phase matching case.

3. Consider a uniaxial crystal cut with its optic axis normal to the entrance plane. If the angle of incidence is \( \theta_i \) and the two refractive indices are \( n_0 \) and \( n_e \), show that the angle of refraction for the \( e \)-wave is given by

\[
\tan \theta_r = \frac{n_e}{n_0} \frac{\sin \theta_i}{\sqrt{n_e^2 - \sin^2 \theta_i}}
\]

4. State with reason whether the following statements are true or false.
   a. A monochromatic plane wave with \( \vec{E} = E_0(1,0,0) \) is incident on a uniaxial crystal at normal incidence. The propagation vector is along the \( z \) axis and the optic axis of the crystal is oriented along the \( y \) axis. This wave will split into two nearly equal components on entering the crystal.
   b. In a material with normal dispersion the frequency conversion process generating frequency \( 2\omega_a - \omega_b \) from two monochromatic plane waves with frequencies \( \omega_a \) and \( \omega_b = 0.9\omega_a \) can be phase matched in a non-collinear configuration.
   c. The transmission of a Fabry Perot resonator is 1 when the round trip optical path is an integral multiple of wavelength.
   d. Thermal changes due to laser absorption leads to substantial second harmonic generation in an inversion symmetric medium.

5. Two lasers with frequencies \( \omega_1 = 2 \times 10^{15} \text{ rad/s} \) and \( \omega_2 = 1.9 \times 10^{15} \text{ rad/s} \) are used as inputs in a
difference frequency generation experiment in a uniaxial crystal with phase matching angle of 50°. The initial powers of the two beams are $P_1 = 1\text{MW}$ and $P_2 = 10\text{ MW}$, respectively.

a. Estimate the maximum power that can be generated at the difference frequency using a single crystal.
b. If the maximum conversion occurs in a crystal of length $l$ what would be the conversion in a crystal of length $2l$?
c. If more crystals of the same type are available can you think of a way to increase the conversion efficiency substantially?

6. (a) Write the phase matching condition in a degenerate four wave mixing process.

(b) Write the corresponding susceptibility tensor.

(c) What is the maximum reflectivity of a Phase Conjugate Mirror formed by a nonlinear medium pumped by two oppositely directed high power lasers.

7. The nonlinear optical crystal $\text{Na}_2\text{BaNb}_5\text{O}_{15}$ (BANANA) has the symmetry 2mm with the following symmetry elements.

\[ xyz \rightarrow x\bar{y}z, x\bar{z}y, \bar{x}yz, \bar{x}\bar{y}z \]

Show that
a. the crystal is biaxial.
b. the second order susceptibility tensor $\chi^{(2)}$ has only the following nonzero elements.
c. which of these components should be equal for second harmonic generation.

8. Two lasers with frequencies $\omega_1 = 4 \times 10^{15} \text{ rad/s}$ and $\omega_2 = 110^{15} \text{ rad/s}$ are used as inputs in a frequency mixing experiment. The initial powers of the two beams are $P_1 = 2\text{MW}$ and $P_2 = 1\text{ MW}$, respectively. Estimate the maximum power that can be generated at the sum and difference frequencies and at the second harmonic of the $\omega_1$ beam.

9. In a phase matched second harmonic generation experiment 1% of the incident laser power is converted to the second harmonic. What fraction of power will be converted if the crystal length is

a. doubled
b. increased 8 times

c. incident intensity is increased 10 times without changing the crystal length.

10. Consider a CO$_2$ laser beam propagating along the direction $(\theta, \phi)$ in a crystal with zinc-blende structure. Find an expression for $\chi^{(2)}(\theta, \phi)$.

11. Consider a DFG process in LiNbO$_3$. Input waves are the second harmonic (532nm) of a Nd:YAG laser and a tunable Ti:sapphire laser 700-900nm.

Find the variation of phase-matching angle as a function of the wavelength of the Ti:sapphire laser. The refractive indices of are given by the following Sellmeier equations:
12. Assume that the above crystal is oriented for phase-matching SHG of Nd:YAG laser at 24.5°C. If this crystal is used in an OPO, what will be the output frequencies when the temperature is 40°C?

\[ n_0^2 = 4.9048 + \frac{0.11775 + 2.2314 \times 10^{-8} F}{\lambda^2 - (0.21802 - 2.9671 \times 10^{-8} F)^2} + 2.1429 \times 10^{-8} F - 0.027153 \lambda^2 \]

\[ n_e^2 = 4.5820 + \frac{0.09921 + 5.2716 \times 10^{-8} F}{\lambda^2 - (0.21090 - 4.9143 \times 10^{-8} F)^2} + 2.2971 \times 10^{-7} F - 0.021940 \lambda^2 \]

\[ F = (T - 24.5)(T + 24.5 + 546) \]

\( \lambda \) in \( \mu \text{m}; T \) in \( ^\circ\text{C} \)