Steady State error of a System

Dr. Bishakh Bhattacharya
Professor, Department of Mechanical Engineering
IIT Kanpur
This Lecture Contains

- Steady state error of a system
- Position, Velocity and Acceleration Constants
- Optimal Steady State Error
Steady state error - Introduction

- **Steady state error** refers to the long-term behavior of a dynamic system.
- The **Type** of a system is significant to predict the nature of this error.
- A system having no pole at the origin is referred as **Type-0** system.
- Thus, **Type-1**, refers to one pole at the origin and so on.
- It will be shown in this lecture that, it is the **type** of a system which can directly determine whether a particular command will be followed by a system or not.
- We will consider three common commands: namely, step, ramp and parabolic ramp and find out the steady state response/error of a system to follow these commands.
- A closed loop control system shows remarkable performance in reducing the steady state error of a system.
Steady state error of a system

• Error in a system: \( E(s) = \frac{U(s)}{1+G(s)} \)

\[
\lim_{t \to \infty} e(t) = e_{ss} = \lim_{s \to 0} \frac{sU(s)}{1 + G(s)}
\]

• For a step input

\[
e_{ss} = \frac{A}{1 + G(0)}
\]

• Plant Transfer function \( G(s) \) is defined as

\[
G(s) = \frac{K \prod_{i=1}^{M} (s + z_i)}{s^k \prod_{j=1}^{N} (s + p_j)}
\]
Error Constants

- Position error constant
  
  \[ K_p = \lim_{s \to 0} G(s) \]

- Steady state error of a step input of magnitude A is
  \[ e_{ss} = \frac{A}{1 + K_p} \]

- Steady state error will be zero for system with type greater than or equal to 1
• For ramp input

\[ e_{ss} = \lim_{s \to 0} \frac{A}{sG(s)} \]

• Define velocity constant as

\[ K_v = \lim_{s \to 0} sG(s) \]

• Hence steady state error is

\[ \frac{A}{K_v} \]

• Error will be zero for \( k \) greater than or equal to 2
## Summary of Steady State Errors

<table>
<thead>
<tr>
<th>Type</th>
<th>Step ((A/s))</th>
<th>Ramp ((A/s^2))</th>
<th>Parabolic Ramp ((A/s^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(E_{ss} = A/(1+K_p))</td>
<td>Inf</td>
<td>Inf</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>(A/K_v)</td>
<td>Inf</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>(A/K_a)</td>
</tr>
</tbody>
</table>
Design of Optimal systems

You can choose optimal gain such that the error constant could be minimized. Consider the following system:
Optimized Gain

\[
T(s) = \frac{2 - ks}{s^2 + 3s + 2}
\]

This is a Type zero system. Hence, consider step input.

\[
E(s) = \frac{A}{s} (1 - T_e(s)) = A\left(\frac{k+2}{s+1} - \frac{k+1}{s+2}\right)
\]

\[
J(k) = \int_0^\infty e^2(t)dt = \frac{A^2}{12}[k^2 + 6k + 11]
\]

Note that the Performance index is a quadratic function of gain \(k\), which can be minimized to obtain \(k\).

\[dJ/dk=0 \quad \text{---} \quad k = -3\]
Special References for this lecture

- *Feedback Control of Dynamic Systems*: Frankline, Powell and Emami-Naeini, Pearson Publisher

- *Control Systems Engineering*: Norman S Nise, John Wiley & Sons

- *Systems Dynamics and Response*: S. Graham Kelly, Thomson Publisher