Lecture 6.3
Transonic Flow Through a NACA0012 Airfoil Cascade
Problem Statement

• Liu considered transonic flow through a NACA0012 airfoil cascade which involves periodic boundaries. The cascade has a pitch to chord ratio of unity. The ratio of stagnation pressure at the inlet to static pressure at the exit is taken as 1.3871. The corresponding isentropic exit Mach number is 0.7. The flow medium is air with ratio of specific heats ($\gamma$) and characteristic gas constant ($R$) as 1.4 and 287 $N-m/Kg-K$, respectively.

• Liu (1991) obtained isentropic Mach number contours for the above problem by using a blend of second and fourth order artificial dissipation terms in conjunction with central difference approximation for modelling the inviscid terms.

• Solve the above problem and compare the isentropic Mach number contours with those reported by Liu (1991).
Mathematical Model

• As the problem of transonic flow through the cascade is similar to internal flow through the channel with a bump (Lecture 6.2), the governing equations and boundary conditions are also similar. However, in the present case the boundaries are periodic.

Geometry Model

• The physical model consists of the geometry with a single NACA0012 airfoil and a pair of periodic lines, one above the airfoil and the other below it. The cascade has a pitch to chord ratio of unity. The ratio of stagnation pressure at the inlet to static pressure at the exit is taken as 1.3871. The distance between the periodic boundary lines is equal to the pitch of the cascade, as shown in Fig. 6.3.1.
Fig. 6.3.1 Geometry model with a NACA0012 airfoil
Mesh Generation

- Three-block hybrid grid system consisting of unstructured and structured grids is shown in Fig. 6.3.2. The number of cells, nodes and edges the grid system are 9253, 8885 and 18133 respectively.

Fig. 6.3.2 A three block hybrid grid
Discretization

- Finite volume method as described in Module 4 has been applied to the multi-block hybrid grid system. Inviscid flux quantities are estimated by the upwind biased Advection Upstream Splitting Method (AUSM).

- The other details are similar to the case of “Transonic Flow Over Circular Bump” (Lecture 6.2).
Verification

- Figures 6.3.3 (a) and (b) show the isentropic Mach number contours obtained from the present study and that reported by Liu (1991).

- Figure 6.3.4 shows the isentropic Mach number variation over the surface of the airfoil as a function of the non-dimensional axial distance from the leading edge.

- The surface isentropic Mach number increases and reaches a maximum value of 1.32 at 71.7% chord, against the value of 1.3 predicted at the same location by Liu (1991).
Fig. 6.3.3 Isentropic Mach number contours of transonic flow through NACA0012 airfoil cascade, $M_{is,exit} = 0.7$ (a) Present prediction (b) Liu (1991)
Fig. 6.3.4 Variation of isentropic Mach number over NACA0012 airfoil surface in a cascade, $M_{is,exit} = 0.7$
Prediction of Shock

• It can be seen that while the location of normal shock wave is in close agreement, the shock predicted by the present analysis is much sharper than Liu (1991).

• The fluid decelerates to Mach number 0.83 through a normal shock wave. It is seen that the shock is captured well with a single interior point.

• This shows that the numerical diffusion involved with the present implementation of AUSM is sufficiently less for predicting sharp shock waves occurring in transonic flow through cascades.
Exercise Problems

• For the NACA0012 airfoil cascade discussed in this lecture, obtain isentropic Mach number contours for the exit Mach number cases of $M = 0.5, 0.3, 0.2, 0.1, 0.05$.

• Consider inviscid flow through a linear cascade of a NACA 65 12a$_{10}$ aerofoil. Compute the isentropic Mach number distribution on the suction and pressure surfaces. Take the following conditions and compare the results given in Fig. 6.3.5.

Space chord ratio ($s/ch$) = 1, Stagger angle $\beta = 31^\circ$
Inlet stagnation pressure $p_0 = 133$ kPa
Inlet stagnation temperature $T_0 = 300$ K
Inlet flow angle $\alpha_1 = 45^\circ$
Exit static pressure = 117.3 kPa.
Fig. 6.3.5: Mach number contours for NACA 65 12a₁₀ aerofoil cascade
END OF LECTURE 6.3