Mechanics of Laminated Composite Structures

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Lecture 19
Behavior of Short-Fiber Composites
Load Transfer Mechanism in Short-Fiber Composites

• Till so far, we have assumed that the matrix material in fiber-matrix interface region is perfectly plastic.

• This is not entirely true. In reality, most matrix materials exhibit elasto-plastic behavior. Developing analytical solutions for such systems is not easy.

• Hence, numerical methods may be used to solve such problems to get better understanding of load transfer mechanisms in short-fiber composites.

• Several such studies have shown that load transfer at fiber ends is not significant, and hence our earlier assumption of $\sigma_{fo}$ being zero, stands validated, though in an approximate sense.
Load Transfer Mechanism in Short-Fiber Composites

- Figure 19.1 is a plot of variation of fiber strength for three different fiber lengths.

\[ \sigma_f \]

\[ \sigma_t \]

\[ (\sigma_f)_{\text{max}} \]

\[ \sigma_f \]

\[ (\sigma_f)_{\text{max}} \]
Load Transfer Mechanism in Short-Fiber Composites

• Following observations can be made from Fig. 19.1.

• If fiber length is less than $l_t$, then the normal stress in fiber is zero at either ends of fiber, and it reaches a peak value at mid-fiber length. In such a case, the longer the fiber, the higher is the value of peak normal stress which occurs at its mid-length.

• If fiber length equals $l_t$, then normal stress in fiber gets maximized. However, the shape of stress plot still remains triangular.

• Finally, if fiber length exceeds $l_t$, then normal stress in fiber:
  – Rises from zero to a maximum value over part of the fiber length.
  – Remains constant once it has maximized.
  – Falls back to zero, over remaining part of fiber length.

• Utilization of fiber strength is maximized in the third configuration.
Modulus of Short-Fiber Composites

- Even though mathematically rigorous solutions can be developed to compute moduli of short-fiber composites, it is usually desirable to have relatively simple relations for estimating the same from a design perspective. Here, we directly cite results of Halpin and Tsai, which help us calculate with reasonable accuracy, longitudinal and transverse moduli of short-fiber composites, with fibers aligned in a single direction.

\[
\frac{E_l}{E_m} = \frac{1 + (l/r)\eta_L V_f}{(1 - \eta_L V_f)} \quad \text{(Eq. 19.1)}
\]

where,
\[
\eta_L = \frac{[(E_f/E_m) - 1]}{[(E_f/E_m) + l/r]}.
\]

Also,
\[
\frac{E_T}{E_m} = \frac{1 + 2\eta_T V_f}{(1 - \eta_T V_f)} \quad \text{(Eq. 19.2)}
\]

where,
\[
\eta_T = \frac{[(E_f/E_m) - 1]}{[(E_f/E_m) + 2]}
\]
Modulus of Short-Fiber Composites

• It may be noted from Eq. 19.3, that transverse modulus of unidirectional short-fiber composites does not depend on $l/r$ ratio, and its value equals that for continuous unidirectional composite.

• Earlier, we had discussed the need for randomly oriented short-fiber composites, since they are isotropic in a plane, and hence are appropriate for omni-directional loads.
Modulus of Short-Fiber Composites

• Such composites do not crack easily in their surface layers in the transverse direction.

• Predicting their modulus is analytically difficult. Hence, we cite a well known empirical result, which predicts modulus for a randomly oriented short-fiber composites.

\[ E_{\text{random}} = \frac{3E_L + 5E_T}{8} \]

Here, moduli \( E_L \) and \( E_T \) may be calculated from relations given earlier, or determined experimentally.

• Above equation is a good engineering tool for designing composites with randomly oriented fibers.
What you learnt in this lecture?

• Background information about short-fiber composites

• Load transfer mechanism in short-fibers

• Longitudinal and transverse moduli for short-fiber composites with unidirectional alignment

• Modulus of randomly oriented short-fiber composites