Mechanics of Laminated Composite Structures

Nachiketa Tiwari

Indian Institute of Technology Kanpur
Lecture 20
Behavior of Short-Fiber Composites
\( V_{\text{min}} \) and \( V_{\text{crit}} \) for Short-Fiber Composites

- The average longitudinal stress in a short-fiber composite with unidirectionally oriented fibers can be calculated using rule of mixtures as:

\[
\sigma_c = V_f \sigma_f + V_m \sigma_m. \tag{Eq. 20.1}
\]

Equation 20.1 is a modified form of Eq. 16.2, where \( \sigma_f \) is average fiber stress, which may be calculated by integrating \( \sigma_f \) over fiber length, and dividing this value by fiber length.

- If stress is distributed linearly over fiber length, as shown in Fig. 19.1, then Eq. 20.1 may be rewritten as:

\[
\sigma_c = V_f (\sigma_f)_{\text{max}} + V_m \sigma_m \quad \text{for} \quad l < l_t \tag{Eq. 20.2a}
\]

\[
\sigma_c = V_f ((1 - l_t/(2l))\sigma_f)_{\text{max}} + V_m \sigma_m \quad \text{for} \quad l > l_t \tag{Eq. 20.2b}
\]
$V_{\text{min}}$ and $V_{\text{crit}}$ for Short-Fiber Composites

- Equation 20.2b can be approximated to Eq. 20.3, when fiber length $l$, is significantly larger than load-transfer length, $l_t$.
  \[ \sigma_c = V_f(\sigma_f)_{\text{max}} + V_m\sigma_m \text{ for } l \gg l_t \]  
  (Eq. 20.3)

- These equations help explain the following phenomenon:
  - When fibers are shorter than load-transfer length, then maximum fiber stress is less than average fiber strength. Hence, in such composites, fibers never fracture, regardless of high the value of externally applied stress is.
  - In such cases, matrix or interface failure coincides with the failure of composite.

- Thus, ultimate strength, $\sigma_{cu}$, of a composite where fiber length is less than critical length $l_c$ can be found using Eq. 20.2a, and incorporating $\tau_y$ and $\sigma_{mu}$ as shown in the following relation.
  \[ \sigma_{cu} = V_f(\tau_y/d) + V_m\sigma_{mu} \text{ for } l < l_c \]  
  (Eq. 20.4)

  - Note: The value of $l_{min}$ is same as $l_c$, which is defined in Eq. 18.4.
$V_{min}$ and $V_{crit}$ for Short-Fiber Composites

- For the case when fiber length exceeds critical length, $l_c$, the ultimate strength, $\sigma_{cu}$, of a composite can be found using the following relation:

$$\sigma_{cu} = V_f(1-l_c/l) (\sigma_f)_{max} + V_m(\sigma_m)_{\varepsilon_{fu}} \quad \text{for } l > l_c \quad \text{(Eq. 20.5)}$$

- For the case when fiber length exceeds critical length, $l_c$ in very significant terms, Eq. 20.5 can be approximated as:

$$\sigma_{cu} = V_f(\sigma_f)_{max} + V_m(\sigma_m)_{\varepsilon_{fu}} \quad \text{for } l >> l_c \quad \text{(Eq. 20.6)}$$

- While developing Eqs. 20.5 and 20.6, it has been assumed that the volume fraction for fiber exceeds a certain minimum threshold, $V_{min}$, such that the breakage of all fibers causes sudden rise in matrix stresses, and this increment is so large that matrix material is unable to bear such an increases stress level. Hence, in such a composite, failure of fibers corresponds with failure of overall composite material.
\( V_{\text{min}} \) and \( V_{\text{crit}} \) for Short-Fiber Composites

- Extending the methodology used for continuous fibers as described in Eqs. 16.6 and 16.7, we can also develop similar relations for \( V_{\text{min}} \) and \( V_{\text{crit}} \) for short-fiber composites.

- Thus relations equivalent to Eqs. 16.6 and 16.7 for short-fiber composites are:

\[
V_{\text{min}} = \frac{\sigma_m \mu - (\sigma_m)_{sfu}}{\sigma_f + \sigma_m \mu - (\sigma_m)_{sfu}} \quad \text{(Eq. 20.7)}
\]

\[
V_{\text{crit}} = \frac{\sigma_m \mu - (\sigma_m)_{sfu}}{\sigma_f - (\sigma_m)_{sfu}} \quad \text{(Eq. 20.8)}
\]
\( V_{\text{min}} \) and \( V_{\text{crit}} \) for Short-Fiber Composites

- Comparing Eqs. 16.6 and 16.7 with Eqs. 20.7 and 20.8, respectively, we find that the values of \( V_{\text{min}} \) and \( V_{\text{crit}} \) for short-fiber composites are higher than those for continuous fiber composites.

- This is because load transfer on fibers in short-fiber composites is not maximized.

- However, as fiber length becomes large, vis-à-vis load transfer length, the values of \( V_{\text{min}} \) and \( V_{\text{crit}} \) for short-fiber composites and continuous fiber composites come very close.

- For cases when fiber volume fraction is less than \( V_{\text{min}} \), composite fracture is not coincidental with fiber fracture. Rather, post fiber fracture, the load borne by fibers get transferred to matrix material, and composite eventually fails when matrix material is unable to support externally applied load.
Failure of Short-Fiber Composites

• When fiber volume is less than \( V_{\text{min}} \), the composite strength, as explained earlier, is not determined by load at which fiber fracture occurs.

• In such a case, the ultimate strength of composite is given by the following relation:

\[
\sigma_{\text{cu}} = (1 - V_f)\sigma_{\text{mu}} \quad \text{for} \quad V_f < V_{\text{min}}
\]

(Eq. 20.9)

• Short-fiber composites tend to have higher stress concentrations in matrix material due to the presence of a very large number of fiber ends. This factor reduces the composite strength even further.

• To determine strength of a randomly oriented short-fiber composite, strength of a symmetric quasi-isotropic laminate \([0/45/-45/90]_s\) is determined and used as an estimate. However, such an estimate is usually found to be higher than experimental data.
References

