Introduction to Composite Materials and Structures

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Lecture 35

Hygroscopic Stresses in Plates
Lecture Overview

• Introduction

• Mechanical and Hygroscopic Strains

• Stiffness Matrix for a Lamina

• Hygrothermal Forces and Moments
Introduction

- **Hygroscopy** is the ability of a substance to absorb water molecules from the surrounding environment. This property is exhibited by a large number of polymers used in composite fabrication including nylon, ABS, polycarbonate, cellulose, and poly-methyl-methacrylate.

- Hygroscopic materials “swell” due to moisture absorption. The amount a particular material is affected by moisture absorption is known as Coefficient of Hygroscopic Expansion (CHE). This is alternatively referred as Coefficient of Moisture Expansion (CME). However, some materials “contact” upon moisture absorption. Such materials have a negative value of CME.

- Coefficient of Moisture Expansion (CME) is defined as the ratio of the proportional length variation of a sample to its to the proportional mass variation due to moisture evaporation or absorption. Thus, symbolically:
  \[ \beta = \frac{(\Delta l_o/l)}{(\Delta m_o/m)} \]  
  (Eq. 35.1)
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• Equation 35.1 is valid for materials with isotropic hygroscopic properties. Orthotropic materials may have three different coefficients of moisture expansion coefficients, \( \beta_L, \beta_T, \) and \( \beta_T' \), corresponding to principal material directions.

• Micromechanical relations for CME for unidirectional lamina are analogous to those for CTE for similar materials. Thus, CME values for unidirectional lamina can be written as:

\[
\beta_L = \frac{(E_f V_f \beta_f + E_m V_m \beta_m)}{E_L} \quad \text{(Eq. 35.2)}
\]

\[
\beta_T = (1+\nu_f)V_f \beta_f + (1+\nu_m)V_m \beta_m - \beta_L \nu_{LT} \quad \text{(Eq. 35.3)}
\]

• Equations 35.2 and 35.3 are valid for composites with fibers having isotropic properties.
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• In case, fibers used in the composite are ortho-hygroscopic (Kevlar), they have different axial and transverse water absorption properties.

For composites made from such orthotropic fibers and isotropic matrix, a different equation for $\alpha_T$, as proposed by Hasin should be used. This equation is given below.

$$\beta_T = (1+\nu_L \beta_L / \beta_f)V_f \beta_T + (1+\nu_m)V_m \beta_m - (\nu_L V_f \beta_f + \nu_m V_m)(E \beta)_L / E_L \quad \text{(Eq. 34.4)}$$

where,

$$E_L = E_{Lf} V_f + E_m V_m$$

and

$$(E \beta)_L = E_{Lf} V_f \beta_L + E_m V_m \beta_m$$
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- However, many fibers (glass, graphite, boron) do not absorb moisture. Hence, the value of $\beta$ for such fibers is zero. In such a case, the relations for longitudinal and transverse moisture coefficients can be written as:

$$\beta_L = E_m V_m \frac{\beta_m}{E_L} \quad \text{(Eq. 35.5)}$$

$$\beta_T = (1+\nu_m)V_m \beta_m - \beta_L V_{LT} \quad \text{(Eq. 35.6)}$$

- Equations 35.5 and 35.6 are valid for composites with fibers having isotropic properties. For a composite with orthotropic fibers, but isotropic matrix, relations for longitudinal and transverse moisture coefficients are:

$$\beta_T = (1+\nu_m)V_m \beta_m - \nu_m V_m (E \beta)_L/E_L \quad \text{(Eq. 35.7)}$$

where,

$$E_L = E_{Lf} V_f + E_m V_m \quad \text{and} \quad (E \beta)_L = E_m V_m \beta_m$$
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• Further, the relations for hygroscopic strains ($\varepsilon^H$) in a unidirectional ply are:
  \[ \varepsilon^H_L = \beta_L \Delta c \]
  and
  \[ \varepsilon^H_T = \beta_T \Delta c \]  \hspace{1cm} (Eq. 35.8)

where,
  \[ \varepsilon^H_L = \text{Hygroscopic strain in longitudinal direction.} \]
  \[ \varepsilon^H_T = \text{Hygroscopic strain in transverse direction.} \]
  \[ \Delta c = c - c_o \]
  \[ c = \text{Ratio of moisture mass in composite and mass of composite sample} \]
  \[ c_o = \text{Ratio of moisture mass in composite and mass of composite sample, at a state when hygroscopic strains in the composite are zero.} \]

It should be noted here that there are no hygroscopic shear strains with respect to L-T axes. Hence, $\gamma^H_{LT} = 0$. 
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• Looking at Equations 35.8, it can be stated that $\beta_L$ and $\beta_T$ are thermal strains in a sample corresponding to hygroscopic moisture concentration increment of unity. Hence, these coefficients follow the same transformation law as that followed by the strain vector.

• Thus, coefficient of thermal expansions measured with respect to an arbitrary coordinate system (x-y) can be written as:

\[
\begin{pmatrix}
\beta_{xx}^H \\
\beta_{yy}^H \\
\frac{1}{2} \beta_{xy}^H
\end{pmatrix} = [T]^{-1} \begin{pmatrix}
\beta_L^H \\
\beta_T^H \\
0
\end{pmatrix}
\]

(Eq. 35.9)

• Here, $[T]^{-1}$ is the inverse of $[T]$ as defined in Eq. 10.7. Using these relations, hygroscopic strains in arbitrary coordinate system can be written as:

\[
\begin{pmatrix}
\varepsilon_{xx}^H \\
\varepsilon_{yy}^H \\
\gamma_{xy}^H
\end{pmatrix} = \begin{pmatrix}
\beta_{xx}^H \\
\beta_{yy}^H \\
\beta_{xy}^H
\end{pmatrix} \Delta c
\]

(Eq. 35.10)
Mechanical and Hygroscopic Strains

- Hygroscopic strains by themselves cannot generate a force or a moment, unless the body is not completely free to deform due to temperature. Thus, at the level of a whole laminate, there are no resultant forces and moments due to moisture effects alone.

- However, at the level of a lamina, the same may not be true. This is because a lamina by itself is not entirely free to bend or twist or expand due to changes in moisture content.

- These stresses in a lamina are attributable to strains which are in excess of moisture strains as defined in Eq. 35.10. These excessive strains are known as mechanical strains, and are denoted by a superscript M. Thus:

\[
\begin{pmatrix}
\varepsilon^M_x \\
\varepsilon^M_y \\
\gamma^M_{xy}
\end{pmatrix} = \begin{pmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{pmatrix} - \begin{pmatrix}
\varepsilon^H_x \\
\varepsilon^H_y \\
\gamma^H_{xy}
\end{pmatrix} = \begin{pmatrix}
\varepsilon^O_x \\
\varepsilon^O_y \\
\gamma^O_{xy}
\end{pmatrix} + z \begin{pmatrix}
k^O_x \\
k^O_y \\
k^O_{xy}
\end{pmatrix} - \begin{pmatrix}
\beta^H_x \\
\beta^H_y \\
\beta^H_{xy}
\end{pmatrix} \Delta c \quad \text{(Eq. 35.11)}
\]
Hygroscopic Stresses

• In Eq. 35.11, we see that mechanical strain is the difference of total strain, and hygroscopic strain.

• Further, hygroscopic stresses at individual ply level may be calculated by multiplying mechanical strain vector with lamina stiffness matrix. Thus:

\[
\begin{bmatrix}
\sigma_{xx}^M \\
\sigma_{yy}^M \\
\gamma_{xy}^M
\end{bmatrix}
= [Q]
\begin{bmatrix}
\varepsilon_{xx}^N \\
\varepsilon_{yy}^N \\
\gamma_{xy}^N
\end{bmatrix}
= \begin{bmatrix}
Q_{11} & Q_{12} & Q_{16} \\
Q_{12} & Q_{22} & Q_{26} \\
Q_{16} & Q_{26} & Q_{66}
\end{bmatrix}
- \begin{bmatrix}
\varepsilon_{xx}^o \\
\varepsilon_{yy}^o \\
\gamma_{xy}^o
\end{bmatrix}
+ z
\begin{bmatrix}
k_{xx}^o \\
k_{yy}^o \\
k_{xy}^o
\end{bmatrix}
- \begin{bmatrix}
\beta_{xx}^H \\
\beta_{yy}^H \\
\beta_{xy}^H
\end{bmatrix}
\Delta c
\] (Eq. 35.12)

• Equation 35.12 may be integrated over thickness (assuming moisture is constant over thickness) to yield relations for force-resultant vector. Similarly, Eq. 35.12 may be multiplied with \( z \), and then integrated over thickness, to yield relations for moment-resultant vector.

• The methodology for this is identical to that discussed while developing Eqs. 34.5 and 34.6.
Equilibrium Equations

• Similar to the case for thermal stresses (as discussed in Lecture 34), these force and moment resultant vectors have a hygroscopic component as well.

• The definitions of hygroscopic force and moment resultants are identical analogous to thermal force and moment resultants as defined in Eqs. 34.7.

• Finally, the equilibrium equations for the plate are essentially the same as defined in Eqs. 27.6, 27.7, and 27.8. This is so, since resultant force and moment equations already include hygroscopic effects on account of their revised definitions.

• These equations can be solved to understand the role of hygroscopic strains on composite stresses.
Hygrothermal Effects

• Composites exhibit hygrothermal stresses and strains when exposed to temperature and moisture. Analysis of such stresses can be conducted by:
  — Finding strains in unidirectional lamina by modifying Eq. 35.8 as shown below:

\[
\varepsilon_{HTl} = \alpha_l \Delta T + \beta_l \Delta c,
\]

\[
\varepsilon_{HTT} = \alpha_T \Delta T + \beta_T \Delta c
\]

(Eq. 35.13)

\[\gamma_{LT} = 0.\]

These strains may next be transformed to x-y coordinate system, which is common for all composite layers.

Going further, by using superposition principle, stresses and strains at laminate level may be calculated by using the approach as discussed earlier in this lecture, and also in Lecture 34.

Further details on these stresses are discussed in following Lecture 36.
References

