Theory of polycrystal deformation

• Microscopically, plastic flow is due to slip on a plane, or a set of planes, within a crystal

• Deformation mechanism in the grains of a polycrystal are same as those in a single crystal.

• It should be possible to relate quantitatively the plastic properties of a polycrystalline material with those of a single crystal
First attempt: Sachs, Cox and Sopwith

- Tensile yield strength of a polycrystal should be the average of the tensile strengths of the crystal in it.

- It is well known that for a single crystal,

\[ \sigma = \left( \frac{1}{m} \right) \tau \]

Schmid factor = \( \cos \varphi \cos \lambda \)

**CRSS for slip**

- Tensile yield strength of a polycrystal should be the average of the tensile strengths of the crystal in it. The average of Schmid factors for randomly oriented single crystals constituting a FCC polycrystal gives,

\[ \left( \frac{1}{m} \right)_{\text{average}} = 2.238 \]
Therefore, *according to Sachs*, tensile yield strength of a randomly oriented FCC polycrystal is given by 
\[ \sigma = 2.238 \tau \]

- A grain deforms on the most highly stressed slip system.
- A fcc crystal deformed in uniaxial tension will rotate until \( <112> \parallel \) tensile axis.
- For compression, it will rotate towards \( <110> \).
- For sheet rolling (compression along ND and tensile along RD), crystal will rotate towards \( \{011\}<211> \).
Limitations:

- With only single slip operating, a circular section of a grain would become elliptical.
- In randomly oriented polycrystal, the major axes of all the ellipse would not be parallel, so the deformation would not be compatible.

Arguments:

- Several slip systems should be activated for in a grain, or at least, in a part of grain – because the grains are require to fit together without voids after deformation.
- Sachs theory only gives a lower bound approximation to the strength in uniaxial tension.
Second attempt: Taylor’s analysis

Assumption: All the grains undergo the same shape change as the entire polycrystal

In a randomly oriented polycrystal under uniaxial tension along X-direction, deformation occurs by axially symmetric flow; so

\[ d\varepsilon_y = d\varepsilon_z = -1/2 \, d\varepsilon_x \]

and \( d\gamma_{yz} = d\gamma_{zx} = d\gamma_{xy} = 0 \)

The work per volume with a grain is:

\[ dw = \sum_i \tau \, d\gamma_i = \tau \sum_i |d\gamma_i| \]

where \( \tau \) (CRSS for slip) is assumed to be the same on all slip systems and \( d\gamma_i \) is incremental slip on individual slip systems.

Replacing \( \sum_i |d\gamma_i| \rightarrow d\gamma \)

\[ dw = \tau \, d\gamma \]
In uniaxial tension, the incremental work per unit volume due to the external stress is \( \sigma_x d \varepsilon_x \),

\[
dw = \tau \, d \gamma = \sigma_x \, d \varepsilon_x,
\]

or

\[
\frac{\sigma_x}{\tau} = \frac{d \gamma}{d \varepsilon_x} = M.
\]

By calculating the value of \( M = d \gamma/d \varepsilon_x \) for all orientations and averaging to find \( \bar{M} \), the flow behaviour of a random polycrystal can be calculated as:

\[
\sigma_x = \bar{M} \tau \quad \text{and} \quad d \varepsilon_x = \frac{d \gamma}{\bar{M}}
\]

where \( \bar{M} \) is the Taylor factor, which depends on the orientation.
Questions

1. How deformation behavior of polycrystalline materials is different from single crystal?

2. Explain the Sachs theory and its limitations?

3. Explain Taylors theory. How it is different from Sachs theory?