Strengthening Mechanisms
Cylindrical single crystal of area $A$ under tensile load $P$.

Area of the slip plane $A/\cos\phi$ and load on the plane $P\cos\lambda$.

Shear stress for slip to occur is called the *critical resolved shear stress* (CRSS), $\tau_R$. 

$$\tau_R = \frac{P\cos\lambda}{A/\cos\phi} = \frac{P}{A} \cos\phi \cos\lambda$$
Effect of Grain boundaries

- Grain boundaries act as obstacles to dislocations and hence, dislocations pile up at the grain boundaries.

- Number of dislocations in the pile-up, \( n = k \pi \tau_s L / Gb \) (\( G \), shear modulus, \( b \), Burger vector, \( \tau_s \), avg. resolved shear stress).

- A pile-up of \( n \) dislocations – One big dislocation with Burger vector \( nb \).

- Stress at the tip of the pile up = \( nb \tau_s \).
Grain boundary strengthening

- For a dislocation source at the center of a grain diameter $D$,
  \[ n = k \pi \tau_s D / 4 Gb \]
- The critical shear stress to cross the grain boundary barrier,
  \[ \tau_c = n \tau_s = \pi \tau_s^2 D / 4 Gb \]
- $\tau_s = \tau - \tau_i$ ($\tau_i$ is the lattice resistance to dislocation motion)
- $\tau_c = \pi (\tau - \tau_i)^2 D / 4 Gb \Rightarrow \tau = \tau_i + \left( \tau_c 4 Gb / \pi D \right)^{1/2} = \tau_i + k' D^{-1/2}$
- Expressing this in terms of normal stresses gives rise to the Hall-Petch relationship
  \[ \sigma_o = \sigma_i + k D^{-1/2} \]
- $k$ is known as “locking parameter”, which is a measure of the relative hardening contribution of grain boundaries and $\sigma_i$ is known as “friction stress” which represents overall resistance of the lattice to dislocation motion.
Solid solution strengthening

- Solute atoms introduce lattice strain as their size is different from the host atoms.
- A larger substitutional solute atom will impose a compressive stress (Fig. a) while a smaller interstitial atom will cause tensile stresses in the lattice (Fig. b).
- Interstitial atoms are often bigger than the interstitial space they occupy, resulting in a compressive stress field.
Solid solution hardening

- Dislocations have strain field at their cores due to lattice distortion
- Solute atoms with a tensile strain field will diffuse to the dislocation core to nullify part of the compressive strain field of the dislocation to reduce the strain energy.
- This hinders motion of the dislocation and hence, the strength increases.

Dislocation strain field

Solid solution hardening
Precipitation hardening

- Strength and hardness of some alloys can be increased by formation of fine precipitates.
- The solute should have increasing solubility with increasing temperature (e.g. Al-Cu) for the precipitation to occur.
- Heat treatment – Solutionizing or heating to single phase region. Quenching - rapid cooling to get a supercooled solid solution (SSS) (Normal cooling – Coarse equilibrium CuAl$_2$ phase below $T_N$). Isothermal holding at certain temperature.
precipitation sequence

- When the supersaturated solid solution is held isothermally (aging), fine particles of precipitates form in several steps.
- The precipitation sequence – Guiner-Preston (GP) zones (cluster of Cu atoms) → \( \theta'' \rightarrow \theta' \rightarrow \theta \)
- \( \theta'' \) and \( \theta' \) have different crystal structures than the parent phase (\( \alpha \)) and are coherent with the parent lattice, while the equilibrium phase \( \theta \), which forms on prolonged aging (Overaging) is not coherent.
Hardening mechanism

- $\theta''$ and $\theta'$ have different crystal structures but maintain coherency with parent lattice resulting in lattice strain. This impedes dislocation motion and hence, the hardness and strength increases.

- Further aging for longer time dissolves the $\theta'$ phase and the equilibrium phase $\theta$ (CuAl$_2$) forms. This phase is no longer coherent with parent lattice and as a result hardness decreases, a phenomenon called overaging.
Strain Hardening

- Increasing strength and hardness by plastic deformation is called *strain hardening* or *work hardening*. Also referred as *cold working* as deformation takes place at RT.
- Extent of strain hardening increases with degree of cold working (% area reduction).
- Since the metal is deformed in a certain direction, grains are elongated in the direction of working.

![Initial structure vs Cold-worked structure](image)
Strain Hardening mechanism

- Cold working generates dislocations – dislocation density increases. Higher density of dislocations impedes their motion due to interactions of dislocation strain fields.
- Hence the strength increases according to the relation \( \sigma_o = \sigma_i + \alpha Gb \rho^{1/2} \). \( \rho \) is the dislocation density, \( b \), Burger vector, \( G \), shear modulus and \( \alpha \) is a constant.
- The ductility decreases after cold working.

![Diagram showing the relationship between hardness, strength, ductility, and percentage of cold work.](image)
Examples

Ex.1 Calculate the resolved shear stress for an FCC single crystal on the (111) [0 1 1] slip system if a stress of 14 MPa is applied in [001] direction.

Solution: The angle between two direction vectors \([u_1v_1w_1]\) and \([u_2v_2w_2]\) is given as

\[
\cos \theta = \frac{u_1u_2 + v_1v_2 + w_1w_2}{\sqrt{u_1^2 + v_1^2 + w_1^2} \sqrt{u_2^2 + v_2^2 + w_2^2}}
\]

Therefore, \(\cos \lambda = \frac{a}{\sqrt{2a}} = 0.707\) and \(\cos \phi = \frac{a}{\sqrt{3a}} = 0.577\)

\(\tau_R = 14 \cos \lambda \cos \phi = 14 \times 0.707 \times 0.577 = 5.7\) MPa
Quiz

1. What is critical resolve shear stress? Derive the expression for CRSS.
2. Calculate the resolved shear stress for an Ni single crystal on the (111) [0 \abar 1 1] slip system if a stress of 15 MPa is applied in [001] direction.
3. A stress of 5 MPa is applied to a single crystal FCC metal in the [0 0 \abar 1] direction. Calculate the CRSS on (1 1 \abar 1) plane in [ \abar 1 0 \abar 1], [0 \abar 1 1] and [ \abar 1 1 0] directions.
4. What is the mechanism of grain refinement strengthening?
5. Show that strength is proportional to $D^{-1/2}$ ($D =$ grain dia).
6. An iron rod has a grain size of 0.01 mm and yield strength of 230 MPa. The strength is 275 MPa at a grain size of 0.006 mm. In order achieve a yield strength of 310 MPa what should be the grain size?
7. Why is strain hardening also called cold working?
8. What kind of microstructure develops after cold working?
Quiz

9. Why does hardness and strength increase on clod working? What is the effect of cold working on ductility?
10. How is dislocation density related to strength?
11. What is the mechanism of strengthening by solid solution?
12. What kind of strain field interstitial atoms generally introduce?
13. What kind of strain fields are associated with dislocations?
14. What are the different stages of precipitation hardening?
15. What kind of alloy system hardens by precipitation?
16. Will precipitation hardening occur if the alloy is cooled slowly from the single phase region?
17. What is supersaturated solid solution?
18. How does the strength increase by precipitation?
19. What is coherent and incoherent precipitate?
20. What is the main strengthening phase in Al-Cu alloys.
References

http://www.soton.ac.uk/~engmats/xtal/deformation/control.htm
http://people.virginia.edu/~lz2n/mse209/Chapter7.pdf

Key words. Strengthening mechanism, Hall-Petch equation, Solid solution hardening, precipitation hardening, strain hardening.