Mechanics of Laminated Composite Structures

Nachiketa Tiwari

Indian Institute of Technology Kanpur
Lecture 18
Behavior of Short-Fiber Composites
Lecture Overview

• Background information about short-fiber composites

• Load transfer mechanism in short-fibers

• Longitudinal and transverse moduli for short-fiber composites with unidirectional alignment

• Modulus of randomly oriented short-fiber composites
About Short-Fiber Composites

• We have seen earlier that unidirectional composites tend to be very stiff and strong in fiber direction, but very weak in the transverse direction. Their weakness in transverse direction is attributable to presence of significant stress concentration at the interface of matrix and fiber.

• Given these attributes, unidirectional composites are very useful in applications where state of stress is well known.
  – In such applications, lamination sequence of composite can be tailor-made to bear external loads optimally.

• However, if externally applied loads are omni-directional, or if their direction can vary in time, then such laminates fabricated by stacking up unidirectional laminae may not necessarily meet our design needs.
About Short-Fiber Composites

• We may still be able design a laminate for such cases (that is when loading is omni-directional) which is equally strong in all directions, but even in such a design, the top and bottom layers will be weak in transverse directions, and failure could get initiated from here.

• Hence, in such applications, it is useful to have lamina which have in-plane isotropy.

• One way to produce such lamina is by using short fibers which are randomly oriented. Such composites, in general are significantly less expensive than unidirectional composites. The fiber lengths in these are between 1 to 8 cm.

• Such composites are used extensively in general purpose applications, such as car body panels, boats, household goods, etc. In most of such applications, glass fiber is used as the reinforcing material for matrix.
Load Transfer Mechanism in Short-Fiber Composites

• In composite materials, fibers are invariably surrounded by matrix material. Hence, external load is directly applied to matrix, and from here, it gets transferred to fibers.

• A part of this load gets transferred to fibers at their ends, while remaining portion of this load gets transferred to fibers through their external cylindrical surfaces.

• For unidirectional composites with continuous fibers, transfer of load at fiber ends may be very small vis-à-vis load transfer through fiber’s external surface.

• This is because fibers are very long, and hence their cylindrical surfaces, across which load gets transferred through shear-mechanism, are sufficiently long. In such fibers, the effect of load transfer through fiber ends may not significantly affect overall mechanics of load transfer.
Load Transfer Mechanism in Short-Fiber Composites

• However, in short-fiber composites the same may not be necessarily true. In such composites, the length of the fiber is not sufficiently long such that much of load transfer happens across cylindrical surfaces of fibers.

• Thus, in such fibers, both the ends, as well as external cylindrical surfaces of fibers play a significant role in matrix-to-fiber load transfer.

• Hence, it is important to understand role of end-effects in context of load transfer to fibers. Without this understanding, our understanding of reinforcing effects in short-fiber composites will be inaccurate and flawed.
Load Transfer Mechanism in Short-Fiber Composites

• Consider a short-fiber of length \( l \) embedded in matrix which is shown in Fig. 18.1. The figure also shows the details of an infinitesimal portion of fiber of length \( dz \), which experiences normal stress in length direction, and shear stress, \( \tau \), along its cylindrical surface.

• Please note that while normal stress at one end of infinitesimal fiber is \( \sigma_f \), it is \( \sigma_f + d\sigma_f \) at its other end.

• This variation in normal stress along the length of infinitesimally long fiber is because some of the load gets transferred from matrix to fiber due to application of shear stress on its cylindrical surface.
Load Transfer Mechanism in Short-Fiber Composites

Fig. 18.1: Force Equilibrium of an Infinitesimal Portion of Discontinuous Fiber Which is Aligned to External Load
Load Transfer Mechanism in Short-Fiber Composites

• From principles of static equilibrium, we can now write equation of force equilibrium for this infinitesimally-sized portion of fiber.

\[
\pi r^2 \sigma_f + (2 \pi r \tau dz) \sigma_f = \pi r^2 (\sigma_f + d\sigma_f)
\]

• Cancelling out term \( \Pi r^2 \sigma_f \) from both sides, and rearranging remaining terms we get:

\[
d\sigma_f / dz = 2\tau / r
\]

• Integrating above equation yields,

\[
\sigma_f = \sigma_{fo} + (2/r) \int \tau \, dz
\]

where the integral limits are 0 to \( z \).

• Quite often, fiber separates from the matrix due to presence of large stress concentration. In other cases, matrix yields at the fiber end. The implication of either cases is that the integration constant for above equation, \( \sigma_{fo} \), is zero. Thus, above equation can be rewritten as:

\[
\sigma_f = (2/r) \int \tau \, dz
\]
Load Transfer Mechanism in Short-Fiber Composites

• The integral equation shown earlier can be evaluated if variation of shear stress, $\tau$, with respect to coordinate $z$, is known. At this point, we make an *assumption* that shear stress at the interface of fiber and matrix is constant along fiber length, and equals matrix yield shear, i.e. $\tau_y$. Such an assumption may be made for a system where matrix material transmits maximum possible stress to fiber, which would be $\tau_y$. For such a case, the integral equation can be simplified as:

$$\sigma_f = 2\tau_y \frac{z}{r}$$  \hspace{1cm} (Eq. 18.1a)

• For short fibers, maximum fiber stress is expected to occur at mid-length, i.e. $z = l/2$, while it will be zero at its extremities for reasons explained earlier. Hence, the equation written above will hold good only for values of $z = 0$ to $l/2$, and for the region $z = l/2$ to $l$, the equation will have to have a negative slope. Further, the maximum value of fiber stress will be, as per above equation:

$$\sigma_{f_{\text{max}}} = \tau_y \frac{l}{r}, \text{ corresponding to } z = l/2.$$  \hspace{1cm} (Eq. 18.1b)
Load Transfer Mechanism in Short-Fiber Composites

• Equation 18.1 places no limit on the upper bound for fiber stress, and can approach very large values if \( l \) is made very large. However, in reality there will indeed be a limit, which will correspond to the stress borne by continuous and infinitely long fibers in unidirectional plies. This stress, as calculated earlier is \( E_f/E_m \sigma_c \). Equating this value to maximum fiber stress in short-fiber (as per Eq. 18.1) gives us a load-transfer length, \( l_t \), which is required to achieve maximum possible stress in fiber. This is shown below.

\[
\sigma_{f_{\text{max}}} = \frac{\tau_y}{E_f/E_m \sigma_c}
\]

or,

\[
\frac{l_t}{r} = \frac{\sigma_{f_{\text{max}}}}{\tau_y} = \left(\frac{E_f}{E_m \sigma_c}\right) / \tau_y
\]

(Eq. 18.2a)

(Eq. 18.2b)

• Thus, a fiber which is at least \( l_t \) long, develops maximum fiber stress \( (E_f/E_m \sigma_c) \) as defined earlier, when the externally applied stress is \( \sigma_c \).
Load Transfer Mechanism in Short-Fiber Composites

• Hence, if we increase external stress $\sigma_c$, we will have to increase $l_t$ to ensure maximum load in fiber, as $\sigma_{f_{\text{max}}}$, which equals $E_f/E_m \sigma_c$, will also increase.

• However, there is a limit beyond which external stress $\sigma_c$ cannot be increased.

• This limit corresponds to a point when the stress in fiber equals its ultimate strength ($\sigma_{uf}$),
  - At this limit, any further stress in external stress will lead to failure of fiber.
  - Thus, the condition for maximum possible stress in fiber is:

$$\sigma_{f_{\text{max}}} = \sigma_{uf} = \frac{E_f}{E_m} \sigma_c$$

(Eq. 18.3)
Load Transfer Mechanism in Short-Fiber Composites

• For such a limiting stress, there is a corresponding *minimum fiber length* which is required to support such a level of stress.

• Mathematically, the value of minimum fiber length can be calculated from Eq. 18.2b and is given below.

\[
\frac{l_{\text{min}}}{r} = \frac{\sigma_{uf}}{\tau_y}
\]  
(Eq. 18.4)

• Thus, any design of a short-fiber composite should ensure that its fiber is at least \(l_{\text{min}}\) long, because in such a system the overall composite strength will be will maximized.

• If fibers are shorter than this critical length, then composite strength would not be at its maximum value, thereby adding weight and cost to the structure. Finally, if \(l\) is very large compared to \(l_{\text{min}}\), then composite increasingly behaves as one with continuous fibers.