Chapter 14: Vibration Generations Mechanisms: Self Excited Vibration

Introduction:

Self-excited systems begin to vibrate of their own accord spontaneously, the amplitude increasing until some nonlinear effect limits any further increase. The energy supplying these vibrations is obtained from a uniform source of power associated with the system which, due to some mechanism inherent in the system, gives rise to oscillating forces. The force acting on a vibrating object is usually external to the system and independent of the motion. However, there are systems in which the exciting force is a function of the motion variables (displacement, velocity or acceleration) and thus varies with the motion it produces (called coupling). Friction-induced vibration (in vehicle clutches and brakes, vehicle-bridge interaction) and flow-induced vibration (circular wood saws, CDs, DVDs, in machining, fluid-conveying pipelines) are examples of self-excited vibration. Self-excited vibration represents an important phenomenon in physical and mechanical systems. There exist different sources of self-excitation, which need different mathematical models describing important properties of the self-excitation. In most cases the self- excited vibration represents a danger for the save run of different systems and devices. Therefore, it is necessary to use means for vibration suppressing or, at least, for reducing the vibration intensity. The occurrence of self-excited vibration in a physical system is intimately associated with the stability of equilibrium positions of the system. If the system is disturbed from a position of equilibrium, forces generally appear which cause the system to move either toward the equilibrium position or away from it.

There are systems where the excitation comes from within, due to its own displacement. When a system is disturbed, the free vibration under certain conditions can cause an excitation that makes the system vibrate further. An increase in vibration thus can cause a further increase in the excitation and therefore the vibratory amplitude runs out of control. When the amplitudes become larger and larger the restoring force goes on increasing further. The increased restoring force fights the increasing amplitudes until a balance is reached amongst the excitation generated by the amplitude and the restoring force. Then the vibration remains sustained at this value, called a limit cycle vibration. The excitation gets removed when the amplitude of vibration becomes zero for some reason and then the system comes to rest. Once a vibration is initiated, an excitation comes into effect and the system runs off with increasing amplitudes until a limit cycle is reached. Such a vibratory motion is called Self Excited Vibration.

The function of many machines and devices, such as power picks, drilling sets, compacting equipment, hammers, etc. is based on self-excited vibrations produced by the action of compressed air. Such machines can hardly be expected to operate efficiently without a thorough understanding of the problems of existence and stability of periodic motions, gained by research studies. These problems arise as a result of the non-linear dependence of pneumatic forces on the motion of the mechanical system. Another strong non-linearity is added to the motion whenever impacts occur in the system.

Machining and measuring operations are invariably accompanied by vibration. To achieve higher accuracy and productivity vibration in machine tool must be controlled. For analysis of dynamic behavior of machine tool rigidity and stability are two important characteristics. Machine tool vibrations may be divided into 3 basic types as Free or transient vibration, Forced vibration and Self excited vibration (Machine tool chatter). Chatter is a self-excited vibration which is induced and maintained by forces generated by the cutting process. It effects surface finish, tool life, production rate and also produces noise. Chatter resistance of a machine tool is usually characterized by a maximum stable (i.e., not causing chatter vibration) depth of cut b_{lim}. Machine-tool chatter is essentially a problem of dynamic stability. A machine tool under vibration-free cutting conditions may be regarded as a dynamical system in steady-state motion. Systems of this kind may become dynamically unstable and break into oscillation around the steady motion. In self-excited vibration the alternating force that sustains the motion is created or controlled by the motion itself; when the motion stops, the alternating force disappears. In a forced vibration the sustaining alternating force exists independent of the motion and persists when the vibratory motion is stopped.

The vibration behaviour of a machine tool can be improved by a reduction of the intensity of the sources of vibration by enhancement of the effective static stiffness and damping. By appropriate choice of cutting regimes, tool design, and work-piece can be designed properly. Abatement of the sources is important mainly for forced vibrations. Stiffness and damping are important for both forced and self excited (chatter) vibrations. Both parameters, especially stiffness, are critical for accuracy of machine tools, stiffness by reducing structural deformations from the cutting forces, and damping by accelerating the decay of transient vibrations. Self-excited vibrations are characterized by the presence of a mechanism whereby a system will vibrate at its own natural or critical frequency, essentially independent of the frequency of any external stimulus. In mathematical terms, the motion is described by the unstable homogeneous solution to the homogeneous equations of motion. In contradistinction, in the case of "forced," or "resonant," vibrations, the frequency of the oscillation is dependent on (equal to, or a whole number ratio of) the frequency of a forcing function external to the vibrating system (e.g., shaft rotational speed in the case of rotating shafts). In mathematical terms, the forced vibration is the particular solution to the non-homogeneous equations of motion.

Self-excited vibrations pervade all areas of design and operations of physical systems where motion or time-variant parameters are involved—aeromechanical systems (flutter, aircraft flight dynamics), aerodynamics (separation, stall, musical wind instruments, diffuser and inlet chugging), aerothermodynamics (flame instability, combustor screech), mechanical systems (machine-tool chatter), and feedback networks (pneumatic, hydraulic, and electromechanical servomechanisms). (Ehrich, 1999)

The mechanisms of self-excitation in rotating machinery, which have been identified, can be categorized as follows:

- Whirling or Whipping
- Hysteretic whirl
- Fluid trapped in the rotor
- Dry friction whip
- Fluid bearing whip
- Seal and blade-tip-clearance effect in turbomachinery
- Propeller and turbomachinery whirl

- Parametric Instability
- Asymmetric shafting
- Pulsating torque
- Pulsating longitudinal loading
- Stick-Slip Rubs and Chatter

Self-excited oscillations are oscillations that are excited by the motion of the system. Self-excited oscillations are induced by nonlinear forms of damping where the damping term is negative over a certain range of motion. Mechanical system that exhibits negative damping, where the free oscillations amplitude grows, is shown in Fig.

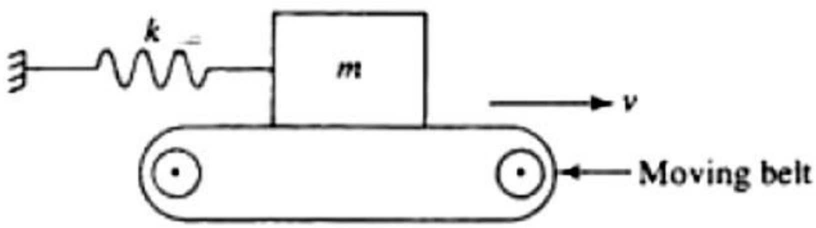
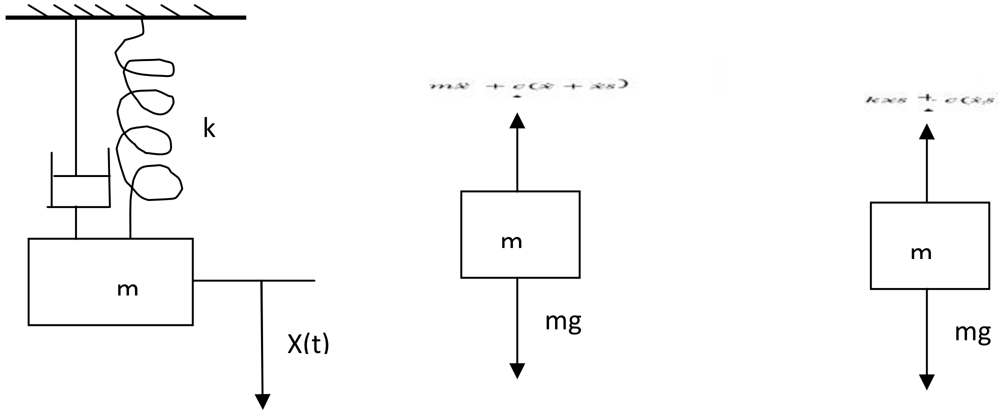


Fig. 4.4 System with negative damping

The instability of rotating shafts, the flutter of turbine blades, the flow induced vibration of pipes, and the automobile wheel shimmy and aerodynamically induced motion of bridges are typical examples of self-excited vibrations. A system is dynamically stable, if the motion (or displacement) converges or remains steady with time. On the other hand, if the amplitude of displacement increases continuously (diverges) with time it is said to be dynamically unstable. The motion diverges and the system becomes unstable if energy is fed into the system through self-excitation. For a damped free vibration system as shown in Fig. 4.4 the characteristics equations becomes;

$$m\ddot{x} + c\dot{x} + kx = 0$$



Substituting, $x(t) = a\,e^{\lambda t}$ in equation, get,

$$a(m\lambda^2 e^{\lambda t} + c\lambda e^{\lambda t} + k e^{\lambda t}) = 0$$

here $\quad a \neq 0$ and $e^{\lambda t} \neq 0$

hence, $\quad m\lambda^2 + c\lambda + k = 0$

$$\lambda^2 + \frac{c}{m}\lambda + \frac{k}{m} = 0$$

The solution of equation 1.8 yields as follows

$$\lambda_{1,2} = -\frac{c}{2m} \pm \frac{1}{2}\sqrt{\frac{c^2}{m^2} - 4\frac{k}{m}}$$

Since, the solution is assumed to be $x(t) = C\,e^{\lambda t}$ the motion will be diverging and aperiodic if the roots s_1 and s_2 are real and positive. This situation can *be* avoided if *c/m* and *k/m* are positive. The motion will also diverge if the roots λ_1 and λ_2 are complex conjugates with positive real parts. Thus, the fundamental criterion of stability in linear systems is that the roots of the characteristic equation have negative real parts, thereby producing decaying amplitudes. The whirling speed at onset of instability is the shaft's natural or critical frequency, irrespective of the shaft's rotational speed (rpm).The direction of whirl may be in the same rotational direction as the shaft rotation (forward whirl) or opposite to the direction of shaft rotation (backward whirl), depending on the direction of the destabilizing force.

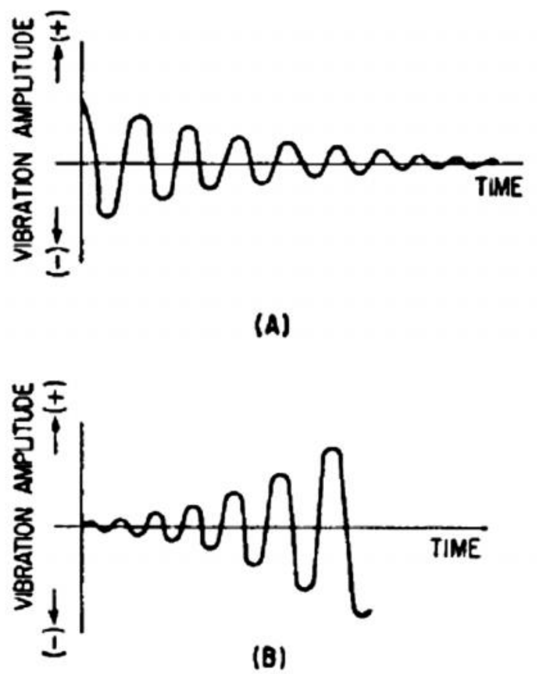
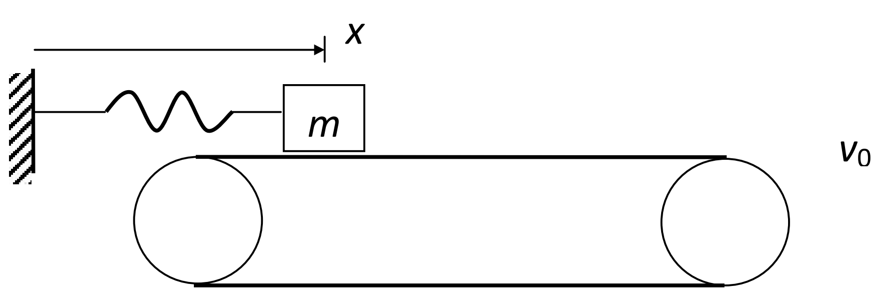
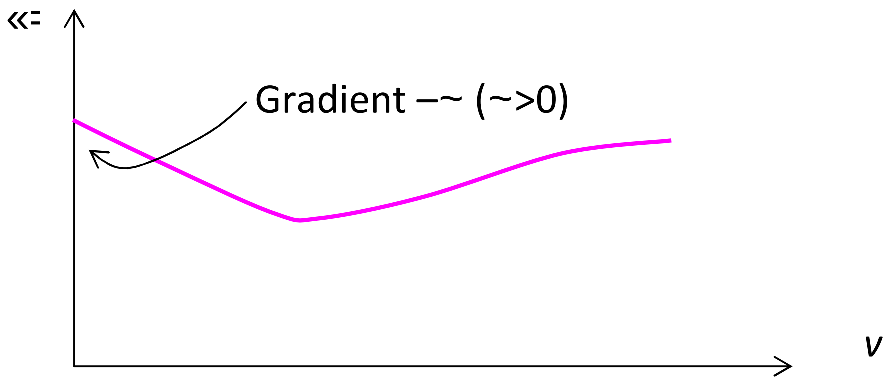


Fig. 4.5 (*A*) Illustration showing a decaying vibration (stable) corresponding to negative real parts of the complex roots (*B*) Increasing vibration corresponding to positive real parts of the complex roots (unstable)

Example 4.1: A mass supported by a spring is carried by a moving belt through friction.



The friction coefficient at the mass and belt interface is a function of the relative velocity between the mass and the belt as



The equation of motion of the mass is

$$m\ddot{x} + kx = -«mg = -«_0[1 - ~(\dot{x} - v_0)]mg = -«_0(1 + ~v_0)mg + «_0~mg\dot{x}$$

or

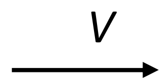
$$m\ddot{x} - «_0~mg\dot{x} + kx = -«_0(1 + ~v_0)mg$$

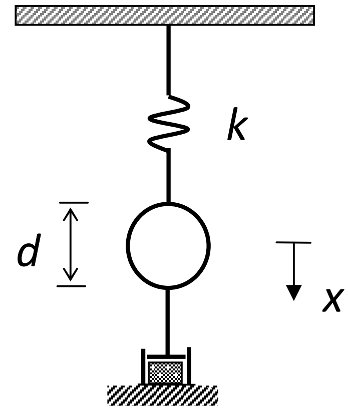
negative damping causing (initially) divergent vibration

As vibration grows, velocity $\dot{x}$ and hence relative velocity $\dot{x} - v_0$. This causes the friction coefficient to decrease (see m – v curve) and then vibration decreases. This cycle of increasing and decreasing vibration repeats itself forever (unless there is structural damping). The moving belt can sustain vibration — **self-excited** vibration.

Example 4.2: a solid oscillating in a fluid can interact with the fluid and produce interesting behaviour. The relative airflow against the oscillating solid modifies the *velocity vector* and thus the lift and drag force acting on the solid.

Consider a long cylinder of length l supported by a spring k and a damper c.



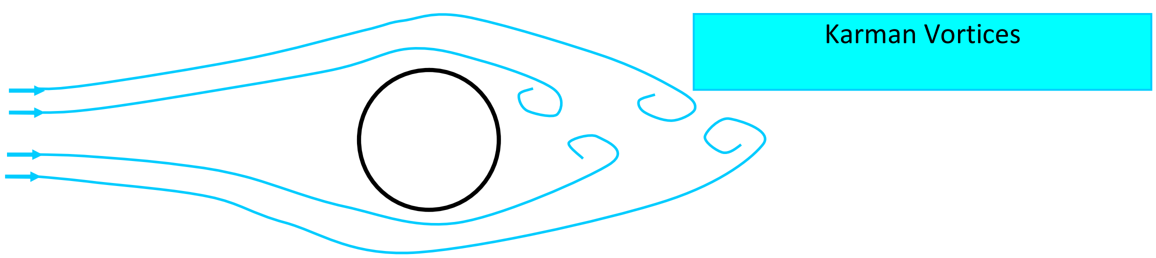


The equation of vertical motion of the cylinder in the flow is

$$m\ddot{x} + c\dot{x} + kx = f(t)$$

The lift force f is

$$f(t) = \frac{1}{2} \circ V^2 dlC_L$$



Due to vibration of the cylinder, the lift coefficient is no longer a constant because of shedding of vortices. It may be expressed as

$$C_L = C_{L0} \sin \mu\, t \qquad (C_{L0} \approx 1 \text{ for cylinder})$$

and

$$\mu = \frac{2\pi k_s V}{d}$$

where k_s is the **Strouhal number**. For $\mathrm{Re} = \frac{{}^{\circ} Vd}{\text{«}} > 1000$, $k_s \cong 0.21$.

So when

$$V = \frac{d}{2\pi k_s}\sqrt{\frac{k}{m}} = \frac{\mu_n d}{2\pi k_s}$$

the cylinder vibrates violently (in resonance) in the flow. When flow velocity becomes high enough, the flow becomes turbulent, and the lift force becomes random.

Flutter is a phenomenon of self-excited vibration.

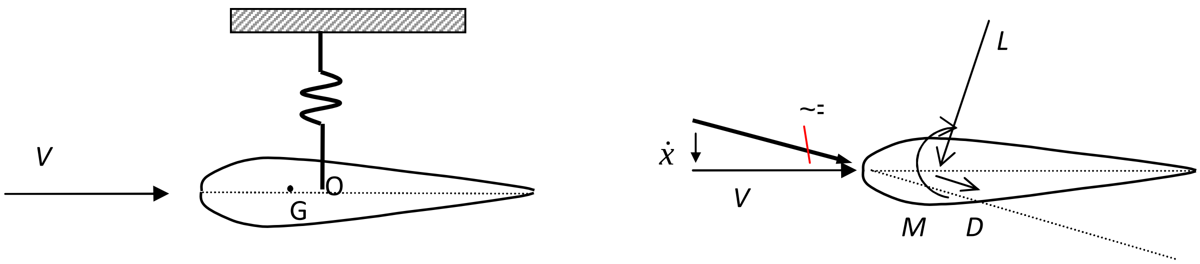


Fig. 4.5 A rigid wing attached to a rigid support through a spring

Vertical motion:

$$m\ddot{x} + kx = -L\cos\sim - D\sin\sim$$

where

$$L = \frac{{}^{\circ}}{2}(V^2 + \dot{x}^2)lcC_L = \frac{{}^{\circ}}{2}(V^2 + \dot{x}^2)lc\frac{\partial C_L}{\partial -}- \qquad D = \frac{{}^{\circ}}{2}(V^2 + \dot{x}^2)lc\frac{\partial C_D}{\partial -}-$$

and

$$- = -_0 - \sim \qquad \sim = \arctan(\frac{\dot{x}}{V})$$

Assume small displacement so that

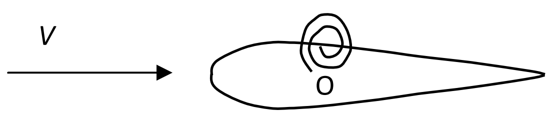
$$\sim\ \approx \tan\sim\ = \frac{\dot{x}}{V} \qquad \cos\sim\ \approx 1 \qquad V^2 + \dot{x}^2 \approx V^2$$

then

$$m\ddot{x} + kx = \frac{\circ}{2} V^2 lc[\frac{\mathrm{d}C_{\mathrm{L}}}{\mathrm{d}\text{¯}}(\text{¯}_0 - \sim)\cos\sim - \frac{\mathrm{d}C_{\mathrm{D}}}{\mathrm{d}\text{¯}}(\text{¯}_0 - \sim)\sin\sim]$$
$$= \frac{\circ}{2} V^2 lc[\frac{\mathrm{d}C_{\mathrm{L}}}{\mathrm{d}\text{¯}}\text{¯}_0 - (\frac{\mathrm{d}C_{\mathrm{L}}}{\mathrm{d}\text{¯}} + \frac{\mathrm{d}C_{\mathrm{D}}}{\mathrm{d}\text{¯}}\text{¯}_0)\sim]$$

finally

$$m\ddot{x} + \frac{\circ}{2} Vlc(\frac{\mathrm{d}C_{\mathrm{L}}}{\mathrm{d}\text{¯}} + \frac{\mathrm{d}C_{\mathrm{D}}}{\mathrm{d}\text{¯}}\text{¯}_0)\dot{x} + kx = \frac{\circ}{2} V^2 lc \frac{\mathrm{d}C_{\mathrm{L}}}{\mathrm{d}\text{¯}}\text{¯}_0$$



Torsional vibration (assuming centre of gravity and aerodynamic centre coincide):

$$(me^2 + I)\ddot{\text{¯}} + K\text{¯} = Le + M$$

where the pitching moment

$$L = \frac{\circ}{2} V^2 lc^2 C_{\mathrm{M}} = \frac{\circ}{2} V^2 lc \frac{\mathrm{d}C_{\mathrm{L}}}{\mathrm{d}\text{¯}}(\text{¯}_0 + \text{¯})$$
$$M = \frac{\circ}{2} V^2 lc^2 C_{\mathrm{M}} = \frac{\circ}{2} V^2 lc^2 \frac{\mathrm{d}C_{\mathrm{M}}}{\mathrm{d}\text{¯}}(\text{¯}_0 + \text{¯})$$

The above equation becomes

$$(me^2 + I)\ddot{\text{¯}} + [K - \frac{\circ}{2} V^2 lc(\frac{\mathrm{d}C_{\mathrm{L}}}{\mathrm{d}\text{¯}} e + \frac{\mathrm{d}C_{\mathrm{M}}}{\mathrm{d}\text{¯}} c)]\text{¯} = \frac{\circ}{2} V^2 lc(\frac{\mathrm{d}C_{\mathrm{L}}}{\mathrm{d}\text{¯}} e + \frac{\mathrm{d}C_{\mathrm{M}}}{\mathrm{d}\text{¯}} c)]\text{¯}_0$$

The static divergence speed is

$$V_{\mathrm{div}} = \sqrt{\frac{2K}{\circ\, lc\left(\frac{\mathrm{d}C_{\mathrm{L}}}{\mathrm{d}-} e + \frac{\mathrm{d}C_{\mathrm{M}}}{\mathrm{d}-} c\right)}}$$

for a symmetric aerofoil

$$V_{\mathrm{div}} = \sqrt{\frac{2K}{\circ\, lc \frac{\mathrm{d}C_{\mathrm{L}}}{\mathrm{d}-} e}}$$

The high the V_{div} , the greater speed capacity the aircraft has.