Module 2 - GEARS

Lecture – 12 HELICAL GEARS-PROBLEMS

Contents

12.1 Helical gears – Problem 1 Force analysis
12.2 Helical gears – Problem 2 Stress analysis
12.3 Helical gears –Problem 3 Reworking of gear dimensions of crossed helical gears
12.4 Helical gears – Problem 4 Design of double helical gears

12.1 HELICAL GEARS – PROBLEM 1

A 75 kW induction motor runs at 740 rpm in clock wise direction as shown in Fig.12.1. A 19 tooth helical pinion with 20° normal pressure angle, 10 mm normal module and a helix angle of 23° is keyed to the motor shaft. Draw a 3-dimensional sketch of the motor shaft and the pinion. Show the forces acting on the pinion and the bearing at A and B. The thrust should be taken out at A.

![Fig.12.1 Helical gear layout diagram](image)

Data: W=75kW, n_1=740rpm, Z_1 = 19, Z_2 = 38, \( \phi_n = 20^\circ \), \( \psi = 23^\circ \), \( m_n = 10 \) mm.
Question: Find reactions at A&B.

Solution: Transverse Pressure angle

\[ \tan \phi_n = \tan \phi \cos \psi \]

\[ \phi = \tan^{-1} \left( \frac{\tan \phi_n}{\cos \psi} \right) \]

\[ = \tan^{-1} \left( \frac{\tan 20^\circ}{\cos 23^\circ} \right) = 21.57^\circ \]

\[ m = \frac{m_n}{\cos \psi} = \frac{10}{\cos 23^\circ} = 10.864 \text{ mm} \]

Pitch diameter of the pinion:

\[ d_1 = mZ_1 = 10.864 \times 19 = 206.4 \text{ mm} \]

Pitch line velocity:

\[ V = \pi d_1 n_1 / 60 = \pi \times 206.4 \times 740 / 60000 = 8 \text{ m/s} \]

Tangential force on the pinion: \( F_t \)

\[ F_t = \frac{1000W}{V} = \frac{1000 \times 7.5}{8} = 9375 \text{ N} \]

\[ F_r = F_t \tan \phi = 9375 \tan 21.57^\circ = 3706 \text{ N} \]

\[ F_a = F_t \tan \psi = 9375 \tan 23^\circ = 3980 \text{ N} \]

\[ F_n = \frac{F_t}{\cos \phi_n \cos \psi} = \frac{9375}{\cos 20^\circ \cos 23^\circ} = 10838 \text{ N} \]

3 forces, \( F_r \), in the \(-y\) direction, \( F_a \) in the \(x\) direction, and \( F_t \) in the \(+z\) direction are acting at the pitch point \(c\) of the pinion as shown in the sketch.

Bearing at A is made to take the Axial reaction \( R_{A_X} = 3980 \text{ N} \)

Taking moments about the \(z\) axis

\[ -F_r (950) + F_a (206.4/2) + R_B^y (750) = 0, \text{ i.e.,} \]

\[ -3706 \times 950 + 3980 \times 103.2 + R_B^y (750) = 0 \]
Fig. 12.2 Reaction the shaft bearings due to forces at the pinion pitch point

\[ R_B^y = 4146.7 \text{ N} \uparrow \]
\[ \Sigma F^y = 0, \text{ from which } R_a^y = 440.7 \text{ N} \downarrow \]

Taking moment about y axis,

\[ R_B^z \times 750 - F_t \times 950 = 0 \]
\[ i.e, 750 R_B^z - 9375 \times 950 = 0 \rightarrow R_B^z = 11875 \text{ N} \]

\[ \Sigma F^z = 0, \text{ from which } R_A^z = 2500 \text{ N} \]

\[ T = F_t \times \frac{206.4}{2} = 9375 \times 103.2 = 96750 \text{ Nmm} = 96.75 \text{ Nm} \]

Fig. 12.3 Reaction on shaft bearings due to forces at the pinion pitch point from calculation

\[ R_A^x = 3980 \text{ N} \]
\[ R_A^y = 440.7 \text{ N} \]
\[ R_A^z = 2500 \text{ N} \]
\[ T = 96.75 \text{ Nm} \]
\[ R_B^z = 11875 \text{ N} \]
\[ R_B^y = 4146.7 \text{ N} \]
\[ F_r = 3706 \text{ N} \]
\[ F_t = 9375 \text{ N} \]
\[ F_a = 3980 \text{ N} \]
12.2 HELICAL GEARS - PROBLEM 2

A helical gear drive shown in Fig.12.4 transmits 20 kW power at 1440 rpm to a machine input shaft running at 360 rpm. The motor shaft pinion has 18 teeth, 20° normal pressure angle and a normal module of 4 mm and 30° right hand helix. Determine all dimensions of the gear and the pinion. \( b = 1.2 \ p_a \). Comment the chosen gears.

The pinion material is made of C45 steel with hardness 380 Bhn and tensile strength \( \sigma_{ut} = 1240 \) MPa. The gear is made of ductile iron grade 120/90/02 of hardness 331 Bhn and tensile strength \( \sigma_{ut} = 974 \) MPa. Both gears are hobbed, HT and OQ&T and ground.

**Given data:**

\( W = 20 \) kW, \( n_1 = 1440 \) rpm, \( Z_1 = 18 \), \( m_n = 4 \) mm, \( \theta_n = 20^\circ \), \( b = 1.2 \ p_a \), \( n_2 = 360 \) rpm, \( \Psi = 30^\circ \) RH Helix

**The following assumptions are made:**

(a) Tooth profiles are std. involutes.
(b) Gears mesh along their pitch circles
(c) All loads are transmitted at the pitch point and mid planes of the gears.
(d) All power losses are neglected.

![Fig.12.4 Helical gear layout diagram](image)
Solution:

\[ \tan \phi_n = \tan \phi \cdot \cos \psi \]

1. Transverse pressure angle \( \phi = \tan^{-1}(\tan \phi_n / \cos \psi) = \tan^{-1}(\tan 20^\circ / \cos 30^\circ) = 22.8^\circ \)

2. Transverse module: \( m = m_n / \cos \psi \)
   
i.e., \( m = 4 / \cos 30^\circ = 4.62 \text{ mm} \)

3. Pinion pitch dia.: \( d_1 = Z_1 \cdot m = 18 \times 4.62 = 83.2 \text{ mm} \)

4. Gear, no. of teeth: \( Z_2 = Z_1 \left( \frac{n_1}{n_2} \right) = 18 \left( \frac{1440}{360} \right) = 72 \)

5. Gear dia.: \( d_2 = Z_2 \cdot m = 72 \times 4.62 = 335.7 \text{ mm} \)

6. \( p = \pi m = \pi \times 4.62 = 14.51 \text{ mm} \)

7. \( p_a = p / \tan \psi = 14.51 / \tan 30^\circ = 25.13 \text{ mm} \)

8. \( b = 1.2 p_a = 1.2 \times 25.13 = 30.16 \text{ mm} \)

9. \( V = \pi d_1 \frac{n_1}{60000} = \pi \times 83.2 \times 1440 \div 60000 = 6.27 \text{ m/s} \)

10. \( d_{b1} = d_1 \cos \phi = 83.2 \cos 22.8^\circ = 76.7 \text{ mm} \)
    \( d_{b2} = d_2 \cos \phi = 335.7 \cos 22.8^\circ = 309.5 \text{ mm} \)

11. Addendum: \( h_a = m_n = 4.0 \text{ mm} \)

12. Dedendum: \( h_f = 1.25 m_n = 1.25 \times 4.0 = 5.00 \text{ mm} \)

13. \( F_t = 1000 \text{ W} / V = 1000 \times 20 / 6.27 = 3190 \text{ N} \)

14. \( F_r = F_t \tan \phi = 3190 \times \tan 22.8^\circ = 1341 \text{ N} \)

15. \( F_a = F_t \tan \psi = 3190 \times \tan 30^\circ = 1842 \text{ N} \)
Fig. 12.5 View of the forces acting on pitch cylinder of the helical drive pinion

**Bending stress on the pinion:**

\[
\sigma_{b1} = \frac{F_t}{b m_n J} K_{v} K_{o} (0.93 K_{m})
\]

\(J = 0.45\) for \(Z_{v1} = Z_1 / \cos^3 \psi = 18 / \cos^3 30^\circ = 27.7\) or 28 and \(\psi = 30^\circ\) from Fig.12.6

\(J\)-multiplication factor from Fig.12.7 = 1.013 from Fig.12.7

\(Z_{v2} = Z_2 / \cos^3 \psi = 72 / \cos^3 30^\circ = 110.9\) or 111 teeth mating gear.

\(J = 0.45 \times 1.013 = 0.4559\)

**HELICAL GEAR - TOOTH BENDING STRESS**

Fig.12.6 Geometry factor J for helical gear with \(\varphi_n = 20^\circ\) and mating with 75 tooth gear
**Fig.12.7 J- factor multiplier when the mating gear has tooth other than 75**

$$K_v = \left[ \frac{78 + (200V)^{0.5}}{78} \right]^{0.5} \approx \left[ \frac{78 + (200 \times 6.27)^{0.5}}{78} \right]^{0.5} = 1.21$$

$K_o = 1.25$ assuming uniform source of power and moderate shock from driven machinery, Table 12.1

$K_m = 1.5$ for $b=30.16$ mm & less rigid mountings, less accurate gears, contact across full face, Table 12.2

**HELICAL GEAR –TOOTH BENDING STRESS (AGMA)**

**Table 12.1 - Overload factor $K_o$**

<table>
<thead>
<tr>
<th>Source of power</th>
<th>Uniform</th>
<th>Moderate Shock</th>
<th>Heavy Shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>1.00</td>
<td>1.25</td>
<td>1.75</td>
</tr>
<tr>
<td>Light shock</td>
<td>1.25</td>
<td>1.50</td>
<td>2.00</td>
</tr>
<tr>
<td>Medium shock</td>
<td>1.50</td>
<td>1.75</td>
<td>2.25</td>
</tr>
</tbody>
</table>
### Table 12.2 Load distribution factor $K_m$

<table>
<thead>
<tr>
<th>Characteristics of Support</th>
<th>Face width (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accurate mountings, small bearing clearances, minimum deflection, precision gears</td>
<td>0 - 50</td>
</tr>
<tr>
<td></td>
<td>150</td>
</tr>
<tr>
<td></td>
<td>225</td>
</tr>
<tr>
<td></td>
<td>400 up</td>
</tr>
<tr>
<td>Less rigid mountings, less accurate gears, contact across the full face</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td>1.7</td>
</tr>
<tr>
<td>Accuracy and mounting such that less than full-face contact exists</td>
<td>Over 2.0</td>
</tr>
<tr>
<td></td>
<td>Over 2.0</td>
</tr>
<tr>
<td></td>
<td>Over 2.0</td>
</tr>
<tr>
<td></td>
<td>Over 2.0</td>
</tr>
</tbody>
</table>

**Bending stress in the pinion is**

$$\sigma_{b1} = \frac{F_t}{b m_n J_v K_v K_o (0.93 K_m)}$$

$$= \frac{3190}{30.2 \times 4.0 \times 0.4559 \times 1.21 \times 1.25 (0.93 \times 1.5)}$$

$$= 122.2 \text{ MPa}$$

- For the gear $J = 0.525$, for $Z_{v2} = 111$ & $\psi = 30^\circ$ from Fig. 12.6
- J-factor multiplier = 0.965 for $Z_{v1} = 28$ & $\psi = 30^\circ$ from Fig. 12.7

For the gear, $J = 0.525 \times 0.965 = 0.5066$

**Bending stress for the gear is**

$$\sigma_{b2} = \frac{F_t}{b m_n J_v K_v K_o (0.93 K_m)}$$

$$= \frac{3190}{30.2 \times 4.0 \times 0.5066 \times 1.21 \times 1.25 (0.93 \times 1.5)}$$

$$= 110 \text{ MPa}$$
Corrected bending fatigue strength of the pinion:

\[ \sigma_e = \sigma_{e'} k_L k_V k_s k_r k_T k_f k_m \]

\[ \sigma_{e'} = 0.5 \sigma_{ut} = 0.5 \times 1240 = 620 \text{ MPa} \]

- \( k_L = 1.0 \) for bending
- \( k_V = 1.0 \) for bending for \( m \leq 5 \) module,
- \( k_s = 0.645 \) for \( \sigma_{ut} = 1240 \text{ MPa} \) from Fig. 12.8
- \( k_r = 0.897 \) for 90% reliability from the Table 12.3
- \( k_T = 1.0 \) with Temp. < 120°C,
- \( k_T = 1.0 \)
- \( k_m = 1.33 \) for \( \sigma_{ut} = 1240 \text{ MPa} \) from the Fig.12.9

\[ \sigma_e = 620 \times 1 \times 1 \times 0.645 \times 1 \times 1 \times 0.897 \times 1.33 = 477 \text{ MPa} \]

**SPUR GEAR – PERMISSIBLE TOOTH BENDING STRESS (AGMA)**

![Fig. 12.8 Surface factor \( k_s \)]

**Table 12.3 Reliability factor \( k_r \)**

<table>
<thead>
<tr>
<th>Reliability factor ( R )</th>
<th>0.50</th>
<th>0.90</th>
<th>0.95</th>
<th>0.99</th>
<th>0.999</th>
<th>0.9999</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor ( k_r )</td>
<td>1.000</td>
<td>0.897</td>
<td>0.868</td>
<td>0.814</td>
<td>0.753</td>
<td>0.702</td>
</tr>
</tbody>
</table>
kf = fatigue stress concentration factor. As this factor is included in J factor, kf = 1 is taken.

km = Factor for miscellaneous effects. For idler gears subjected to two way bending, km = 1. For other gears subjected to one way bending, the value is taken from the Fig. 12.9. Use km = 1.33 for σut less than 1.4 GPa.

Corrected fatigue strength of the gear:

\[ \sigma_e = \sigma_e' k_L k_V k_s k_r k_T k_f k_m \]

\[ \sigma_e' = 0.35\sigma_{ut} = 0.35 \times 974 = 340.9 \text{ MPa} \]

\[ k_L = 1.0 \text{ for bending} \]

\[ k_V = 1.0 \text{ for bending for } m \leq 5 \text{ module,} \]

\[ k_s = 0.673 \text{ for } \sigma_{ut} = 974 \text{ MPa from Fig. 12.8} \]

\[ k_r = 0.897 \text{ for 90% reliability from the Table 12.3} \]

\[ k_T = 1.0 \text{ with Temp. } < 120^\circ \text{C}, \]

\[ k_f = 1.0 \]

\[ km = 1.33 \text{ for } \sigma_{ut} = 974 \text{ MPa from Fig. 12.9} \]

\[ \sigma_e = 340.9 \times 1 \times 1 \times 0.673 \times 0.897 \times 1 \times 1 \times 1.33 = 273.7 \text{ MPa} \]
Factor of safety for the pinion on bending:

\[ s_{b1} = \frac{\sigma_e}{\sigma_{b1}} = \frac{477}{122.2} = 3.9 \]

Factor of safety for the gear on bending:

\[ s_{b2} = \frac{\sigma_e}{\sigma_{b2}} = \frac{273.7}{110} = 2.49 \]

### Table 12.4 Guidance on the necessary safety factor

<table>
<thead>
<tr>
<th>Factor of safety against</th>
<th>Long life gearing</th>
<th>Finite life gearing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tooth breakage ( S_p \geq )</td>
<td>1.8 ( \cdots ) 4</td>
<td>1.5 ( \cdots ) 2</td>
</tr>
<tr>
<td>Pitting ( S_G )</td>
<td>1.3 ( \cdots ) 2.5</td>
<td>0.4 ( \cdots ) 1</td>
</tr>
<tr>
<td>Scoring ( S_F )</td>
<td>3 ( \cdots ) 5</td>
<td>3 ( \cdots ) 5</td>
</tr>
</tbody>
</table>

As per Niemen Table 12.4, the minimum factor of safety for infinite life in bending fatigue is 1.8. Since both the case the factor of safety exceeds this value, the gears will have infinite life.

**Ans:** The gear is weaker among the two in bending fatigue as its factor of safety is lower.

Contact stress on helical gears is given by:

\[
\sigma_H = C_p \sqrt{F_i \left( \frac{\cos \psi}{0.95CR} \right) K_v K_o (0.93K_m)}
\]

\[ C_p = 166 \text{ (MPa)}^{0.5} \] for steel pinion vs cast iron gear from Table 12.5.

\[
I = \frac{\sin \phi \cos \phi}{2} \frac{i}{i+1} = \frac{\sin 22.8^\circ \cos 22.8^\circ}{2} \frac{4}{4+1} = 0.143
\]
Table 12.5 Elastic coefficient $C_p$ for spur gears, in $\sqrt{\text{MPa}}$

<table>
<thead>
<tr>
<th>Pinion Material ((\mu = 0.3) in all cases)</th>
<th>Gear Material</th>
<th>Steel</th>
<th>Cast Iron</th>
<th>Al Bronze</th>
<th>Tin Bronze</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel, $E = 207 \text{ GPa}$</td>
<td>191</td>
<td>166</td>
<td>162</td>
<td>158</td>
<td></td>
</tr>
<tr>
<td>Cast Iron, $E = 131 \text{ GPa}$</td>
<td>166</td>
<td>149</td>
<td>149</td>
<td>145</td>
<td></td>
</tr>
<tr>
<td>Al Bronze, $E = 121 \text{ GPa}$</td>
<td>162</td>
<td>149</td>
<td>145</td>
<td>141</td>
<td></td>
</tr>
<tr>
<td>Tin Bronze, $E = 110 \text{ GPa}$</td>
<td>158</td>
<td>145</td>
<td>141</td>
<td>137</td>
<td></td>
</tr>
</tbody>
</table>

Contact ratio is given by:

$$CR_t = \left( \frac{\sqrt{(r_1 + a)^2 - r_{b1}^2} + \sqrt{(r_2 + a)^2 - r_{b2}^2} - (r_1 + r_2) \sin \phi}{\pi m \cos \phi} \right)$$

Using standard tooth system with $a = 1m_n$, $CR_t$:

$$CR_t = \left( \frac{\sqrt{(41.6 + 4.0)^2 - 38.35^2}}{\pi \times 4.62 \cos 22.8^\circ} \right) + \left( \frac{\sqrt{(167.85 + 4.0)^2 - 154.75^2}}{\pi \times 4.62 \cos 22.8^\circ} \right) - \left( \frac{(41.6 + 167.85) \sin 22.8^\circ}{\pi \times 4.62 \cos 22.8^\circ} \right) = 1.365$$

$K_v = 1.21$, $K_o = 1.25$, $K_m = 1.5$

$$\sigma_H = \sqrt{C_p \frac{F_t}{b d l} \left( \frac{\cos \psi}{0.95 \, CR} \right) K_v K_o (0.93 K_m)}$$

$$= 166 \sqrt{\frac{3190}{30.2 \times 83.2 \times 0.143 \left( \frac{\cos 30^\circ}{0.95 \times 1.365} \right)}} \times 1.21 \times 1.25 \times (0.93 \times 1.5)$$

$$= 587 \text{ MPa}$$
Surface fatigue strength of pinion is:

\[ \sigma_{sf} = \sigma_{sf}' K_L K_H K_R K_T \]

\[ \sigma_{sf}' = \text{surface fatigue strength of the material} \]

\[ = 2.8 \text{ (Bhn)} - 69 \quad \text{from Table 12.6} \]

\[ = 2.8 \times 380 - 69 \]

\[ = 995 \text{ MPa} \]

**HELICAL GEAR – SURFACE FATIGUE STRENGTH**

\[ K_L = 0.9 \quad \text{for } 10^8 \text{ cycles from Fig.12.10} \]

\[ K_H = 1.005 \quad \text{for } K = 380/331 = 1.14 \text{ & } i = 4 \text{ from Fig.12.11} \]

\[ K_R = 1.0 \quad \text{for 99% reliability from Table 12.7} \]

\[ K_T = 1.0 \quad \text{assuming temp. } < 120^0\text{C} \]

For the pinion material,

\[ \sigma_{sf1} = \sigma_{sf}' K_L K_H K_R K_T = 995 \times 0.9 \times 1 \times 1.005 \times 1 = 900 \text{ MPa} \]

**Table 12.6 Surface fatigue strength \( \sigma_{sf}' \) (MPa) for metallic spur gears**

*(10^7 cycle life, 99% reliability and temperature <120°C)*

<table>
<thead>
<tr>
<th>Material</th>
<th>( \sigma_{sf}' ) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>( 2.8\text{(Bhn)} - 69 \text{ MPa} )</td>
</tr>
<tr>
<td>Nodular Iron</td>
<td>( 0.95[2.8\text{(Bhn)} - 69] \text{ MPa} )</td>
</tr>
<tr>
<td>Cast Iron, grade 20</td>
<td>379</td>
</tr>
<tr>
<td>Cast Iron, grade 30</td>
<td>482</td>
</tr>
<tr>
<td>Cast Iron, grade 40</td>
<td>551</td>
</tr>
<tr>
<td>Tin Bronze, AGMA 2C (11% Sn)</td>
<td>207</td>
</tr>
<tr>
<td>Aluminium Bronze (ASTM B 148 – 52) (Alloy 9C – H.T.)</td>
<td>448</td>
</tr>
</tbody>
</table>
Fig. 12.10 Life Factor $K_L$

Fig. 12.11 Hardness ratio factor, $K_H$

$K = \text{Brinell hardness ratio of pinion and gear}, K_H = 1.0 \text{ for values of } K \text{ below 1.2}$

<table>
<thead>
<tr>
<th>Reliability (%)</th>
<th>$K_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>1.25</td>
</tr>
<tr>
<td>99</td>
<td>1.00</td>
</tr>
<tr>
<td>99.9</td>
<td>0.80</td>
</tr>
</tbody>
</table>

$K_T (\text{Temperature factor}) = 1 \text{ for } T \leq 120^\circ\text{C} \text{ based on Lubricant temperature.}$
Above 120°C, it is less than 1 to be taken from AGMA standards.

For gear: \(\sigma_{sf}' = 0.95[2.8(Bhn)-69] = 0.95[2.8 \times 331 - 69] = 815\) MPa

\(K_L = 0.9\) \ for 10^8 cycles from Fig.12.10

\(K_H = 1.005\) \ for \(K = 380/331 = 1.14\) \& \(i=4\) \ from Fig.12.11

\(K_R = 1.0\) \ for 99% reliability from Table 12.7

\(K_T = 1.0\) \ assuming temp. < 120°C

\(\sigma_{sf}^2 = \sigma_{sf}' K_L K_H K_R K_T = 815 \times 0.9 \times 1.005 \times 1 \times 1 = 795\) MPa

**HELICAL GEAR – ALLOWABLE SURFACE FATIGUE STRESS (AGMA)**

Allowable surface fatigue stress for design is given by

\[
[\sigma_H] = \frac{\sigma_{sf}}{s_H}
\]

Factor of safety \(s_H = 1.1\) to 1.5

Design equation is: \(\sigma_H \leq [\sigma_H]\)

Factor of safety for the pinion against pitting:

\(s_{H1} = \sigma_{sf1} / \sigma_H = 900 / 587 = 1.53\)

Factor of safety for gear against pitting:

\(s_{H2} = \sigma_{sf2} / \sigma_H = 795 / 587 = 1.35\)

In both case the factor of safety is more than 1.3 against pitting (Table 12.4) and the design is adequate. Among these, gear is slightly weaker than pinion and is likely to fail first.

The factor of safety in surface fatigue is proportional to square root of load and that in bending fatigue is directly proportional to load. Hence, the equivalent bending factor of safety for corresponding surface fatigue \((s_{H2})^2 = 1.35^2 = 1.81\) is compared with \((S_{b2})\) and is <2.49. So the gears are likely to fail due to surface fatigue and not due to bending fatigue.

------------------
12.3 HELICAL GEARS - PROBLEM 3

In a crossed helical gear drive, the shaft angle is 90° and the gear ratio is 1:1 with the helix angle $\psi_1 = \psi_2 = 45^\circ$. The normal module is 4 mm and the number of teeth in the gears are $Z_1 = Z_2 = 50$. The above identical gears are to be so changed that the driven gear has a pitch diameter of around 200 mm in the new arrangement.

**Data:** $\Sigma = \psi_1 + \psi_2 = 90^\circ; \psi_1 = \psi_2 = 45^\circ; m_n = 4 \text{ mm}$; $Z_1 = Z_2 = 50$ and $d_2 \approx 200 \text{ mm}$.

**Solution:**

$$d_1 = \frac{m_n Z_1}{\cos \psi_1} \quad \text{and} \quad d_2 = \frac{m_n Z_2}{\cos \psi_2}$$

Centre distance: $C = 0.5 (d_1 + d_2) = 0.5 m_n (Z_1 + Z_2) / \cos \psi$

$$= 0.5 \times 4 \times (2 \times 50) / \cos 45^\circ$$

$$= 282.84 \text{ mm}$$

Also

$$Z = \frac{d_2 \cos \psi_2}{m_n}$$

Therefore

$$C = \frac{m_n}{2} \times \frac{d_1 \cos \psi_1}{m_n} \times \left( \frac{\sin \psi_1 + \cos \psi_2}{\sin \psi_1 \cos \psi_2} \right) = \frac{d_2}{2} \left( 1 + \cot \psi_2 \right)$$

Or

$$\cot \psi_2 = \frac{2C}{d_2} - 1 = \frac{2 \times 282.84}{200} - 1 = 1.828, \quad \psi_2 = 28.675^\circ$$

Hence,

$$Z_2 = \frac{d_2 \cos \psi_2}{m_n} = \frac{200 \times \cos 28.675^\circ}{4} = 43.86$$
Taking an integral value for \( Z = 44 \) and substituting

\[
\frac{2C}{m_n Z} = \frac{\sin \psi_2 + \cos \psi_2}{\sin \psi_2 \cos \psi_2}
\]

or

\[
\frac{C}{m_n Z} = \frac{282.84}{4 \times 44} = \frac{\sin \psi_2 + \cos \psi_2}{2 \sin \psi_2 \cos \psi_2}
\]

Squaring:

\[
2.5826 = \frac{1 + \sin 2\psi_2}{\sin^2 2\psi_2}
\]

Solving we get \( \psi_2 = 28.9^\circ \)

Final values \( d_1 = 4 \times 44 / \sin 28.85^\circ = 364.75 \text{ mm} \)

\( d_2 = 4 \times 44 / \cos 28.85^\circ = 200.94 \text{ mm} \) which is near to 200 mm

\( C = 0.5 \times (d_1 + d_2) = (364.75 + 200.94) = 282.84 \text{ mm} \) equal to original centre distance.

-------------------

### 12.4 HELICAL GEARS - PROBLEM 4

In a turbine drive 300 kW power is transmitted using a pair of double helical gear. The pinion speed is 2950 rpm and that of the gear is about 816.5 rpm. There are no space constraints on the gear drive. Selecting suitable materials, design the pinion and the gear to last for \( 10^8 \) cycles.

Data: \( W = 300 \text{kW}; n_1 = 2950 \text{rpm}; n_2 = 816.5 \text{ rpm}; \) Life \( 10^8 \) cycles.
**Solution:** Since there are no constraints for the drive design, the number of teeth on the pinion is assumed as $Z_1 = 29$. Helix angle of $35^\circ$ and normal pressure angle $\phi_n = 20^\circ$ are taken for the gears and $b = 1.2 \ p_a$ is assumed.

$$\omega_1 = \frac{2\pi n_1}{60} = \frac{2\pi \times 2950}{60} = 308.77 \text{ rad/s}$$

$$i = \frac{n_1}{n_2} = \frac{2950}{816.5} = 3.612$$

$$Z_2 = i \ Z_1 = 3.612 \times 29 = 104.8 \text{ rounded to 105}$$

**Torque:**

$$T_1 = \frac{1000W}{\omega} = \frac{1000 \times 300}{308.77} = 971.6 \text{Nm}$$

The double helical gear is considered as two single helical gears coupled together sharing the torque equally. Torque on each half is $T_1 = 971.6/2 = 485.8 \text{ Nm}=485800 \text{ Nmm}$.

**The AGMA bending stress equation:**

$$\sigma_b = \frac{F_t}{b m \ J} \ K_p \ K_o \ (0.93K_m)$$

$p = \pi m = \pi m_n / \cos \psi = \pi m_n / \cos 35^\circ = 3.833m_n$

$p_a = p / \tan \psi$.

Assuming $b = 1.2p_a = 1.2 \ p / \tan \psi = 1.2 \times 3.833m_n / \tan 35^\circ = 6.569m_n$

$F_t = 2T_1 / d_1 = 2T_1 / mZ_1 = 2T_1 \cos \psi / m_n \ Z_1 = 2 \times 485800 \times \cos 35^\circ / m_n \times 29$

$$= 27444 / m_n \ \text{N}$$

J for the pinion with teeth $Z_{v1} = Z_1 / \cos^3 \psi = 29 / \cos^3 35^\circ = 82$, $\psi=35^\circ$ is: $J=0.47$ from Fig. 12.6

J multiplier for mating with $Z_{v2} = Z_2 / \cos^3 \psi = 105 / \cos^3 45^\circ = 297$, is $=1.015$ from Fig. 12.7

For pinion $J = 0.47 \times 1.015 = 0.4771$
HELICAL GEAR - TOOTH BENDING STRESS

Fig. 12.6 Geometry factor J for helical gear with \( \phi_n = 20^\circ \) and mating with 75 tooth gear.

Fig. 12.7 J-factor multiplier when the mating gear has tooth other than 75

J factor for the gear with teeth \( Z_{v2} = 297 \) and \( \psi = 35^\circ \) is \( J = 0.495 \) from Fig. 12.6.

J multiplier for mating with \( Z_{v1} = 82 \) is \( 1.003 \) from Fig. 12.7.

For gear \( J = 0.495 \times 1.003 = 0.4965 \)

\[ K_o = 1.25 \text{ assumed since } V \text{ is not known.} \]

\[ K_o = 1.25 \text{ assuming uniform source of power and moderate shock from driven machinery, Table 12.1.} \]
K_m = 1.3 expecting b=150 mm Accurate mountings, small bearing clearances, minimum
deflection, precision gears, Table 12.2.

**Helical Gear – Tooth Bending Stress (AGMA)**

### Table 12.1 - Overload factor K_o

<table>
<thead>
<tr>
<th>Source of power</th>
<th>Uniform</th>
<th>Moderate Shock</th>
<th>Heavy Shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>1.00</td>
<td>1.25</td>
<td>1.75</td>
</tr>
<tr>
<td>Light shock</td>
<td>1.25</td>
<td>1.50</td>
<td>2.00</td>
</tr>
<tr>
<td>Medium shock</td>
<td>1.50</td>
<td>1.75</td>
<td>2.25</td>
</tr>
</tbody>
</table>

### Table 12.2 Load distribution factor K_m

<table>
<thead>
<tr>
<th>Characteristics of Support</th>
<th>0 - 50</th>
<th>150</th>
<th>225</th>
<th>400 up</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accurate mountings, small bearing clearances, minimum deflection, precision gears</td>
<td>1.2</td>
<td>1.3</td>
<td>1.4</td>
<td>1.7</td>
</tr>
<tr>
<td>Less rigid mountings, less accurate gears, contact across the full face</td>
<td>1.5</td>
<td>1.6</td>
<td>1.7</td>
<td>2.0</td>
</tr>
<tr>
<td>Accuracy and mounting such that less than full-face contact exists</td>
<td>Over 2.0</td>
<td>Over 2.0</td>
<td>Over 2.0</td>
<td>Over 2.0</td>
</tr>
</tbody>
</table>

For the pinion:

\[
\sigma_{b1} = \frac{F_t}{b m_n j v} \cdot K_v \cdot K_o \cdot (0.93K_m)
\]

\[
= \frac{27444}{6.569 m_n^3 \times 0.4771} \times 1.25 \times 1.25 \times (0.93 \times 1.3) = \frac{16542}{m_n^3}
\]
For the gear:

\[ \sigma_{b2} = \frac{F_t}{b \cdot m_n} J K_v K_o (0.93 K_m) \]

\[ = \frac{27444}{6.569 m_n^3 \times 0.4965} \times 1.25 \times 1.25 \times (0.93 \times 1.3) \]

\[ = \frac{15895}{m_n^3} \]

The pinion material is made from C45 steel with hardness 380 Bhn and tensile strength \( \sigma_{ut} = 1240 \text{ MPa} \). The gear is made from ductile iron grade 120/90/02 of hardness 331 Bhn and tensile strength \( \sigma_{ut} = 974 \text{ MPa} \). Both gears are hobbed, HT and OQ&T and ground.

**Corrected bending fatigue strength of the pinion:**

\[ \sigma_e = \sigma_{e'} k_L k_v k_s k_r k_T k_f k_m \]

\[ \sigma_{e'} = 0.5 \sigma_{ut} = 0.5 \times 1240 = 620 \text{ MPa} \]

- \( k_L = 1.0 \) for bending
- \( k_V = 1.0 \) for bending for \( m \leq 5 \) module,
- \( k_s = 0.645 \) for \( \sigma_{ut} = 1240 \text{ MPa} \) from Fig.12.8
- \( k_r = 0.897 \) for 90% reliability from the Table 12.3
- \( k_T = 1.0 \) with Temp. \(< 120^\circ \text{C}\), \( k_f = 1.0 \)
- \( k_m = 1.33 \) for \( \sigma_{ut} = 1240 \text{ MPa} \) from the Fig.12.9

\[ \sigma_e = 620 \times 1 \times 1 \times 0.645 \times 1 \times 1 \times 0.897 \times 1.33 = 477 \text{ MPa} \]

**Corrected bending fatigue strength of the gear:**

\[ \sigma_e = \sigma_{e'} k_L k_v k_s k_r k_T k_f k_m \]

\[ \sigma_{e'} = 0.35 \sigma_{ut} = 0.35 \times 974 = 340.9 \text{ MPa} \]

- \( k_L = 1.0 \) for bending
- \( k_V = 1.0 \) for bending for \( m \leq 5 \) module,
- \( k_s = 0.673 \) for \( \sigma_{ut} = 974 \text{ MPa} \) from Fig.12.8
- \( k_r = 0.897 \) for 90% reliability from the Table 12.3
$k_T = 1.0$ with Temp. $< 120^\circ C$, $k_f = 1.0$

$k_m = 1.33$ for $\sigma_{ut} = 974$ MPa from Fig.12.95

$\sigma_e = 340.9 \times 1 \times 1 \times 0.673 \times 0.897 \times 1 \times 1 \times 1.33 = 273.7$ MPa

Permissible stress for the pinion in bending fatigue with factor of safety 1.6 for finite life gearing from Table 12.4:

$[\sigma_{b1}] = \frac{\sigma_e}{s_b} = 477/1.6 = 298$ MPa

Permissible stress for the pinion in bending fatigue with factor of safety 1.6,

$[\sigma_{b2}] = \frac{\sigma_e}{s_b} = 273.7/1.6 = 171$ MPa

For the pinion,

$\sigma_{b2} = \frac{16542}{m_n^3} = [\sigma]_2 = 298$

$m_n = 3.81$ mm

For the gear,

$\sigma_{b2} = \frac{15895}{m_n^3} = [\sigma]_2 = 171$

$m_n = 4.53$ mm

Take a standard value of 5 mm as given in Table 12.8.

<table>
<thead>
<tr>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.25</td>
<td>1.5</td>
<td>1.75</td>
<td>2.0</td>
<td>2.25</td>
<td>2.5</td>
<td>3</td>
</tr>
<tr>
<td>3.5</td>
<td>4</td>
<td>4.5</td>
<td>5</td>
<td>5.5</td>
<td>6</td>
<td>6.5</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>14</td>
<td>15</td>
<td>16</td>
<td>18</td>
<td>20</td>
<td>22</td>
<td>24</td>
</tr>
<tr>
<td>26</td>
<td>28</td>
<td>30</td>
<td>33</td>
<td>36</td>
<td>39</td>
<td>42</td>
</tr>
<tr>
<td>45</td>
<td>50</td>
<td>Further increase is in terms of 5 mm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
m = \frac{m_n}{\cos35^\circ} = \frac{5}{\cos35^\circ} = 6.104 \text{ mm}

d_1 = mZ_1 = 6.104 \times 29 = 177.01 \text{ mm}

d_2 = mZ_2 = 6.104 \times 105 = 640.92 \text{ mm}

p = 3.833m_n = 3.833 \times 5 = 19.165 \text{ mm}

p_a = \frac{p}{\tan \phi} = \frac{19.165}{\tan 35^\circ} = 27.37 \text{ mm}

b = 1.2p_a = 1.2 \times 27.37 = 32.84 \text{ mm}, \text{ take } 35 \text{ mm}

da_1 = d_1 + 2m_n = 177.01 + 2 \times 5 = 187.01 \text{ mm}

da_2 = d_2 + 2m_n = 640.92 + 2 \times 5 = 650.92 \text{ mm}

Transverse pressure angle: \tan \phi_n = \tan \phi \cos \psi

\phi = \tan^{-1} \left( \frac{\tan \phi_n}{\cos \psi} \right) = \tan^{-1} \left( \frac{\tan 20^\circ}{\cos 35^\circ} \right) = 23.96^\circ

d_{b1} = d_1 \cos \phi = 177.01 \cos 23.96^\circ = 161.76 \text{ mm}

d_{b2} = d_2 \cos \phi = 640.92 \cos 23.96^\circ = 585.69 \text{ mm}

C = 0.5(d_1+d_2) = 0.5(177.01+640.92) = 408.97 \text{ mm}

V = 0.5\omega d_1 = 0.5 \times 308.77 \times 177.01 \times 10^{-3} = 27.33 \text{ m/s}

F_t = 2T_1/d_1 = 2 \times 485800 / 177.01 = 5489 \text{ N}

Contact stress on the gears is given by:

\sigma_H = C_p \sqrt{\frac{F_t}{bdl} \left( \frac{\cos \psi}{0.95 \text{CR}} \right) K_v K_o (0.93K_m)}

C_p = 166 (\text{MPa})^{0.5} \text{ for steel pinion vs cast iron gear from Table 12.5.}

I = \frac{\sin \phi \cos \phi}{2} \frac{i}{i+1}

= \frac{\sin 23.96^\circ \cos 23.96^\circ}{2} \frac{3.621}{3.621+1} = 0.1454
Contact ratio is given by:

\[
CR_t = \frac{\sqrt{(r_1+a)^2 - r_b^2} + \sqrt{(r_2+a)^2 - r_b^2} - (r_1+r_2) \sin \phi}{\pi m \cos \phi}
\]

Using standard tooth system with \(a = 1m_n\), \(CR_t\):

\[
CR_t = \frac{\sqrt{(93.51^2 - 80.88^2)} + \sqrt{(325.46^2 - 292.85^2)}}{\pi x 6.104 \cos 23.96^\circ} - \frac{408.97 \sin 23.96^\circ}{\pi x 6.104 \cos 23.96^\circ} = 1.3044
\]

\[
K_v = \left[ \frac{78 + (200V)^{0.5}}{78} \right]^{0.5} = \left[ \frac{78 + (200 \times 27.33)^{0.5}}{78} \right]^{0.5} = 1.396
\]

\(K_v = 1.396, \ K_o = 1.25, \ K_m = 1\).

\[
\sigma_H = C_p \sqrt{\frac{F_t (\cos \psi)}{bdl (0.95CR)}} K_v K_o (0.93K_m)
\]

\[
= 166 \sqrt{\frac{5489}{35 \times 177.01 \times 0.1454 \times (0.95 \times 1.3044)}} \times 1.396 \times 1.25 (0.93 \times 1.5)
\]

\[
= 519.8 \text{ MPa}
\]
Contact fatigue strength of pinion is:

$$\sigma_{sf} = \sigma_{sf}' K_L K_H K_R K_T$$

$$\sigma_{sf}' = \text{surface fatigue strength of the material} = 2.8 \text{ (Bhn)} - 69 \quad \text{From Table 12.6}$$

$$= 2.8 \times 380 - 69$$

$$= 995 \text{ MPa}$$

**HELICAL GEAR – SURFACE FATIGUE STRENGTH**

Table 12.6 Surface fatigue strength $\sigma_{sf}'$ (MPa), for metallic spur gears, (10$^7$ cycle life 99% reliability and temperature < 120° C)

<table>
<thead>
<tr>
<th>Material</th>
<th>$\sigma_{sf}'$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>2.8 (Bhn) - 69 MPa</td>
</tr>
<tr>
<td>Nodular Iron</td>
<td>0.95 [ 2.8 (Bhn) - 69 MPa ]</td>
</tr>
<tr>
<td>Cast Iron, grade 20</td>
<td>379</td>
</tr>
<tr>
<td>Cast Iron, grade 30</td>
<td>482</td>
</tr>
<tr>
<td>Cast Iron, grade 40</td>
<td>551</td>
</tr>
<tr>
<td>Tin Bronze, AGMA 2C (11% Sn)</td>
<td>207</td>
</tr>
<tr>
<td>Aluminium Bronze (ASTM B 148 = 52) (Alloy 9C - H.T.)</td>
<td>448</td>
</tr>
</tbody>
</table>

$$K_L = 0.9 \quad \text{for 10}^8 \text{ cycles from Fig.12.10}$$

$$K_H = 1.005 \quad \text{for } K = 380/331 = 1.14 \& i=4 \text{ from Fig.12.11}$$

$$K_R = 1.0 \quad \text{for 99% reliability from Table 12.7}$$

$$K_T = 1.0 \quad \text{assuming temp. < 120° C}$$

$$\sigma_{sf} = \sigma_{sf}' K_L K_H K_R K_T = 995 \times 0.9 \times 1.005 \times 1 \times 1$$

$$= 900 \text{ MPa}$$
**Fig. 12.10** Life Factor $K_L$

**Fig. 12.11** Hardness ratio factor, $K_H$. $K = Brinell hardness ratio of pinion and gear, $K_H = 1.0$ for values of $K$ below 1.2

**Table 12.7** Reliability factor $K_R$

<table>
<thead>
<tr>
<th>Reliability (%)</th>
<th>$K_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>1.25</td>
</tr>
<tr>
<td>99</td>
<td>1.00</td>
</tr>
<tr>
<td>99.9</td>
<td>0.80</td>
</tr>
</tbody>
</table>
KT = temperature factor,

= 1 for T \leq 120^\circ C based on Lubricant temperature.

Above 120^\circ C, it is less than 1 to be taken from AGMA standards.

HELICAL GEAR – ALLOWABLE SURFACE FATIGUE STRESS (AGMA)

Allowable surface fatigue stress for design is given by

$$[\sigma_H] = \sigma_{sf} / s_H$$

Design equation is: $$\sigma_H \leq [\sigma_H]$$

For gear: $$\sigma_{sf}' = 0.95[2.8(\text{Bhn})-69] = 0.95[2.8\times331-69] = 815 \text{ MPa}$$

$$K_L = 0.97$$ for 2.5x10^7 cycles from Fig.12.10

$$K_H = 1.005$$ for K = 380/331 = 1.14 & i=4 from Fig.12.11

$$K_R = 1.0$$ for 99\% reliability from Table 12.10

$$K_T = 1.0$$ assuming temp. < 120^\circ C

$$\sigma_{sf} = \sigma_{sf}' K_L K_H K_R K_T = 815 \times 0.97 \times 1.005 \times 1 \times 1 = 795 \text{ MPa}$$

Factor of safety for the pinion against pitting:

$$s_{H1} = \sigma_{sf} / \sigma_H = 900 / 519.8 = 1.73$$

Factor of safety for gear against pitting:

$$s_{H2} = \sigma_{sf} / \sigma_H = 795 / 519.8 = 1.53$$

Table 12.4 Guidance on the necessary factor of safety

<table>
<thead>
<tr>
<th>Factor of safety against</th>
<th>Long life gearing</th>
<th>Finite life gearing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tooth breakage $S_B \geq$</td>
<td>1.8 ... 4</td>
<td>1.5 ... 2</td>
</tr>
<tr>
<td>Pitting $S_C$</td>
<td>1.3 ... 2.5</td>
<td>0.4 ... 1</td>
</tr>
<tr>
<td>Scoring $S_F$</td>
<td>3 ... 5</td>
<td>3 ... 5</td>
</tr>
</tbody>
</table>
As per the Niemen guidance for factor of safety given in Table 12.4, for long life gearing the factor of safety has to be more than 1.3 in pitting. Since for both gear and pinion the factor of safeties is more than 1.3, the design is adequate.

The final specifications of the pinion and gear are:
20° pressure angle involute teeth with helix angle of 35°, \( h_a = 1m_n \), \( h_f = 1.25m_n \)

<table>
<thead>
<tr>
<th></th>
<th>( Z )</th>
<th>( m_n ) mm</th>
<th>( d ) mm</th>
<th>( d_a ) mm</th>
<th>( d_b ) mm</th>
<th>( d_r ) mm</th>
<th>( m_t ) mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pinion</td>
<td>29</td>
<td>5</td>
<td>177.01</td>
<td>187.01</td>
<td>161.76</td>
<td>164.51</td>
<td>6.104</td>
</tr>
<tr>
<td>Gear</td>
<td>105</td>
<td>5</td>
<td>640.92</td>
<td>650.92</td>
<td>585.69</td>
<td>628.42</td>
<td>6.104</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>( \Phi_n )</th>
<th>( \varphi_t )</th>
<th>( b ) mm</th>
<th>( p_t ) mm</th>
<th>( p_a ) mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pinion</td>
<td>20°</td>
<td>23.96°</td>
<td>35</td>
<td>19.165</td>
<td>27.37</td>
</tr>
<tr>
<td>Gear</td>
<td>20°</td>
<td>23.96°</td>
<td>35</td>
<td>19.165</td>
<td>27.37</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>( CR_t )</th>
<th>( CR_a )</th>
<th>( CR )</th>
<th>( FS_{s_b} )</th>
<th>( FS_{s_H} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pinion</td>
<td>1.3044</td>
<td>1.2787</td>
<td>2.583</td>
<td>1.99</td>
<td>1.73</td>
</tr>
<tr>
<td>Gear</td>
<td>1.3044</td>
<td>1.2787</td>
<td>2.583</td>
<td>1.89</td>
<td>1.53</td>
</tr>
</tbody>
</table>
Fig. 12.12 Dimensional sketch of the pinion and the gear. (All dimensions are in mm and not to scale.)

Fig. 12.13 Assembly drawing of the double helical gearbox