MODULE 10

Mass Transfer
What is Mass Transfer?

“Mass transfer specifically refers to the relative motion of species in a mixture due to concentration gradients.”

Analogy between Heat and Mass Transfer

Since the principles of mass transfer are very similar to those of heat transfer, the analogy between heat and mass transfer will be used throughout this module.
Mass transfer through Diffusion

**Conduction**

\[ q'' = -k \frac{dT}{dy} \left[ \frac{J}{m^2 \text{s}} \right] \]

*(Fourier’s law)*

**Mass Diffusion**

\[ j_A'' = -\rho D_{AB} \frac{d\xi_A}{dy} \left[ \frac{\text{kg}}{m^2 \text{s}} \right] \]

*(Fick’s law)*

\[ \rho \] is the density of the gas mixture

\[ D_{AB} \] is the diffusion coefficient

\[ \xi_A = \frac{\rho_A}{\rho} \] is the mass concentration of component A
Mass transfer through Diffusion

The sum of all diffusion fluxes must be zero: $\sum j_i'' = 0$

\[
\xi_A + \xi_B = 1
\]

\[
\frac{d}{dy} \xi_A = -\frac{d}{dy} \xi_B
\]

$D_{BA} = D_{AB} = D$
• Consider unsteady diffusive transfer through a layer

*Heat conduction, unsteady, semi-infinite plate*

\[
\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right)
\]

\[
\frac{\partial T}{\partial t} = \frac{k}{\rho c} \frac{\partial^2 T}{\partial x^2} = \alpha \frac{\partial^2 T}{\partial x^2}
\]

**Similarity transformation:**

**PDE** → **ODE**

\[
\frac{d^2 \theta}{dy^2} + 2\eta \frac{d \theta^*}{d\eta} = 0 \quad \eta = \frac{x}{\sqrt{4\alpha t}}
\]
Heat and Mass Diffusion: Analogy

Solution: \[ \frac{T - T_0}{T_u - T} = 1 - \text{erf} \left( \frac{x}{\sqrt{4\alpha t}} \right) \]

Temperature field

Heat flux \[ \phi \bigg|_{x=0} = -k \frac{dT}{dx} \bigg|_{x=0} = \frac{k}{\sqrt{\pi \alpha t}} (T_u - T_0) = \sqrt{\frac{k c \rho}{\pi t}} (T_u - T) \]
Heat and Mass Diffusion: Analogy

Diffusion of a gas component, which is brought in contact with another gas layer at time \( t=0 \)

Differential equation:

\[
\frac{\partial \rho_i}{\partial t} = \rho D \frac{\partial^2 \xi_i}{\partial x^2}
\]

\[
\frac{\partial \xi_i}{\partial t} = D \frac{\partial^2 \xi_i}{\partial x^2}
\]

Initial and boundary conditions:

\[
\xi_i(t = 0, x) = \xi_{i,o}
\]

\[
\xi_i(t > 0, x = 0) = \xi_{i,u}
\]

\[
\xi_i(t > 0, x \to \infty) = \xi_{i,o}
\]
Heat and Mass Diffusion: Analogy

Solution: \[
\frac{\xi_i - \xi_{i,o}}{\xi_u - \xi_{i,o}} = 1 - \text{erf}\left(\frac{x}{\sqrt{4Dt}}\right)
\]

Concentration field

Diffusive mass flux \[
j_i\big|_{x=0} = \frac{\rho D}{\sqrt{\pi Dt}} \left(\xi_{i,ph} - \xi_{i,o}\right)
\]
Diffusive mass transfer on a surface (Mass convection)

Fick's Law, diffusive mass flow rate:

\[ j''_A = -\rho D \frac{\partial \xi}{\partial y} \bigg|_{y=0} = -\rho D \frac{\xi_{\infty} - \xi_w}{L} \frac{\partial \xi^*}{\partial y^*} \bigg|_{y^*=0} \]

mass transfer coefficient \( h_{mass} \)

\[ j''_A = h_{mass} (\xi_w - \xi_{\infty}) \]

Dimensionless mass transfer number, the Sherwood number \( Sh \)

\[ \frac{h_{mass} L}{\rho D} = Sh = \frac{\partial \xi^*}{\partial y^*} \bigg|_{y^*=0} = f(Re, Sc) \]

\[ Sh = C \text{ Re}^m \text{ Sc}^n \]

Note: Compare with energy eqn. and Nusselt No.: The constants \( C \) and the exponents \( m \) and \( n \) of both relationships must be equal for comparable boundary conditions.
Diffusive mass transfer on a surface.

Dimensionless number to represent the relative magnitudes of heat and mass diffusion in the thermal and concentration boundary layers

**Lewis No.**

\[
Le = \frac{Sc}{Pr} = \frac{\alpha}{D} = \frac{\text{Thermal diffusivity}}{\text{Mass diffusivity}}
\]

**Analogy between heat and mass transfer**

Comparing the correlation for the heat and mass transfer

\[
\frac{Sh}{Nu} = \left(\frac{Sc}{Pr}\right)^n
\]

Hence,

\[
\frac{h_{\text{mass}}}{h / c_p} = \left(\frac{Sc}{Pr}\right)^{n-1}
\]

For gases, Pr \approx Sc, hence:

\[
\frac{h_{\text{mass}}}{h / c_p} = 1 \quad \text{Lewis relation}
\]