Lecture 1.1
Introduction to Fluid Dynamics
Introduction

- A thorough study of the laws of fluid mechanics is necessary to understand the fluid motion within the turbomachinery components.

- In this introductory lecture, some relevant concepts of fluid mechanics are revised, before entering the realms of computational fluid dynamics.

- Fluid mechanics has two parts:
  - Kinematics
  - Dynamics
• The kinematics is about describing the fluid motion without taking into account the forces that cause the motion.

• In fluid dynamics, the forces are considered for the analysis of fluid motion. Governing equations are derived by considering the balance of these forces.

• Computational fluid dynamics is a subject where the fluid motion is studied by solving the governing differential equations using numerical methods.
Microscopic versus Macroscopic Approaches

• Microscopic and macroscopic are the two approaches using which the fluid flows are analyzed and the flow properties like density, velocity, etc. are determined at each point of the flow domain.

• In the microscopic approach, the molecular mean free path is far larger than the characteristic dimension.

• In the macroscopic approach, the fluid is considered as a continuum, that is, the fluid in a region is assumed to occupy every geometric point of that region. The point here means a very small volume in the fluid region whose size is zero in the limit.
• Flow studies in machines working at very low pressures such as a turbo molecular pump require microscopic view. The flows in turbomachines, which are of interest in this course, are all in the continuum and hence the macroscopic approach is appropriate.

• The course objectives are, therefore, to know
  – The formulation of equations governing the fluid flow in turbomachines
  – Physical modeling of the flow domains in a turbomachine
  – The methodology to solve the governing equations and
  – The analysis of the numerical results.

• The results obtained for a turbomachine model can be extended to another geometrically similar prototype machine when the kinematic and dynamic similarity rules are respected between the model and the prototype.
Eulerian and Lagrangian Approaches

• In the Eulerian approach, the flow properties are described as functions of space and time, like in cinematography.

• The complete state of motion is described by a succession of instantaneous states of flow. Thus if $\phi$ is a flow property, then $\phi(x, y, z, t)$ or $\phi(\vec{r}, t)$ is the value of $\phi$ at $\vec{r}$, the position occupied by the particle and at the instant of time $t$.

• At a later time the fluid particle occupying the position $(x, y, z)$ will be different. If the flow is steady then “$\phi$” is independent of $t$ and is a function of $x$, $y$ and $z$ alone.
• In the Lagrange method, fluid particles are identified by their position \( \vec{r}_o \) (or \( x_o, y_o, z_o \)) at any time \( t_o \) and describe their trajectories. The trajectory of the line along which a particle moves is called a path line.

• The instantaneous velocity and acceleration \( \vec{v} \) and \( \vec{a} \) can be expressed as

\[
\vec{v} = \left( \frac{\partial \vec{r}}{\partial t} \right)_{\vec{r}_o=const.} = \vec{v}(\vec{r}_o,t) \quad \text{and} \quad \vec{a} = \left( \frac{\partial \vec{v}}{\partial t} \right)_{\vec{r}_o=const} = \left( \frac{\partial^2 \vec{r}}{\partial t^2} \right)_{\vec{r}_o=const} = \vec{a}(\vec{r}_o,t)
\]

• Except for multi-phase flows such as gas-solid flows, where one or both of the phases may be considered for the Lagrangian description, the Eulerian approach is more appropriate for describing the fluid motion. The governing equations in this course are therefore written using only the Eulerian approach.
Velocity of a Fluid Particle

• In the Eulerian description, the particles which occupy a chosen point are going to change from time $t$ to $t + \delta t$.

• Let $P$ be the point whose position vector is $\vec{r}$ at time $t$ with respect to some fixed point ‘O’. That is,
  $$\overrightarrow{OP} = \vec{r}$$

• After a time $\delta t$ the same particle is moved to $P'$, as shown in the Fig. 1.1.1, having a distance $\delta r$ from $P$. Therefore, the particle velocity $\vec{v}$ at $P$ is given by
  $$\vec{v} = \lim_{\delta t \to 0} \frac{\delta r}{\delta t} = \frac{dr}{dt}$$  (1.1.1)
• That is, the velocity is dependent on $\vec{r}$ and $t$.

• The velocity in Cartesian coordinates is given by

$$\vec{v} = u\hat{i} + v\hat{j} + w\hat{k}$$

(1.1.2)

where $u = \frac{dx}{dt}$, $v = \frac{dy}{dt}$, $w = \frac{dz}{dt}$

• $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors in the coordinate directions $x$, $y$, $z$, respectively.

Note that $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

(1.1.3)
Substantial Derivative

- In order to compute the time rate of change of any property at the chosen point, therefore, the local rate of change of the property and also the change of its position have to be used. That is

\[
\frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z} = \frac{\partial}{\partial t} + \vec{v} \cdot \nabla \quad (1.1.4)
\]

- The derivative D/Dt is called by several names such as substantial derivative, material derivative, particle derivative or total derivative.
Acceleration of a Fluid Particle

• The first term on R.H.S. of Eq. (1.1.4), is called the local derivative which indicates the unsteady time variation of fluid property at a point. The sum of the last three terms is called convective derivative of the particle.

• Using the substantial derivative, the acceleration of a fluid particle is written as

\[
\ddot{a} = \frac{D\ddot{v}}{Dt} = \frac{\partial\ddot{v}}{\partial t} + (\ddot{v} \cdot \nabla)\ddot{v} = \frac{\partial\ddot{v}}{\partial t} + u\frac{\partial\ddot{v}}{\partial x} + v\frac{\partial\ddot{v}}{\partial y} + w\frac{\partial\ddot{v}}{\partial z}
\]  

(1.1.5)

• Note that the acceleration \(\frac{D\ddot{v}}{Dt}\) is called total acceleration, comprising of local acceleration (partial derivative w.r.t. time) and convective acceleration (caused due to the fluid motion itself). \((x, y, z)\) is the inertial frame of reference.
Streamline

• Streamline is an imaginary curve in the flow such that the tangent at each point on the curve coincides with the direction of the velocity at that point.

• When the flow is steady, the streamline coincides with the path line of any fluid particle. The equation of a streamline is

\[ \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \]  \hspace{1cm} (1.1.6)

• However, in unsteady motion, the flow pattern varies with time. Therefore, the streamlines and path lines differ with each other. (refer Fig. 1.1.2).

• The turbomachinery flows, in general, are unsteady.
Fig. 1.1.2 Streamlines and pathlines
Stream Surface and Stream Tube

• A stream sheet or stream surface is generated by a group of streamlines passing through some given curve, C. If the curve C is closed, the streamlines form a stream tube.

• (Since a stream tube is like a virtual solid surface, a particle can have no motion perpendicular to the streamline.)

• The space that the fluid occupies in a turbomachine can be considered as a stream tube. Even the flow between the blades of the turbomachines may be treated approximately in this manner.

• This approach gives an average behavior of fluid flow and is known as a quasi-one dimensional treatment.
Stream Function

• As there can be no flow across a streamline, stream function is a constant over a given streamline.

• From the equation of streamline Eq. (1.1.6), the velocity components can be readily derived as derivatives of stream function with respect to the spatial coordinates.

• For incompressible flow, the change in stream function signifies the change in mass flow rate.

• For inviscid and incompressible flow, (also irrotational flows) the stream function satisfies the Laplace’s equation. A flow net, with the mutually orthogonal stream function and velocity potential functions, can be constructed for such flows.
Vorticity and Irrotational Flow

• The curl of the velocity vector is called vorticity, \( \vec{\omega} (= \nabla \times \vec{v}) \)

• If \( \vec{\omega} = \xi \hat{i} + \eta \hat{j} + \zeta \hat{k} \), then,

\[
\xi = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}; \quad \eta = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}; \quad \text{and} \quad \zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \quad (1.1.7)
\]

• Vorticity is a vector and its magnitude is twice the angular velocity of the fluid particle at a given position in the flow field.
• A flow field is called irrotational when the vorticity is zero i.e. $\xi = \eta = \zeta = 0$.

• Velocity potential $\phi$ exists in such a flow and hence is called potential flow.

• The velocity components in this case are given by

$$u = -\frac{\partial \phi}{\partial x}; \quad v = -\frac{\partial \phi}{\partial y}; \quad w = -\frac{\partial \phi}{\partial z}$$

(1.1.8)
Vortex Line, Vortex Sheet and Vortex Tube

• Vortex line is an imaginary curve lying in the flow field such that its tangent at any point coincides with the direction of vorticity at that point.

• The equation of a vortex line is
  \[ \frac{dx}{\xi} = \frac{dy}{\eta} = \frac{dz}{\zeta} \]  (1.1.9)

• Vortex sheet is a surface composed of vortex lines.

• Vortex tube is generated by vortex lines, drawn through each point of a closed curve C. A vortex tube is also referred simply as a vortex.

• A vortex tube of a very small cross-section is called a vortex filament.
Circulation

- The instantaneous line integral of the tangential velocity around any closed curve \( C \) in a flow field is called circulation, denoted by \( \Gamma \).

- It is taken as positive when \( C \), as shown in Fig. 1.1.3, is traversed such that the area enclosed by \( C \) lies to the left.

\[
\Gamma = \oint_{C} \vec{v} \cdot d\vec{s} = \oint_{C} (u\,dx + v\,dy + w\,dz)
\]

\[
= \iint_{A} (\nabla \times \vec{v}) \cdot n\,dA \quad \text{(by Stokes theorem)}
\]

- Stokes theorem links circulation with vorticity.
• The circulation around any curve \( C \), bounding an area \( A \) (simply or multiply connected), is the sum of the circulations around all the lesser areas in to which \( A \) may be arbitrarily divided.

• Strength of a vortex tube is the circulation along a circuit lying on the surface of the vortex tube and passing around it only once.

• Strength of a vortex is same throughout its length.

• A vortex cannot have an end within the fluid i.e. vortex filament either forms a closed curve or extends to the fluid boundaries.

• The above points can be applied to develop a simple theory called isolated blade element theory for turbomachines, where the number of blades is not very large.
Summary of Lecture 1.1

In this lecture introductory aspects of fluid dynamics such as Microscopic versus Macroscopic approaches, Eulerian versus Lagrangian approaches and basic definitions of velocity stream function, vorticity and circulation are presented. The understanding of the concept of circulation is required to develop blade element theory used for turbomachines.

END OF LECTURE 1.1