Energy Theorems and Weak form of the Governing Equation

Lecture 27
Smart and Micro Systems
Introduction to Theory of Elasticity

- There are 15 unknowns in Theory of Elasticity, which are:
  - 6 stress components: $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \tau_{xy}, \tau_{yz}, \tau_{zx}$
  - 6 strain components: $\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}, \gamma_{xy}, \gamma_{yz}, \gamma_{zx}$
  - 3 displacement components: $u, v, w$

- We need 15 equations to solve for these 15 unknowns.
These 15 equations are

3 Equations of Equilibrium

\[
\frac{2\sigma_{xx}}{\varepsilon_x} + \frac{\sigma_{xy}}{\varepsilon_y} + \frac{\sigma_{xz}}{\varepsilon_z} = \delta u = \frac{\sigma_{zz}}{\varepsilon_z} \rightarrow x \text{ direction}
\]

\[
\frac{\sigma_{yy}}{\varepsilon_y} + \frac{2\sigma_{yz}}{\varepsilon_z} = \delta v \rightarrow y \text{ direction}
\]

\[
\frac{\sigma_{zz}}{\varepsilon_z} + \frac{\sigma_{yx}}{\varepsilon_y} + \frac{2\sigma_{xy}}{\varepsilon_x} = \delta w \rightarrow z \text{ direction}
\]
6 Stress-Strain relations

Hooke's Law

\[
\{\sigma\} = \begin{bmatrix} C \end{bmatrix} \{\varepsilon\}
\]

\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz} \\
\tau_{xy} \\
\tau_{yz} \\
\tau_{zx}
\end{bmatrix} = 
\begin{bmatrix}
C_{11} & C_{12} & \cdots & C_{16} \\
C_{21} & C_{22} & \cdots & C_{26} \\
. & . & \ddots & . \\
. & . & . & C_{61} \\
. & . & . & . & C_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\varepsilon_{zz} \\
\gamma_{xy} \\
\gamma_{yz} \\
\gamma_{zx}
\end{bmatrix}
\]

\[
C_{12} = C_{21}
\]

Isotropic Stress

\[
\frac{E}{\nu}
\]

9
6-Strain displacement relations

\[ \varepsilon_{xx} = \frac{\partial u}{\partial x}, \quad \varepsilon_{yy} = \frac{\partial u}{\partial y}, \quad \varepsilon_{zz} = \frac{\partial w}{\partial z} \]

\[ \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}, \quad \gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \]
In addition, the obtained solution should satisfy both the essential and Natural boundary conditions

\[ \sigma = \sigma_0 \]

\[ u = u_0 \]

\[ t = t_1 \]
Work and Complimentary Work

Consider a Body subjected to a force system

\[ \hat{F} = F_x i + F_y j + F_z k \]

Body undergoes infinitesimal deformation

\[ d\hat{u} = dui + dvj + dwk \]

Work done is a dot product of force and deformation

\[ dW = \hat{F} \cdot d\hat{u} = F_x du + F_y dv + F_z dw \]

Total Work done

\[ W = \int_{u_1}^{u_2} \hat{F} \cdot d\hat{u} \]
Example

- Consider a 1-D system where the force in the x-direction is given by \( F_x = ku^n \)

- The work done will then be

\[
W = \int_0^u F_x \, du = \int_0^u ku^n \, du = \frac{ku^{n+1}}{n+1} = \frac{F_x u}{n+1}
\]  

(1)

- Consider the upper part of the curve

The work definition=

\[
W^* = \int_{F_1}^{F_2} \hat{u} \bullet d\hat{F}
\]
Consider the same 1-D system with \( F_x = ku^n \)

Deformation \( u=(1/k)F_x^{(1/n)} \)

Using in the second definition of work, we have

\[
W^* = \hat{u}dF_x = \int_0^F (1/k)F_x^{(1/n)}dF_x = \frac{F_x^{(1/n+1)}}{k(1/n+1)} = \frac{F_xu}{(1/n+1)} \tag{2}
\]

Eqn (1) and (2), although are definition of work, their values are different. Eqn(1) is called Work and Eqn (2) is called Complementary Work

Former is used in the Displacement based analysis and later in the Forced based analysis
Strain Energy

By Second law of Thermodynamics, we have total energy of the system $E$,

$$E = W_E + W_H$$

Assuming the process is adiabatic, $W_H=0$ and the mechanical work $W_E= U+T$, where $U$ is Strain Energy, and $T$ is Kinetic energy. If the loads are gradually applied, then $T=0$ and $E=W_E$

Let us now consider 1-D state of stress and derive the expression

For Strain energy in terms of stresses and strain. Let us consider a small volume of a body $dV$
The Change in strain energy due to differential stresses on the left and the right faces is given by

\[ dU = -\sigma_{xx} dy \, dz \, du + \left( \sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x} dx \right) dy \, dz \, d \left( u + \frac{\partial u}{\partial x} dx \right) + B_x dy \, dx \, dz \]

The above equation can be simplified as

\[ dU = \sigma_{xx} d \left( \frac{\partial u}{\partial x} \right) dx \, dy \, dz + du \, dx \, dy \, dz \left( \frac{\partial \sigma_{xx}}{\partial x} + B_x \right) \]

The last term in the brackets is the Equilibrium Equation.

Hence

\[ dU = \sigma_{xx} d \left( \frac{\partial u}{\partial x} \right) dx \, dy \, dz = \sigma_{xx} \, d \varepsilon_{xx} \, dV \]
We will now introduce the term **Strain Energy Density** as Strain Energy per unit Volume. Hence, the above expression will become

\[ dS_D = \sigma_{xx} d\varepsilon_{xx} \]

Integrating the above expression

\[ S_D = \int_0^{\varepsilon_{xx}} \sigma_{xx} d\varepsilon_{xx} \]

Hence, we have the

\[ U = \int_V S_D dV \]

The above definition is called Strain energy. This is the area under the stress strain curve.

Similarly, we can define complementary Strain energy, by taking the upper part of the curve, which is given by

\[ U^* = \int_V S^*_D dV, \quad S^*_D = \int_0^{\varepsilon_{xx}} \varepsilon_{xx} d\sigma_{xx} \]
Extension to 3-D state of stress

\[
S_D = \frac{1}{2E} \left( \sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2 \right) - \frac{\nu}{E} \left( \sigma_{xx} \sigma_{yy} + \sigma_{yy} \sigma_{zz} + \sigma_{zz} \sigma_{xx} \right)
\]

or

\[
S_D = \frac{1}{2(1+\nu)(1-2\nu)} \left[ (\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})^2
+ \frac{1}{2} \left( \varepsilon_{xy}^2 + \varepsilon_{yz}^2 + \varepsilon_{zx}^2 \right)
+ \frac{1}{2} \left( \gamma_{xy}^2 + \gamma_{yz}^2 + \gamma_{zx}^2 \right) \right]
\]
Energy Functional and Variational Operator
Energy Functional

- Formulation of finite elements require energy functional.
- Any DE that is positive definite and self-adjoint, will have a functional represented by $I(w)$, where $w$ is the dependent variable of DE.

$$I(w) = \int_{a}^{b} F\left(x, w, \frac{dw}{dx}, \frac{d^2w}{dx^2}\right) dx$$  \hspace{1cm} (28)

- “a” and “b” are two points on the boundary.
A Functional is said to be linear if it satisfies the following condition
\[ F(\alpha w + \beta v) = \alpha F(w) + \beta F(v) \]

Where \( \alpha \) and \( \beta \) are some scalars and \( w \) & \( v \) are the dependent variables.

A functional is called **quadratic functional**, when the following relation exist
\[ I(\alpha w) = \alpha^2 I(w) \]

If there are two functions \( p \) and \( q \), their inner product over the domain \( V \) can be defined as
\[ (p, q) = \int_V pq \, dV \]

Inner product itself can be thought of as a functional.
Any DE can be represented as

\[ Lu = f, \quad \text{over the domain } V \]

\[ u = u_0, \quad \text{over } \tau \]

\[ q = q_0, \quad \text{over } \tau_2 \]

Here, \( L \) is the Differential operator. If \( u_0 \) is zero, then we call the essential boundary conditions as \textit{homogenous}. For non zero \( u_0 \), the essential boundary condition becomes \textit{non-homogenous}. 
Properties of Differential operator

- There is always a functional for a given differential equation provided the differential operator $L$ satisfies the following conditions:
  - The differential operator $L$ requires to be **self-adjoint** or **symmetric**. That is, $(Lu,v) = (u,Lv)$, where $u$ and $v$ are any two functions that satisfy the same appropriate boundary conditions.
  - The differential operator $L$ requires to be **positive definite**. That is $(Lu,u) \geq 0$ for functions $u$ satisfying appropriate boundary conditions. The equality will hold only when $u=0$ everywhere in the domain.
  - One can actually construct the functional for ODE $Lu = f$ having a selfadjoint differential Operator $L$ with dependent variable $w$ as

$$I(w) = (Lw,w) - 2(w,f)$$
Derivation of Energy Functional for a Beam

- **Governing ODE for a Beam**

\[
E I \frac{d^4 w}{dx^4} + q = 0, \quad L = E I \frac{d^4}{dx^4}
\]

- **Integrating this by parts, we get**

\[
(Lw, w) = \int_0^l EI \frac{d^4 w}{dx^4} w \, dx
\]

\[
(Lw, w) = wEI \frac{d^3 w}{dx^3} \bigg|_{x=0}^{x=l} - \int_0^l EI \frac{d^3 w}{dx^3} \frac{dw}{dx} \, dx
\]

- The first term is the boundary term that has two parts, one is the displacement boundary condition and the second part \((EI \frac{d^3 w}{dx^3})\) is the force boundary condition and in the present case, it represents the shear force. For a right hand coordinate system, it is denoted by

- \(-V\).
\[(Lw, w) = -w(0)(V(0) + w(l)V(l)) - \int_0^l EI \frac{d^3w}{dx^3} \frac{dw}{dx} \, dx\]

- Integrating again the last part of the above equation by parts, we get

\[(Lw, w) = -w(0)V(0) + w(l)V(l) - \frac{dw}{dx} EI \frac{d^2w}{dx^2} \bigg|_{x=0}^{x=l} + \int_0^l EI \frac{d^2w}{dx^2} \frac{d^2w}{dx^2} \, dx\]

\[= -w(0)V(0) + w(l)V(l) - \phi(l)M(l) + \phi(0)M(0) + \int_0^l EI \left[ \frac{d^2w}{dx^2} \right]^2 \, dx\]

- Here, \( \phi \) is the rotation of cross section (also called the slope) and \( M \) is moment resultant.

- Fixed end condition where \( w = \frac{dw}{dx} = \phi = 0 \)

- Free boundary condition where \( V = -EI \frac{d^3w}{dx^3} = M = EI \frac{d^2w}{dx^2} = 0 \)

- Hinged boundary condition where \( w = M = EI \frac{d^2w}{dx^2} = 0 \)
All these boundary conditions are exactly satisfied for a beam with any boundary. Hence the energy functional becomes

\[
(L \ w \ , \ w) = 2 \cdot \frac{1}{2} \int_0^l EI \left[ \frac{d^2 w}{dx^2} \right]^2 dx
\]

and

\[
I (w) = 2 \left( \frac{1}{2} \int_0^l EI \left[ \frac{d^2 w}{dx^2} \right]^2 dx + \int_0^l q w \ dx \right)
\]

Variational Symbol \( \delta \)

This is an important symbol in variational method and functional analysis. It represents the variation of a dependent variable and it works like a differential operator in Calculus. It is normally used in the minimization Process required for the operation of energy theorems.
Energy Theorems
Principle of Virtual Work (PVW)

- This principle states that a continuous body is in equilibrium if and only if the virtual work done by all the external forces is equal to the virtual work done by internal forces when the body is subjected to infinitesimal virtual displacement. If $W_E$ is the work done by the external forces and $U$ is the internal energy (also called the strain energy), then the PVW can be mathematically represented as

$$\delta W_E = \delta U$$
PVW Derivation

\[ \mathbf{WF} = \int_{\mathcal{S}} \mathbf{h} \cdot \mathbf{n} \, d\mathbf{S} + \int_{\mathcal{V}} \mathbf{B} \cdot \mathbf{i} \, d\mathbf{V} \]

\[ \Delta i = \mathcal{S} \eta \gamma \]

\[ \Delta = \mathcal{S} \gamma \eta \mu \mathbf{h} \mathbf{d} \mathbf{S} + \int_{\mathcal{V}} \mathbf{B} \mathbf{i} \, d\mathbf{V} \]

\[ \mathbf{\nabla} = \frac{\partial \mathbf{c}}{\partial x} + \mathbf{e}_y + \mathbf{e}_z \]

\[ \mathbf{\nabla} \cdot \mathbf{u} = \int_{\mathcal{S}} \mathbf{n} \cdot \mathbf{u} \, d\mathbf{S} \]

\[ \delta \mathbf{W}_{\mathbf{E}} = \int \left( \mathbf{h} (\gamma \mathbf{B} \mathbf{d} \mathbf{S}) + \int_{\mathcal{V}} \mathbf{B} \mathbf{i} \, d\mathbf{V} \right) \]

\[ = \frac{\partial}{\partial x} (\mathbf{e}_y \mathbf{B} \mathbf{d} \mathbf{V}) + \int_{\mathcal{V}} \mathbf{B} \mathbf{i} \, d\mathbf{V} \]
Principle of Total Potential Energy (PMPE)

- This principle states that of all the displacement field which satisfies prescribed constraint conditions, the correct state is that which makes the total potential energy of the structure, minimum.

\[ \delta(U + V) = 0, \quad V = -W_E \]

- The above principle is the backbone for the finite element development. In addition, this principle can be used to derive the governing differential equation of the system, especially for static analysis, and also their associated boundary conditions. This aspect is demonstrated here by deriving the governing equation for a beam starting from the energy functional.
Derivation of Governing Equation for a beam Using PMPE

The energy functional in a beam is given by

$$ U = \frac{1}{2} \int_{0}^{L} EI \left( \frac{d^2 w}{dx^2} \right)^2 dx, \quad V = -\int_{0}^{L} q w dx $$

By PMPE, we have

$$ \delta \left( \frac{1}{2} \int_{0}^{L} EI \left( \frac{d^2 w}{dx^2} \right)^2 dx - \int_{0}^{L} q w dx \right) = 0 $$

Invoking the variational operation, we have

$$ \left( \int_{0}^{L} EI \left( \frac{d^2 w}{dx^2} \right) \delta \left( \frac{d^2 w}{dx^2} \right) dx - \int_{0}^{L} q \delta w dx \right) = 0 $$

$$ = \left( \int_{0}^{L} EI \left( \frac{d^2 w}{dx^2} \right) \left( \frac{d^2 (\delta w)}{dx^2} \right) dx - \int_{0}^{L} q \delta w dx \right) = 0 $$
Integrating the first term by parts two times and identifying the boundary terms as was done earlier, we get

$$
\delta w(0)V(0) - \delta w(L)V(L) - \delta \phi(L)M(L) - \delta \phi(0)M(0) + \int_0^L \left( EI \frac{d^4 w}{dx^4} + q \right) \delta wdx = 0
$$

Since the variation of displacements at the specified locations (boundary) is always zero, and is arbitrary, the only non zero term contained in the big bracket should be the governing differential equation of the beam.
Derivation of Castigliano’s Theorem From PMPE

Both Strain energy and applied forces are functions of generalized deformation or degrees of freedom $q_n$. Using PMPE, we have

$$\delta \left( U(q_n) - \sum_{n=1}^{N} P_n q_n \right) = 0$$

Taking the first variation, we have

$$\frac{\partial U}{\partial q_1} \delta q_1 + \frac{\partial U}{\partial q_2} \delta q_2 + \ldots + \frac{\partial U}{\partial q_n} \delta q_n - P_1 \delta q_1 - P_2 \delta q_2 - \ldots - P_n \delta q_n = 0$$
Grouping the terms together, we have

\[ \left( \frac{\partial U}{\partial q_1} - P_1 \right) \delta q_1 + \left( \frac{\partial U}{\partial q_2} - P_1 \right) \delta q_2 + \cdots + \left( \frac{\partial U}{\partial q_n} - P_n \right) \delta q_n = 0 \]

Since the all the are arbitrary, terms contained in each bracket should be equal to zero. Hence, we have

\[ \frac{\partial U}{\partial q_n} - P_n = 0, \quad \text{or} \quad \frac{\partial U}{\partial q_n} = P_n \]

The above statement is essentially the Castigliano’s theorem, which states that, if a reaction force at a generalized degree of freedom is required, then differentiating the strain energy with respect to the said degree of freedom will give the required reaction force.
Hamilton’s Principle

This principle is extensively used to derive the governing equation of motion for a structural system under dynamic loads. In fact, this principle can be thought of as PMPE for dynamic system. This principle was first formulated by an Irish mathematician and physicist, Sir William Hamilton. Similar to PMPE, HP is an integral statement of a dynamic system under equilibrium.

Consider a body of Mass $m$ acted by a 3-D force system

$$F(t) = F_x(t)i + F_y(t)j + F_z(t)k$$

The position vector of the mass w.r.t. global coordinates is

$$r(t) = xi + yj + zk$$

The force is further made of conservative force and non conservative Forces, that is

$$F(t) = F_c(t) + F_{nc}(t)$$
If this mass is given a small virtual displacement

$$\delta r(t) = \delta ui + \delta vj + \delta wk$$

This mass moves from position 1 at time to a position 2 at time according to Newton’s second law. Such a path is called the Newtonian Path.

The Equilibrium of the mass can be written as

$$F_x(t) - m\ddot{u}(t) = 0, \quad F_y(t) - m\ddot{v}(t) = 0, \quad F_z(t) - m\ddot{w}(t) = 0$$

Invoking the PVM, which essentially states that the total virtual work done by the infinitesimal virtual displacement should be zero, we have

$$\left[ F_x(t) - m\ddot{u}(t) \right] \delta u(t) + \left[ F_y(t) - m\ddot{v}(t) \right] \delta v(t) + \left[ F_z(t) - m\ddot{w}(t) \right] \delta w(t) = 0$$
Rearranging the terms and integrating the equation between the time and time , we have

\[
\int_{t_1}^{t_2} -m \left[ \ddot{u}(t) \delta u(t) + \dot{v}(t) \delta v(t) + \dot{w}(t) \delta w(t) \right] + \int_{t_1}^{t_2} \left[ F_{x}(t) \delta u(t) + F_{y}(t) \delta v(t) + F_{z}(t) \delta w(t) \right]
\]

Consider the first integral ( ), which can be written after integrating by parts as

\[
I_1 = -m \left[ \ddot{u}(t) \delta u(t) - m \dot{v}(t) \delta v(t) - m \dot{w}(t) \delta w(t) \right]_{t=t_1}^{t=t_2} + \int_{t_1}^{t_2} m \left[ \ddot{u} \delta \dot{u} + \dot{v} \delta \dot{v} + \dot{w} \delta \dot{w} \right] dt
\]

Recognizing that the virtual displacement must vanish at the beginning and end of this varied path, we can write the first integral as

\[
I_1 = \int_{t_1}^{t_2} m \left[ \ddot{u} \delta \dot{u} + \dot{v} \delta \dot{v} + \dot{w} \delta \dot{w} \right] dt = \int_{t_1}^{t_2} m \delta \left[ \dot{u}^2 + \dot{v}^2 + \dot{w}^2 \right] dt = \delta \int_{t_1}^{t_2} T dt
\]
Here, \( T \) represents the total kinetic energy of the system. Now, let us consider the second integral \( (I_2) \)

\[
I_2 = \int_{t_1}^{t_2} \left[ F_{cx}(t)\delta u(t) + F_{cy}(t)\delta v(t) + F_{cz}(t)\delta w(t) \right] dt + \\
\int_{t_1}^{t_2} \left[ F_{ncx}(t)\delta u(t) + F_{ncy}(t)\delta v(t) + F_{ncz}(t)\delta w(t) \right] dt
\]

The second integral in the above integral is nothing but the variation of the work done by the non conservative forces and can be written as

\[
\delta \int_{t_1}^{t_2} W_{nc} \, dt = \delta \int_{t_1}^{t_2} \left[ F_{ncx}(t)u(t) + F_{ncy}(t)v(t) + F_{ncz}(t)w(t) \right] dt
\]

The first integral in is the work done due to internal forces. From the Castigliano’s first theorem,

\[
F_{cx} = -\frac{\partial U}{\partial u}, \quad F_{cy} = -\frac{\partial U}{\partial v}, \quad F_{cz} = -\frac{\partial U}{\partial w}
\]
The negative sign is given to indicate that these forces resist the deformation.

The Integral $I_2$ now becomes

$$I_2 = -\int_{t_1}^{t_2} \left[ \frac{\partial U}{\partial u} \delta u(t) + \frac{\partial U}{\partial v} \delta v(t) + \frac{\partial U}{\partial w} \delta w(t) \right] dt + \delta \int_{t_1}^{t_2} W_{nc} dt$$

$$= \delta \int_{t_1}^{t_2} \left[ -U + W_{nc} \right] dt$$

Hence, the Hamilton’s Principle can be mathematically represented as

$$\delta \int_{t_1}^{t_2} \left[ T - U + W_{nc} \right] dt = 0$$
Weak Form of the Governing ODE
Strong Form

The set of governing PDE’s, with boundary conditions, is called the “strong form” of the problem.

Hence, our strong form is

\[ EI \frac{d^4 w}{dx^4} + q = 0, \quad w(0) = w_0, \quad \left. \frac{dw}{dx} \right|_{x=0} = \theta_0 \]

\[ EI \left. \frac{d^2 w}{dx^2} \right|_{x=L} = M_0, \quad EI \left. \frac{d^3 w}{dx^3} \right|_{x=L} = V_0, \]
Weak Form

We now reformulate the problem into the weak form.

The weak form is a *variational statement* of the problem in which we integrate against a *test function*. The choice of test function is up to us.

This has the effect of relaxing the problem; instead of finding an exact solution everywhere, we are finding a solution that satisfies the strong form on average over the domain.
Now we are looking for an approximate solution.

The residue becomes

\[ EI \frac{d^4 \bar{w}}{dx^4} + q = e_1 \]

- If we weight this with another function \( v \) and integrate over the domain of length \( L \), we get

\[
\int_0^L \left( EI \frac{d^4 \bar{w}}{dx^4} + q \right) v \ dx
\]

Why is it “weak”?

It is a weaker statement of the problem.

A solution of the strong form will also satisfy the weak form, but not vice versa.
Choosing the test function:

We can choose any \( v \) we want, so let's choose \( v \) such that it satisfies \textit{homogeneous} boundary conditions wherever the actual solution satisfies \textit{Dirichlet} boundary conditions. We’ll see why this helps us, and later will do it with more mathematical rigor.

So in our example, \( w(0)=w_0 \) so let \( v(0)=v_0 \),

\[
\left. \frac{dw}{dx} \right|_{x=0} = \theta_0 \quad \text{and so let} \quad \left. \frac{dv}{dx} \right|_{x=0} = \theta_0
\]
Integrating the above expression by parts twice, we will get the boundary terms, which are a combination of both essential and natural boundary condition along with the weak form of the equation. We get the following expression

$$v(0)\overline{V}(0) - v(l)\overline{V}(l) - \phi(l)\overline{M}(l) + \phi(0)\overline{M}(0) + \int_0^l \left[ EI \frac{d^2w}{dx^2} \frac{d^2v}{dx^2} + qv \right] dx$$

Where

$$\overline{V} = -EI d^3w / dx^3, \quad \overline{M} = EI d^2w / dx^2, \quad \phi = dw / dx$$

The above equation is the weak form of the differential equation as it requires reduced continuity requirement compared to the original differential equation. That is, original equation is fourth order equation and it requires functions that are third order continuous, while the weak order requires solutions that are just second order continuous. This aspect is exploited fully in the finite element method.
Summary

- In this Lecture, we studied the concept of work and energy and the associated energy theorems required for finite element formulation.
- We studied the PVW, PMPE, Castigliano’s Theorem and the Hamilton’s principle.
- We also studied the method to construct the weak form of the Governing differential Equation.
THANK YOU