Module 12

Crystal defects in metals III

Lecture 12

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**Keywords:** Forces acting on dislocation, Dislocation interaction, Interaction between parallel screw & parallel edge dislocations, Interaction with forest dislocations, Jogs & kinks, Dislocation locks

**Introduction**

In the last two modules the concepts of point defect and line defects in crystals have been introduced. The presence of such defects is associated with lattice distortion. The extent of this is in fact a measure of local (internal) stresses within the crystal. Point defects usually have a hydrostatic stress field whereas line defects depending on its character may have only shear stress field (as in the case of a screw dislocation) or both shear and hydrostatic stress field (as in the case of an edge dislocation). We have also looked at the glide and the climb motions of a dislocation. Look at the slide 1 to recollect the differences between edge & screw dislocation. However so far we have considered these in isolation. In a crystal however there are several defects. All of them have their own stress field. In this module we shall learn about their interactions.

**Slide 1:** The table summarizes the major differences between edge & screw dislocation. Note that \( \mathbf{b} \) = Burgers vector; \( \mathbf{t} \) = unit vector along the dislocation; and \( \mathbf{n} \) = unit vector normal to the plane containing the dislocation and its Burgers vector.

<table>
<thead>
<tr>
<th>Slip plane</th>
<th>edge ( n=b\mathbf{X}t )</th>
<th>screw ( n=b\mathbf{X}t=0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathbf{b} ), ( \mathbf{t} )</td>
<td>( \mathbf{0} )</td>
<td>( \mathbf{b} )</td>
</tr>
<tr>
<td>Stress field</td>
<td>( \sigma_{11}, \sigma_{22}, \sigma_{33} ) &amp; ( \sigma_{13} ) &amp; ( \sigma_{23} )</td>
<td></td>
</tr>
<tr>
<td>Energy</td>
<td>( \alpha G \mathbf{b}^2/(1-\nu) )</td>
<td>( \alpha G \mathbf{b}^2 )</td>
</tr>
<tr>
<td>Movement</td>
<td>Glide &amp; climb</td>
<td>glide &amp; cross slip</td>
</tr>
</tbody>
</table>

**Force acting on a dislocation:**

In the previous lecture we have seen that if the resolved shear stress on a plane is \( \tau \); the force on the dislocation is \( \mathbf{t}\mathbf{b} \); where \( \mathbf{b} \) denotes Burgers vector. However, often the local stress field around a defect is given in the form of a stress matrix (or stress tensor \( \sigma_{ij} \)). In such a situation we need a more generalized approach to find out the force acting on a dislocation. It has to be a function of \( \sigma \), \( \mathbf{b} \) & \( \mathbf{t} \). Note that \( \mathbf{t}\mathbf{b} \) denotes a product of stress and Burgers vector where \( \tau \) is one of the components of a stress tensor. The product is a vector having magnitude and direction. If the local stress field is given by \( \sigma_{ij} \), a tensor of second rank, the force on the dislocation should be given by \( \mathbf{\bar{\sigma}} \cdot \mathbf{\bar{b}} \). The double bar is a sign of second rank tensor whereas a single bar denotes a vector. This truly represents a product of 3x3 matrix with 3x1 vector giving a 3x1 vector. Its vector product with the unit vector along the dislocation...
direction at a point represents the force acting on it at that point. This can be mathematically represented as follows:

\[
\bar{F} = \bar{a} \times \bar{b} \quad \text{or} \quad F_i = \epsilon_{ijk} a_j b_k
\]

Note that the both the forms of the equation are the same. The former is in vector notation whereas the latter is in tensor notation. The use of repeated suffix represents summation. For example the subscript \('l'\) in equation 1 appears in \(a_l\) and \(b_l\). This means the suffix \(l\) can have values from 1 to 3. More explicitly this is given by:

\[
\sigma_{ll} b_l = \alpha_{1l} b_1 + \alpha_{2l} b_2 + \alpha_{3l} b_3
\]

Physically this represents the component of the vector along the axis \(i\). Using this we could get the magnitude of the vector along the three orthogonal axes. The force on the dislocation is given by its cross product with vector \(t\) representing dislocation direction. The symbol \(\epsilon_{ijk}\) is a coefficient given by the following equation:

\[
\epsilon_{123} = \epsilon_{231} = \epsilon_{312} = +1 \quad \& \quad \epsilon_{132} = \epsilon_{321} = \epsilon_{213} = -1
\]

\[
\epsilon_{ijk} = 0 \text{ if any two or more of the subscripts are the same}
\]

Fig 1: A sketch showing a slip plane with a dislocation (the dark line on the shaded plane). The slip plane is perpendicular to \(x_3\) axis. Shear stress \(\tau\) acts along the direction \(x_2\) on the top face of the crystal.

If the crystal is loaded as shown the only non-zero component of the stress is \(\tau = \sigma_{23} = \sigma_{32}\). Note that the dislocation is aligned along \(x_1\) axis. Therefore the nonzero component of \(t\) representing dislocation direction is \(t_1\). If we assume this to be an edge dislocation the only non-zero component of Burgers vector \(b\) is \(b_2\). Therefore the force on the dislocation is given by

\[
F_i = \epsilon_{ij1} \sigma_{jj} b_2 t_1 = \epsilon_{ij1} \sigma_{j3} b_2 t_1
\]

Thus the only non-zero component of the force is \(F_2 = \epsilon_{231} \sigma_{32} b_2 t_1 = \tau b\). In this case the derivation looks complex. However this does help understand the effect of any arbitrary state of stress on a dislocation.

Interaction between two parallel screw dislocations:
The equation 1 can be used to find the force exerted by a screw dislocation located along the axis $x_3$ on another located at a distance $r$ from it. Figure 2 shows the locations of the two parallel screw dislocations with respect to the reference axes $x_1, x_2, x_3$.

**Fig 2:** A sketch showing locations of two screw dislocations lying along $x_3$ axis. A clockwise circuit has been drawn in dotted line while looking against the direction $x_3$. The arrow to complete the circuit is the Burgers vector of the dislocation. Note dislocation & burgers vector are aligned in opposite direction.

The only non-zero components of stress field of a screw dislocation are $\sigma_{13} & \sigma_{23}$. The Burgers vector & the direction of the dislocation at a distance $r$ are $[0 \ 0 \ b'] & [0 \ 0 \ -1]$; note that only the components along $x_3$ are non-zero. Therefore the stress acting on the second dislocation due to the stress field of the first is given by

$$F_i = \epsilon_{i3j} \sigma_{j3} b_3 t_3 \quad (6)$$

Note that the suffix $i$ & $j$ must have different values. Thus

$$F_1 = \epsilon_{123} \sigma_{23} b_3 t_3 \quad & \quad F_2 = \epsilon_{213} \sigma_{13} b_3 t_3 \quad (7)$$

Recall that if $b$ is the Burgers vector of the dislocation along $x_3$. The two of its non-zero stress components are

$$\sigma_{13} = \frac{Gb}{2\pi} \frac{x_2}{x_1^2+x_2^2} \quad & \quad \sigma_{23} = -\frac{Gb}{2\pi} \frac{x_1}{x_1^2+x_2^2} \quad (8)$$

Substitute these in equation 8 and put $t_3 = -1$ & $b_3 = b'$ to get

$$F_1 = \frac{Gb b'}{2\pi} \frac{x_1}{x_1^2+x_2^2} \quad & \quad F_2 = \frac{Gb b'}{2\pi} \frac{x_2}{x_1^2+x_2^2} \quad (9)$$

Using cylindrical coordinate you can get a still simpler expression for the forces. See the sketch in fig 3 & replace $X_1$ & $X_2$ by $r, \theta$ terms. This gives $F_1 = \frac{Gb b'}{2\pi r} cos \theta$ & $F_2 = \frac{Gb b'}{2\pi r} sin \theta$.

**Fig 3:** A sketch showing the locations of dislocations. The distance between the two is $r$ whereas the angle between $r$ & $X_1$ axis is $\theta$. The relations between the coordinates are also displayed.

The resultant force is thus $F = \frac{Gb b'}{2\pi r}$. Note its direction. In this case it is repulsive. There will be similar repulsive force acting on the other dislocation. As a result the two would move away from each other. In other words like dislocations repel each other. If the dislocations are of opposite nature they attract each other.
Parallel edge dislocation having parallel Burgers vector:

Following the steps described in the previous case you could easily find the forces acting on an edge dislocation due the stress field on another edge dislocation.

The sketch in slide 3 shows that $x_1$ & $x_2$ are the coordinates of the point of intersection of the second dislocation with plane $x_3 = 0$. If $x_2 = 0$ the two dislocations lie on the same plane & $F_2 = 0$. The sign of $F_1$ denotes that the two would repel each other. In short two like dislocation on the same plane would repel whereas unlike dislocations would attract each other. The direction and magnitudes of $F_1$ & $F_2$, the two components of the force are functions of the coordinates $x_1$ & $x_2$. Let us substitute $x_1 = r \cos \theta$ and $x_2 = r \sin \theta$ where $r$ is the distance between the dislocations and $\theta$ is the angle between vector $r$ and the axis $x_2$. The expressions for these after the above substitution are given in slide 4.
The illustration given in slide 4 shows the directions of the force $F_2$ on the second dislocation depending on its locations. If it is above the slip plane of the first (the one at the origin) it would tend to climb up; whereas if it is below the slip plane it would tend to climb down. The dotted lines represent positions where the force $F_1 = 0$. Note that these are at 45° (it follows from the expressions given in polar coordinates). The directions along which $F_2$ acts on either side of these lines are different. The points on the dotted lines represent positions of unstable equilibrium. This is further illustrated by the sketch on the bottom left of slide 4. This gives a plot of force $F_1$ as a function of the distance $x_1$ on a plane $x_2 = \text{constant}$. The location $x_1 = 0$ denotes a position of stable equilibrium. If the dislocation is forced to move a little on either of the two sides the force $F_1$ will help it come back to its initial position. At locations $x_1 = \pm x_2 \text{too } F_1 = 0$. In this case if it moves a little the force $F_1$ would tend to move it away from its position of unstable equilibrium. The most stable configuration of a set of parallel edge dislocations with parallel Burgers vector is therefore one over the other. Later we would look at such an array called sub-grain boundary in a little more detail.

**Interaction with forest dislocation:**

When a dislocation glides on a plane, it might come across other dislocations passing through its slip plane at an angle. Some of these may even be normal to the slip plane. Such dislocations are often referred to as forest dislocation. Slide 5 shows one such interaction between two edge dislocations at right angles with perpendicular Burgers vector. In this case the steps that form are known as kinks. It is aligned in the direction of its Burgers vector. Therefore they are screw dislocations and they can move along the length of the dislocation.
Slide 5: Illustrates what happens when an edge dislocation moving on a plane intersects another edge dislocation perpendicular to it (as shown). Note that the Burgers vectors of the two dislocations are normal to each other. The sketch on the left gives the initial positions of the two whereas the one on the right is after they cross each other. As a result steps are formed on the two dislocations. They are parallel to the respective Burgers vectors. Such steps having screw characters are known as kinks.

Slide 6: Illustrates what happens when an edge dislocation intersects a screw dislocation which is perpendicular to the slip plane. The sketch on the left shows the way the dislocations are oriented with respect to the slip plane. The sketch on the right shows their positions once they cross each other. Here as well steps are formed on the dislocations perpendicular to the respective Burgers vectors. Both of these are edge dislocations. These are known as jogs.

Slide 6 shows a type of interaction resulting in the formation of steps having edge character. The slip plane on which they could glide is perpendicular to the plane shown in the sketch. Often these may be the planes on which dislocations cannot glide. Formation of such steps called jogs hinders dislocation glide and leads to work hardening.

**Dislocation locks:**

Dislocations moving on two slip planes may meet along the line of their intersection. Such interaction may give rise to an altogether a different dislocation. One such illustration is shown in slide 7.

Slide 7: Illustrates what happens when two dislocations moving on two slip planes meet along the line of intersection. There are four \{111\} slip planes in an fcc crystal. These may be represented by the 4 faces of a tetrahedron (ABCD). Consider two slip planes \(\{111\}\) & \(\{1\overline{1}1\}\). The Burgers vectors of the dislocations on these are \(\frac{a}{2}[10\overline{1}]\) & \(\frac{a}{2}[01\overline{1}]\). They meet & give rise to a dislocation having \(\frac{a}{2}[1\overline{1}0]\) as its Burgers vector. Note the dislocation reaction as shown. It is energetically favorable.
The line of intersection between the two slip planes as shown in slide 7 is \( \frac{a}{2}[1\bar{1}0] \). This also represents the direction of the resultant dislocation having \( \frac{a}{2}[110] \) as its Burgers vector. The two are at right angles. This shows that the resultant dislocation is an edge dislocation. Its slip plane is (001). This is not the normal slip plane of the crystal. Therefore the dislocation cannot glide on it. It remains as an immobile dislocation. It has its own stress field which would inhibit dislocation movement on both the planes. Such a dislocation is known as Cottrell- Lomer lock.

**Summary**

Every dislocation has its own stress field. It exerts forces on any nearby dislocations. A simple way of estimating the forces between dislocations has been introduced. It has been used to find the interaction between two parallel screw and edge dislocations. In general like dislocations experience repulsion whereas unlike dislocations experience attraction. The magnitude of the forces of interaction is inversely proportional to the distance between the two. The interaction between two parallel edge dislocations having parallel Burgers vector has been explained in detail. In this case there are two forces. One that helps it glide and the other that helps it climb. Around every edge dislocations there are several locations where the glide force on another edge dislocation is zero. One of these is a stable configuration whereas the other is an unstable configuration. The climb forces are zero only if the dislocations are on the same slip plane. We have also looked at what happens when dislocation moving on a slip plane intersects dislocations passing through it at right angle. It leaves behind steps on the dislocation. These are called kink or jog depending on whether they can glide or not. Dislocations may also interact to form immobile dislocations. They act as lock and hinder dislocation movement. Mechanism of formation of one such lock named after Cottrell – Lomer has been introduced.

**Exercise:**

1. Assume that dislocations are arranged in an array in three dimensions described as a cubic lattice. If average number of dislocations intersecting a plane is \( p \) /unit area show that the average distance between two dislocations is proportional to \( \sqrt{1/p} \).
2. Dislocation density of annealed metal is \( 10^{13} \text{ m}^{-2} \). Find out elastic stored energy per unit volume.
3. Elastic stored energy / unit length of an edge dislocation is given by \( E^e = \frac{Gb^2}{4\pi(1-v)} \ln \left( \frac{r}{r_0} \right) \). Does this mean it can approach infinity as \( r \) becomes very large?
4. What is the difference between a kink and a jog? An edge dislocation crosses another dislocation which is perpendicular to the slip plane. Show with neat diagram the effect of such an interaction.
5. A perfect dislocation moving on plane \((11\bar{1})\) interacts with another moving on\((1\bar{1}1)\). What are the different reactions possible? Which of these are Lomer locks?
6. On what planes can a screw dislocation having Burgers vector \( \frac{a}{2}[111] \) could move in a BCC crystal? What will be the slip plane if it were an edge dislocation?
Answer:

1. Let \( L \) represents edge of a cube and each of these represent a dislocation line of length \( L \). Repeated array of such a cube would represent a net work of dislocation. This is schematically shown as follows:

   ![Diagram of dislocation array]

   Volume of cube = \( L^3 \) Since each edge denotes dislocation of length \( L \) the total length of dislocation within the cube = \( 12L/4 \). This is because each edge belongs to 4 adjacent cubes. Therefore dislocation density \( \rho = \) total length of dislocation / volume of cube = \( 3L/L^3 \). Or; \( \rho = \frac{3}{L^2} \).

   Therefore the distance between two dislocation is inversely proportional to the square root of dislocation density.

2. Previous problem shows that the average distance between two dislocation = \( L \). The stress field of a dislocation can be assumed to extend over a distance = \( L/2 \). Or; \( R = \frac{L}{2} = \frac{3}{\sqrt{2\rho}} = 0.86\rho^{-0.5} \)

   which is approximately: \( \approx \frac{L}{\sqrt{\rho}} \) Energy of a dislocation consists of two parts. \( U = U_{\text{core}} + U_{\text{strain}} \)

   & \( U_{\text{core}} = \frac{Gb^2}{10} \) The strain energy = \( \frac{Gb^2(1-\nu \cos^2 \alpha)}{4\pi(1-\nu)} \ln \left( \frac{R}{r_0} \right) \) where \( \alpha \) represents angle between dislocation and Burgers vector. For edge dislocation \( \alpha = \pi/2 \) whereas for screw \( \alpha = 0 \). To estimate energy of a dislocation let us assume \( b=0.25 \text{nm} \) and \( r_0 = 5b \) and \( R = 0.05 \times 10^6 \). Therefore \( U = \frac{Gb^2}{10} + \frac{Gb^2}{4\pi} \ln \left( \frac{10^{-6}}{5 \times 0.25 \times 10^{-9}} \right) = Gb^2 \left( \frac{1}{10} + \frac{1}{4\pi} \ln 800 \right) = 0.63Gb^2 \approx 0.5Gb^2 \) Assume \( G=50 \text{GPa} \) Energy / unit length of dislocation = \( 0.5 \times 50 \times 10^9 \times 0.0625 \times 10^{-18} \) J/m = \( 1.56 \times 10^{-9} \) J/m Therefore elastic stored energy / unit volume = \( U_p = 1.56 \times 10^{-9} \times 10^{12} = 1.56\, \text{kJ/m}^3 \)

3. No. Because dislocations do not occur in isolation. If the average distance between dislocations is \( L \) average value of \( r = 0.5L \). Therefore energy of dislocations is always finite. Approximately this is equal to 0.5 \( Gb^2 \).

4. Both kink & jog represent a step on the dislocation. The kink can glide on the slip plane of the parent dislocation. However its direction of motion is different. Glide plane of a jog is different from that of the parent dislocation.

   ![Jog on an edge dislocation](image)
   ![Kink on an edge dislocation](image)

   Note the jog is also an edge dislocation. Its slip plane is perpendicular to the glide plane of parent dislocation. The kink is a screw dislocation. Its direction of motion is along the length of the dislocation.

   Here kinks are created on both.

   These have screw character.
5. The Burgers vectors of dislocations on these planes are given in the following table:

<table>
<thead>
<tr>
<th>No.</th>
<th>Reactions</th>
<th>Energy</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \frac{a}{2} [110] ) + ( \frac{a}{2} [011] ) = a[101]</td>
<td>( \frac{a^2}{2} + \frac{a^2}{2} &lt; 2a^2 )</td>
<td>Unfavorable</td>
</tr>
<tr>
<td>2</td>
<td>( \frac{a}{2} [110] ) + ( \frac{a}{2} [111] ) = a[211]</td>
<td>( \frac{a^2}{2} + \frac{a^2}{2} &lt; \frac{3}{2}a^2 )</td>
<td>Unfavorable</td>
</tr>
<tr>
<td>3</td>
<td>( \frac{a}{2} [110] ) + ( \frac{a}{2} [1\overline{1}0] ) = a[011]</td>
<td>( \frac{a^2}{2} + \frac{a^2}{2} &gt; \frac{a^2}{2} )</td>
<td>Favorable</td>
</tr>
<tr>
<td>4</td>
<td>( \frac{a}{2} [110] ) + ( \frac{a}{2} [011] ) = a[101]</td>
<td>( \frac{a^2}{2} + \frac{a^2}{2} &gt; \frac{a^2}{2} )</td>
<td>Favorable</td>
</tr>
<tr>
<td>5</td>
<td>( \frac{a}{2} [110] ) + ( \frac{a}{2} [011] ) = a[1\overline{1}2]</td>
<td>( \frac{a^2}{2} + \frac{a^2}{2} &lt; \frac{3}{2}a^2 )</td>
<td>Unfavorable</td>
</tr>
<tr>
<td>6</td>
<td>( \frac{a}{2} [1\overline{1}0] ) + ( \frac{a}{2} [110] ) = a[001]</td>
<td>( \frac{a^2}{2} + \frac{a^2}{2} &gt; \frac{a^2}{2} )</td>
<td>Favorable</td>
</tr>
<tr>
<td>7</td>
<td>( \frac{a}{2} [1\overline{1}0] ) + ( \frac{a}{2} [110] ) = a[211]</td>
<td>( \frac{a^2}{2} + \frac{a^2}{2} &lt; \frac{3}{2}a^2 )</td>
<td>Unfavorable</td>
</tr>
<tr>
<td>8</td>
<td>( \frac{a}{2} [1\overline{1}0] ) + ( \frac{a}{2} [110] ) = a[010]</td>
<td>( \frac{a^2}{2} + \frac{a^2}{2} = a^2 )</td>
<td>Unfavorable</td>
</tr>
<tr>
<td>9</td>
<td>( \frac{a}{2} [1\overline{1}0] ) + ( \frac{a}{2} [\overline{1}10] ) = a[100]</td>
<td>( \frac{a^2}{2} + \frac{a^2}{2} = a^2 )</td>
<td>Unfavorable</td>
</tr>
<tr>
<td>10</td>
<td>( \frac{a}{2} [1\overline{1}0] ) + ( \frac{a}{2} [011] ) = a[2\overline{1}0]</td>
<td>( \frac{a^2}{2} + \frac{a^2}{2} = a^2 )</td>
<td>Unfavorable</td>
</tr>
<tr>
<td>11</td>
<td>( \frac{a}{2} [\overline{1}10] ) + ( \frac{a}{2} [011] ) = a[100]</td>
<td>( \frac{a^2}{2} + \frac{a^2}{2} &gt; \frac{a^2}{2} )</td>
<td>Favorable; Lomer Lock</td>
</tr>
<tr>
<td>12</td>
<td>( \frac{a}{2} [011] ) + ( \frac{a}{2} [101] ) = a[112]</td>
<td>( \frac{a^2}{2} + \frac{a^2}{2} &lt; \frac{3}{2}a^2 )</td>
<td>Unfavorable</td>
</tr>
<tr>
<td>13</td>
<td>( \frac{a}{2} [011] ) + ( \frac{a}{2} [\overline{1}0] ) = a[\overline{1}10]</td>
<td>( \frac{a^2}{2} + \frac{a^2}{2} &gt; \frac{a^2}{2} )</td>
<td>Favorable: it can glide on</td>
</tr>
<tr>
<td>14</td>
<td>( \frac{a}{2} [011] ) + ( \frac{a}{2} [110] ) = a[121]</td>
<td>( \frac{a^2}{2} + \frac{a^2}{2} &lt; \frac{3}{2}a^2 )</td>
<td>Unfavorable</td>
</tr>
<tr>
<td>15</td>
<td>( \frac{a}{2} [011] ) + ( \frac{a}{2} [\overline{1}10] ) = a[\overline{1}01]</td>
<td>( \frac{a^2}{2} + \frac{a^2}{2} &gt; \frac{a^2}{2} )</td>
<td>Favorable; Lomer Lock</td>
</tr>
<tr>
<td>16</td>
<td>( \frac{a}{2} [011] ) + ( \frac{a}{2} [011] ) = a[010]</td>
<td>( \frac{a^2}{2} + \frac{a^2}{2} = a^2 )</td>
<td>Unfavorable</td>
</tr>
</tbody>
</table>

Note: if Burgers vector \( b = \frac{a}{n} [hkl] \) then \( |b|^2 = \frac{a^2}{n^2} (h^2 + k^2 + l^2) \)

Lomer lock: The two planes intersect along [101]
Favorable reactions producing edge dislocation are Lomer locks. It is immobile because the plane on which it lies is not a close packed plane on
6. Let the slip plane be one of the 12 \{110\} planes. Dislocation & the Burgers vector must lie on slip plane. The Possible glide planes are \((1\bar{1}0), (10\bar{1}), (01\bar{1})\). The best way to check if dot product of Burgers vector & plane normal is equal to zero \((b.n = 0)\). If the slip plane were of type \{112\} the slip plane for this case would be \((\bar{2}11), (\bar{1}21) \& (1\bar{1}2)\). Try to find out the possible slip plane of type \{123\}.

If it were an edge dislocation one must specify its direction. It could glide only if it lies on one of the slip planes. For example an edge dislocation \(\frac{a}{2}[111]\) lying along \([\bar{2}11]\) could glide on \((01\bar{1})\). Find other possibilities.