Electromagnetic Induction

Lecture 29: Electromagnetic Theory

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Mutual Inductance

In the last lecture, we enunciated the Faraday's law according to which if there is a changing magnetic flux in a circuit, there is an emf induced. Incidentally, the circuit that we are talking about could be an imaginary circuit in space as well.

Consider two circuits, we call them loop 1 and loop 2. Suppose the current in loop 1 is changing with time. This means that the magnetic field produced by the current in loop 1 is also time varying. If loop 2 intercepts the flux of this changing magnetic field, accrding to Faraday's law an emf would be generated in this circuit.

We can express the magnetic field due to the current I in loop 1, using the Biot Savart's law,

$$\vec{B}_1 = \frac{\mu_0}{4\pi} I_1 \oint \frac{d\vec{l}_1 \times \vec{r}}{r^3}$$

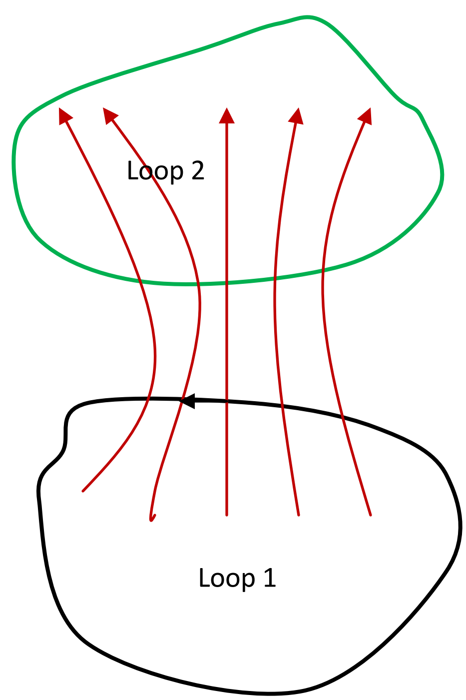
where $\vec{r}$ is the position vector of the point of observation (i.e. the point where the field is calculated) with respect to the current element $I_1 d\vec{l}_1$. This gives rise to flux in loop 2 which can be written as

$\Phi_2 = \int \vec{B}_1 \cdot d\vec{S}_2$

It can be seen that the flux in the second loop is proprtional to the current in the first loop. We can therefore wite,

$$\Phi_2 = M_{21} I_1$$

The proportinaliity constant M_{21} is known as the mutual inductance of the two circuits and it depends on geomery of the circuits and their relative positions and orientations.



The emf in the second circuit due to a change in the current in the first circuit is

$$\mathcal{E}_2 = -\frac{d\Phi_2}{dt} = -M_{21}\frac{dI_1}{dt}$$

We can show that the mutual inductance is symmetric, i.e., $M_{21} = M_{12}$. To show this, let us reexpress the flux in the second loop in terms of the vector potential of the first loop,

$$\Phi_2 = \int \vec{B}_1 \cdot d\vec{S}_2 = \int \left(\nabla \times \overrightarrow{A_1}\right) \cdot d\vec{S}_2 = \oint \vec{A}_1 \cdot d\vec{l}_2$$

We have seen that the vector potential of loop 1 can be expressed as

$$\vec{A}_1 = \frac{\mu_0 I_1}{4\pi}\oint \frac{d\vec{l}_1}{r}$$

where r is the distance from the length element $d\vec{l}_1$ to the point where the vector potential is calculated. If we express with respect to a fixed origin, then the vector potential at a position $\vec{r}_2$ on the loop 2 is given by

$$\vec{A}_1(\vec{r}_2) = \frac{\mu_0 I_1}{4\pi}\oint \frac{d\vec{l}_1}{|\vec{r}_1 - \vec{r}_2|}$$

Thus the expression from the flux through the second loop is

$$\Phi_2 = \oint \vec{A}_1 \cdot d\vec{l}_2 = \frac{\mu_0 I_1}{4\pi}\oint\oint \frac{d\vec{l}_1 \cdot d\vec{l}_2}{|\vec{r}_1 - \vec{r}_2|}$$

This shows that

$$M_{21} = \frac{\mu_0}{4\pi}\oint\oint \frac{d\vec{l}_1 \cdot d\vec{l}_2}{|\vec{r}_1 - \vec{r}_2|}$$

Which is manifestly symmetric in the indices 1 and 2, The above relation is known as Neumann's formula.

Self Inductance

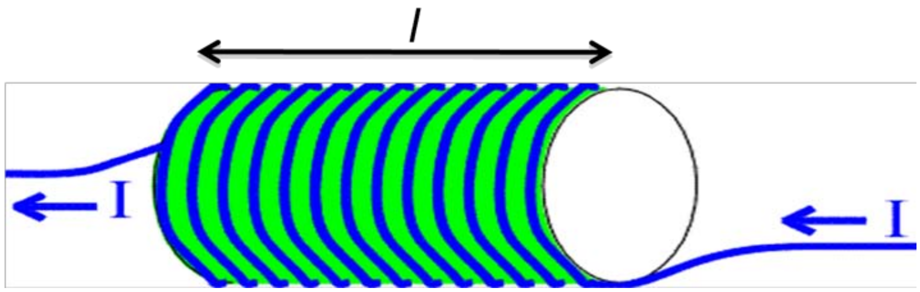
We come to an interesting consequence of the above. Suppose, instead of two circuits, I have just a single loop and we change current in that loop. This change in current changes the magnetic field associated with the current and the flux changes. The given loop uitself will intercept the flux and the changing flux would result in an emf in the circuit itself, the current induced opposing the changing current. The loop does not care about what caused the changing of flux and an emf develops in the loop as per Faraday's law. We are talking about a self effect and the emf is known as the "**back emf**".

The emf is given by

$$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{\partial\Phi}{\partial I}\frac{dI}{dt} = -L\,\frac{dI}{dt}$$

where L is known as the self inductance of the loop.

Example : Self inductance of a solenoid



Consider a solenoid of length L. Neglecting edge effects, the field of a solenoid is confined to inside of the solenoid and is directed along the axis (z direction) ,

$$\vec{B} = \mu_0 n I\,\hat{k}$$

where n is the number of turns per unit length of the solenoid. This field threads each turn of the loop and the flux "linked" with each turn is given by multiplying the above with the area of the turn,

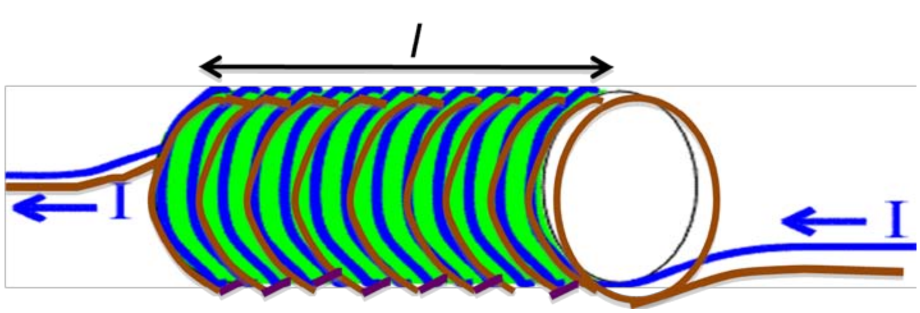
$$\Phi = \pi r^2 \mu_0 n I$$

If l is the length of the solenoid, there are nlnumber of turns in the solenoid, and the total flux linked is $N\Phi = \pi r^2 \mu_0 n^2 I\, l$. The self inductance is thus given by

$$L = N\frac{\partial\Phi}{\partial I} = \pi r^2 \mu_0 n^2 l$$

Example : Mutual Inductance of two tightly wound solenoids

Consider two solenoids which fit snugly with each other so that all the magnetic field produced by any of the solenoids is intercepted by the other.



The flux linked with the second loop when the current I flows through the first loop,

$$\Phi_2 = (\mu_0 n_1 I)\pi R^2 n_2 l$$

This gives the mutual inductance to be

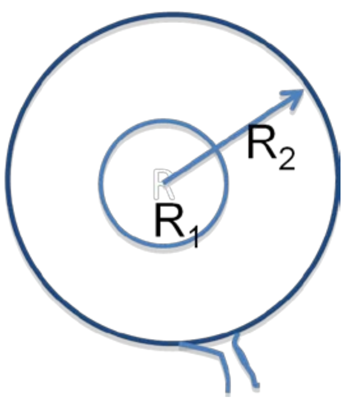
$$M_{21} = \frac{\Phi_2}{I} = \mu_0 n_1 n_2 \pi R^2 l$$

From the previous example, we can obtain the self inductances of each of the solenoids,

$$L_1 = \pi R^2 \mu_0 n_1^2 l$$
$$L_2 = \pi R^2 \mu_0 n_2^2 l$$

One can see that we get, in this case, $M_{12} = \sqrt{L_1 L_2}$. This is not a general expression but in general there is such a relationship valid with a proportionality constant known as the "coefficient of coupling",$M_{12} = \kappa\sqrt{L_1 L_2}$, the coefficient of coupling depends on the relative orientation of the two loops.

Example : Two coplanar and concentric loops



Let us assume $R_1 \ll R_2$ (figure not to scale). The field at the centre of the bigger loop is $\vec{B}_2 = \frac{\mu_0 I_2}{2R_2}\hat{k}$. We can suume that this is the field all over the smaller loop and the flux through the smaller loop is $\Phi_1 = B_2 \pi R_1^2 = \frac{\mu_0 I_2}{2R_2}\pi R_1^2$. The mutual inductance in this case is $M_{12} = \frac{\Phi_1}{I_2} = \frac{\mu_0 \pi R_1^2}{2R_2}$.

Energy of a current distribution

Recall the way we calculated the energy of a charge distribution. We assumed that all the charges were first at infinity so that there was no electric field in space. We then moved the first charge to its position without any energy cost. The second charge was then moved to its position in the field created by the first charge and so on. We cannot remove all the current distribution to infinity and so we must adopt a new apprach for calculation of energy in this case.

When we establish a current in a circuit, we have seen that a back emf develops because of changing current. Work has to be done to compensate this and establish the value of the steady cUrrent that we wish to develop .

If there is more than one circuit, when current change occurs in any of the circuit, it causes an emf in all others. To maintain the current distribution, work has to be done as well.

We have seen how to write the flux through a given loop when there is a current in that loop or in another loop. Generalizing this to a large number of loops, we can write the flux through the i-th loop is

$$\Phi_i = L_i I_i + \sum_{j\neq i} M_{ij} I_j$$

where L_i is the self inductance of the i-th loop and M_{ij} is the mutual inductance between the ith and the j-th loops. The emf through the i-th loop when the currents in each loop changes is given by

$$\mathcal{E}_i = -\frac{d\Phi_i}{dt} = L_i \frac{dI_i}{dt} + \sum_{j\neq i} M_{ij} \frac{dI_j}{dt}$$

The rate of doing work to overcome this emf is

$$\mathcal{E}_i\, I_i = L_i I_i \frac{dI_i}{dt} + \sum_{j\neq i} M_{ij} I_i \frac{dI_j}{dt}$$

Thus, when currents in all the circuits change the rate at which work must be done in maintaining the currents in all the circuits is

$$\begin{aligned}\sum_i \mathcal{E}_i\, I_i &= \sum_i L_i I_i \frac{dI_i}{dt} + \sum_i \sum_{j\neq i} M_{ij} I_i \frac{dI_j}{dt} \\ &= \frac{1}{2}\sum_i \frac{d}{dt}(L_i I_i^2) + \frac{1}{2}\sum_i \sum_{j\neq i} M_{ij} \frac{d}{dt}(I_i I_j)\end{aligned}$$

The last step followed because we could introduce a factor of ½ and write the second term as a symmetric term in terms of I and j using the fact that $M_{ij} = M_{ji}$.

The total work done is obtained by integrating the above over time,

$$\begin{aligned}W &= \sum_i \int \mathcal{E}_i\, I_i dt \\ &= \frac{1}{2}\sum_i L_i I_i^2 + \frac{1}{2}\sum_i \sum_{j\neq i} M_{ij} I_i I_j \\ &= \frac{1}{2}\sum_i I_i \Phi_i = \frac{1}{2}\sum_i I_i \int \vec{B}\cdot d\vec{S}_i\end{aligned}$$

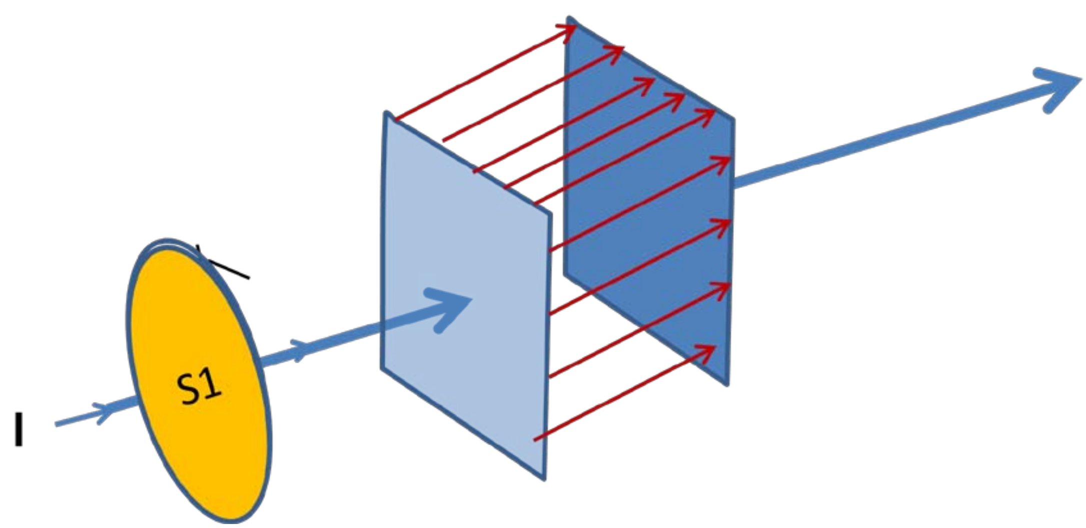
$$= \frac{1}{2}\sum_i I_i \int \left(\nabla \times \vec{A}\right) \cdot d\vec{S}_i$$
$$= \frac{1}{2}\sum_i I_i \int \overrightarrow{A_i} \cdot d\vec{l}_i$$

This is for a discrete current distribution. For a continuous current distribution, this can be easily generalized to give, $W = \frac{1}{2}\int \vec{A} \cdot \vec{J} d^3 r$.

Displacement Current

We are still left with one issue. We understand the assymmetry in the Gauss's law of magnetism and electrostatics because of absence of magnetic monopoles. The time dependent phenomena requires little more thought. If a changing magnetic field could induce an electric field, what about the corresponding effect where there is a changinfg electric field exists? The effect was not detected for long because of reasons that would become clear later. Maxwell had thought about this problem and had concluded that such an effect does indeed exist. This was actually Maxwell's contribution to the set of electromagnetic frield equations which bear his name.

We will illustrate the effect by considering charging of a capacitor plates in a circuit. Assume that we have a circuit which a source and a capacitor, which, for simplicity we take to be a parallel plate capacitor. When the key is closed, current momentarily flows from the battery charging the capacitor plates. We know that there is no current through the gap of the capacitor. Nevertheless, during the operiod when the charging is taking place, there is a changing electric field inside the capacitor plates.

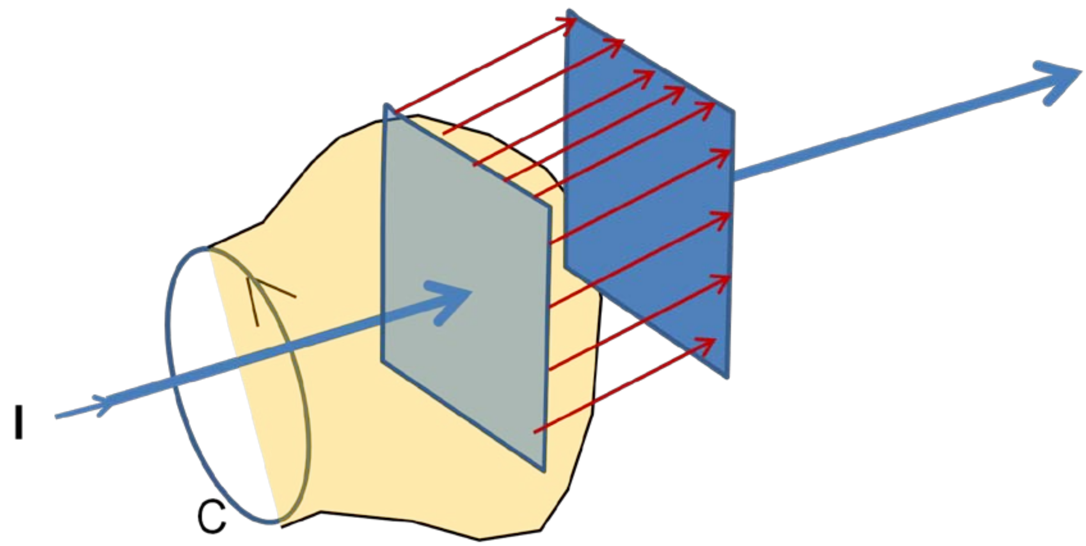


During the process of charging a current exists in the external circuit and we can calculate the magnetic field by using Ampere's law,

$$\oint \vec{B}(t) \cdot dl = \mu_0 I(t)$$

Conser the loop to be directed along the magnetic field direction. If we take a disk as a srface defined by this loop, the flux lines pass through this disk and we get $\nabla \times \vec{B}(t) = \mu_0 \vec{J}(t)$, as has been done before.

However, we now come to a curious anomaly. Suppose, instead of the disk, we take another surface defined by the same loop as its boundary but a pot slike shape which does not insersect the outsdie wire and passes through the capacitor gap, as shown.



Since there is no current passes through the surface, we would get $\nabla \times \vec{B} = 0$.

This is not understandable as both the surface integral represent a loop integral through the same loop and must give unique answer.

The way Maxwell resolved this apparent anomaly is to postulate that just as there is an induced electric field associated with a changing magnetic flux, there is an induced magnetic field associated with a changing electric flux. This has since been verified experimentally though the effect is much smaller than that of Faraday's law.

To visualize this, let us assume that the space between the capacitor plate is filled with a dielectric. In an applied electric field the bound charges within the dielectric would be pushed. This would not of course give rise to a current because such charges would never leave the dielectric. However, Maxwell imagined that the effect would provide something akin to a current which would help remove the anomaly talked above.

The electric flux through the second surface is given by

$$\Phi_E = \int_S \vec{D} \cdot d\vec{S}$$

Let us calculate the rate of change of this flux.

$$\frac{d\Phi_E}{dt} = \frac{d}{dt}\int_S \vec{D} \cdot d\vec{S} = \frac{d}{dt}\int \nabla \cdot \vec{D} dV$$
$$= \frac{d}{dt}\int \rho_{free} dV = \frac{dQ_{free}}{dt} \equiv i_D$$

The rate of change of free charges is clearly the current flowing in to charge the plates. Maxwell called i_D as the displacement current which is equal to the conduction current flowing in to charge the capacitor. This is the "current" which provides continuity of the current flow and is given by

$$i_D = \frac{d\Phi_E}{dt}$$

Maxwell then proposed that the Ampere's law be modified by including this term on the right hand side of the curl equation. In the outside circuit, the electric flux ios zero and only the conduction current exists. Inside the capacitor plates, the conduction current is zero but there is a changing electric flux giving rise to a term which is like a current.

We have then, for Ampere's law,

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

because the displacement current density is obviously given by the second term.

Sincedivergence of a curl is zero, if we take divergence of theabove equation, we get,

$$\nabla \cdot \vec{J} + \frac{\partial}{\partial t}\left(\nabla \cdot \vec{D}\right) = \nabla \cdot \vec{J} + \frac{\partial}{\partial t}\rho = 0$$

which is just the equation of continuity. Thus all the Maxwell's equations are in place now. They are

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \Leftrightarrow \nabla \cdot \vec{D} = \rho_{free}$$
$$\nabla \cdot \vec{B} = 0$$
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\nabla \times \vec{H} = \vec{J}_{free} + \frac{\partial \vec{D}}{\partial t}$$

We supplement these with two “constitutive relations”

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

Magnetostatics IV

Lecture 29: Electromagnetic Theory

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Tutorial Assignment

1. Calculate the self inductance of a long wire of radius R carrying a current I uniformly distributed over its cross section.
2. A toroidal coil of N turns is tightly wound over a doughnut of inner radius a and outer radius b. A long straight wire passes through the centre of the doughnut and carries a time varying current $I(t)$. Determine the mutual inductance between the coil and the straight wire.
3. Two circuits A and B have a mutual inductance M. At t=0, the current in the coil A is switched on and it increases with times as $I_A(t)$. The emf induced in the coil B as a result is found to change with time as given by the relation $\mathcal{E} = \mathcal{E}_0 + \alpha t$, where $\mathcal{E}_0$ and α are constants. Obtain an expression for $I_A(t)$.

4. A rectangular loop of dimensions $a \times b$ lies with its longer sides parallel to a long straight wire, the distance of the wire from the nearer side is d. Calculate the mutual inductance of the pair.
5. An electromagnetic wave travels in a medium of relative permeability 4 and dielectric constant 5. The displacement current density associated with the electromagnetic field varies with space and time as

$$J_d = 5\sin(kz - \omega t)\hat{\imath}$$

(in μA/m^2). Find the electric field, the magnetic field and show that Ampere’s law remains satisfied if $\frac{\omega}{k} = \frac{1}{\sqrt{\mu\epsilon}}$.

6. Using the relation $W = \frac{1}{2}\int \vec{A}\cdot\vec{J} d^3r$ for a current distribution, show that the magnetic energy can be expressed as $W = \frac{1}{2}\int \vec{B}\cdot\vec{H} d^3r$. A coaxial cable consists of an inner conductor of radius a and an outer conductor of radius b. The current in the inner conductor is uniformly distributed while the outer conductor has negligible thickness and provides a return path to the current. Determine the energy of the system.

Solutions to Tutorial Assignments

1. There are two ways of doing this, the first (and by far the, easiest) is to calculate the energy contained in the magnetic field and equate it to $\frac{1}{2}LI^2$. In doing so one has to be careful and discard a self energy contribution from the field outside the wire which gives a divergent term. Consider a strip of unit length and width dr lying between r and r+dr. The field strength on this strip is $B = \frac{\mu_0 I}{2\pi R^2} r$, so that the flux enclosed by this region per unit length is

$$d\Phi = \frac{\mu_0 I}{2\pi R^2} r dr$$

The flux "linked" with this strip is this amount multiplied by a contour of radius r, which is $\frac{I(\pi r^2)}{\pi R^2}$, so that the total flux linked with a unit length of the wire is

$$\frac{\mu_0 I}{2\pi R^4} \int_0^R r^3 dr = \frac{\mu_0 I}{8\pi}$$

Thus the self inductance is $\frac{\mu_0}{8\pi}$

2. The magnetic field due to the straight conductor is $\frac{\mu_0 I}{2\pi r}$. The flux linked with each turn of the toroidal coil is $\Phi = \frac{\mu_0 I}{2\pi} h \int_a^b \frac{dr}{r} = \frac{\mu_0 I}{2\pi} h \ln \frac{b}{a}$. where h is the height of the doughnut. The self inductance is given by $M_{12} = N\frac{\Phi}{I} = N\frac{\mu_0}{2\pi} h \ln \frac{b}{a}$.
3. The emf in B as a result of change in the current in the circuit A is given by

$$\mathcal{E} = -M\frac{dI_A}{dt} = \mathcal{E}_0 + \alpha t$$

which gives $I_A = -\frac{\mathcal{E}_0}{M} t - \frac{\alpha}{M}\frac{t^2}{2}$. As the current is zero at t=0, the constant of integration is zero.

4. When a current I passes through the long wire, a field $\frac{\mu_0 I}{2\pi r}\hat{\phi}$ gets established at a distance r from the wire, which is perpendicular to the loop. The flux through the loop can be calculated by considering a strip of width dr at a distance r from the wire,

$$\Phi = \int d\Phi = \frac{\mu_0 I}{2\pi} \int_d^{d+b} a \frac{dr}{r} = \frac{\mu_0 I a}{2\pi} \ln \frac{d+b}{d}$$

Thus the mutual inductance is

$$M = \frac{\mu_0 a}{2\pi} \ln \frac{d+b}{d}$$

.

5. The displacement current density is

$$J_d = 5\sin(kz - \omega t)\hat{\imath} = \frac{\partial \vec{D}}{\partial t}$$

which gives $\vec{D} = \frac{5}{\omega}\cos(kz - \omega t)\hat{\imath}$. The corresponding electric field is

$$\vec{E} = \frac{5}{\omega\epsilon}\cos(kz - \omega t)\hat{\imath} = \frac{1}{\omega\epsilon_0}\cos(kz - \omega t)\hat{\imath}$$

Taking curl of the electric field we get

$$\nabla \times \vec{E} = \frac{\partial E_x}{\partial z}\hat{\jmath} = -\frac{k}{\omega\epsilon_0}\sin(kz - \omega t)\,\hat{\jmath} = -\frac{\partial \vec{B}}{\partial t}$$

from which we get,

$$\vec{B} = \frac{k}{\omega^2\epsilon_0}\cos\,(kz - \omega t)\hat{\jmath}$$

The H- field is then given by

$$\vec{H} = \frac{k}{\omega^2\epsilon_0\mu}\cos(kz - \omega t)\hat{\jmath} = \frac{k}{4\omega^2\epsilon_0\mu_0}\cos(kz - \omega t)\hat{\jmath}$$

Taking curl of the H field we get

$$\nabla \times \vec{H} = -\frac{\partial H_y}{\partial z}\hat{\imath} = \frac{k^2}{4\omega^2\mu_0\epsilon_0}\sin(kz - \omega t)\hat{\imath}$$

Using $\frac{\omega^2}{k^2} = \frac{1}{\mu\epsilon} = \frac{1}{20\mu_0\epsilon_0}$, we get

$$\nabla \times \vec{H} = 5\sin\,(kz - \omega t)\hat{\imath} = \vec{J}_d = \frac{\partial \vec{D}}{\partial t}$$

6. Use the identity,

$$\begin{aligned}\nabla \cdot (\vec{H} \times \vec{A}) &= \vec{A} \cdot (\nabla \times \vec{H}) - \vec{H} \cdot (\nabla \times \vec{A}) \\ &= \vec{A} \cdot \vec{J} - \vec{H} \cdot \vec{B}\end{aligned}$$

Substituting this and using the divergence theorem we can ignore the surface term and get

$$W = \frac{1}{2}\int \vec{H} \cdot \vec{B} d^3r = \frac{1}{2\mu}\int |B^2| d^3r$$

for linear magnetic material.

For the coaxial cable, with the field being distributed in the inner conductor, the fields are as follows:

$$\vec{B} = \begin{cases} \dfrac{\mu I r}{2\pi a^2}\hat{\phi} & r < a \\ \dfrac{\mu_0 I}{2\pi r}\hat{\phi} & a \le r \le b \\ 0 & r > b \end{cases}$$

The contribution to the energy (per unit length) from the inner conductor is (volume element for unit length is $d^3r = r\,d\theta dr$

$$W = \frac{\mu I^2}{8\pi^2 a^4}\int r^2 d^3r = \frac{\mu I^2}{8\pi^2 a^4}\int_0^a r^2(2\pi r dr) = \frac{\mu I^2}{16\pi}$$

Contribution from the region between the conductors is

$$W = \frac{\mu_0 I^2}{8\pi^2}\int \frac{1}{r^2}\,d^3r = \frac{\mu_0 I^2}{8\pi^2 a^4}\int_0^a \frac{1}{r^2}(2\pi r dr) = \frac{\mu_0 I^2}{4\pi}\ln\frac{b}{a}$$

The total energy is

$$W = \frac{I^2}{4\pi}\left[\mu_0 \ln\frac{b}{a} + \frac{\mu}{4}\right]$$

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Self Assessment Questions

1. A coaxial cable consists of an inner conductor of radius a and an outer conductor of radius b. The current in the inner conductor is uniformly distributed while the outer conductor has negligible thickness and provides a return path to the current. Calculate the self inductance of the cable.
2. Find the mutual inductance of two coplanar squares with a common centre assuming that the square located inside is much smaller in dimension *b* than the bigger square which has a side *a*.
3. The current through a straight wire varies with time as $I(t) = 30 + 10^3 t$ where the current is in Amperes and time in seconds. If the radius of cross section of the wire is 50 (mm)^2 and the resistivity $10^{-8}\ \Omega m$, estimate the displacement current density and compare it with the conduction current density.
4. A capacitor plate of area 0.3 m^2 is being charged at a uniform rate so that the electric field inside the plate varies with time as $\frac{dE}{dt} = 10^{13}$ V/m-s. Calculate the displacement current and estimate the magnetic field strength at a distance 5 cm from the centre of the capacitor plate along a line parallel to both the plates.

Solutions to Self Assessment Questions

1. The self inductance of the inner conductor was calculated in Problem 1. Because the thickness of the outer conductor is negligible, the flux linked is zero and there is no self inductance contribution from the outer shell. There is however, flux linked between the two conductors. The field in this region is $\frac{\mu_0 I}{2\pi r}$, so that flux passing through a strip of width dr between r and r+dr and of unit length is $\mu_0 I \frac{dr}{2\pi r}$. The total flux linked is $\frac{\mu_0 I}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 I}{2\pi}\ln\frac{b}{a}$. Thus the total self inductance is

$$L = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \ln\frac{b}{a}\right]$$

2. The magnetic field at the centre of the square (taken in xy) plane) is given by $\frac{2\sqrt{2}\mu_0 I}{\pi a}\hat{k}$. Since the dimension of the second coil is much smaller than that of the first, it can be assumed that this field exists over the entire square of side b, as a result of which the flux through the second loop is $\frac{2\sqrt{2}\mu_0 I}{\pi a}b^2$, which gives the mutual inductance to be $\frac{2\sqrt{2}\mu_0}{\pi a}b^2$.
3. Consider a length L of the wire. The resistance is $R = \frac{\rho L}{A} = \frac{10^{-8}L}{50\times 10^{-6}} = 2\times 10^{-4}L$. If E is the electric field in the wire, the potential difference between the ends of the wire is $V = EL$ so that the current $I(t) = \frac{V}{R} = \frac{EL}{R} = 5\times 10^5 E(t)$. The displacement current density is
$$J_d = \frac{\epsilon_0 dE}{dt} = 2\times 10^{-4}\epsilon_0\frac{dI}{dt} = 2\times 10^{-1}\epsilon_0 = 1.8\times 10^{-12}\ A/m^2$$
Thus one sees that the displacement current density is very small, the corresponding current density at t=0 is $\frac{30}{50\times 10^{-6}} = 6\times 10^5$ A/m^2 which keeps on increasing with time
4. The displacement current density

$$J_d = \epsilon_0\frac{dE}{dt} = 88.5\ A/m^2$$

The magnetic field at a distance r from the centre is given by

$$2\pi r\ B = \mu_0 88.5\times \pi r^2$$

which gives, substituting r= 0.5 m,$B = 2.78\times 10^{-5}\ T$.