Tutorial problems and questions

1. Derive the lower limit on the wavelength λ for spinodal decomposition including the effect of coherency elastic stress eects.

Answer

Let us consider the coherency strain associated with composition fluctuations: the change in lattice paramter with composition is given by $\frac{1}{a}\frac{da}{dx}$; thus, for a total composition change of Δx, molar volume V_m, Young's modulus E and Poisson's ratio ν, the coherency strain energy is given by

$$\Delta G_{coh} = \eta^2 (\Delta x)^2 V_m E' \tag{10}$$

where

$$\eta = \frac{1}{a}\frac{da}{dx} \tag{11}$$

and

$$E' = \frac{E}{1-\nu} \tag{12}$$

Adding the above energy ΔG_{coh} to Eq. 7, one obtains the equation

$$\begin{aligned} \Delta G &= \Delta G_{chem} + \Delta G_{grad} + \Delta G_{coh} \\ &= \left(\frac{d^2 G}{dx_B^2} + \frac{2\kappa}{\lambda^2} + 2\eta^2 E' V_m \right) \frac{(\Delta x)^2}{2} \end{aligned} \tag{13}$$

from which the condition for spinodal is derived as

$$-\frac{d^2 G}{dx_B^2} > \frac{2\kappa}{\lambda^2} + 2\eta^2 E' V_m \tag{14}$$