9.1 An aircraft instrument weighing 100 N is to be isolated from engines vibrations ranging from 25Hz-40Hz. What statical deflection must the isolators have for achieving 85% isolation.

mass of instrument m=100N

frequency of isolation= 25-40 Hz

isolation =85%

hence transmissibility=1-0.85=0.15

\[
Tr = \frac{1}{\sqrt{1 + \left(\frac{2\xi\omega}{\omega_n}\right)^2}} \left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right) + \left(2\xi\omega\right)^2
\]

If we assume \(\xi = 0\) (very low damping)

\[
Tr = \frac{1}{\left(\frac{\omega}{\omega_n}\right)^2 - 1}
\]

\[0.15 = \frac{1}{\left(\frac{\omega}{\omega_n}\right)^2 - 1}
\]

solving

\[
\frac{\omega}{\omega_n} \geq 2.768
\]

If the transmissibility at lower frequency is less, higher frequencies also even lesser force transmission will be there.

Now let \(\omega = 25\)Hz

\[
\therefore \omega_n = \frac{(2\pi)(25)}{2.768} = 56.75 \text{rad/s}
\]

For a spring-mass model

let \(\delta_{st}\) be the static

deflection of the spring under the weight of instrument.
9.2 An industrial machine of 450kg is supported on springs with a static deflection of 0.50cm. If the machine has rotating unbalance of 0.2kgm. Determine a) the force transmitted to floor at 1200rpm and b) the dynamic amplitude at this speed.

Given : mass of machine \( m = 450 \text{kg} \)

\[ \Delta = 0.5 \text{mm} \]

\[ K = \frac{mg}{\Delta} = \frac{450 \times 9.81}{0.005} = 882900 \text{ N/m} \]

\[ \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\left(\frac{l}{m}\right) \left(\frac{mg}{\delta_{st}}\right)} = \sqrt{\frac{g}{\delta_{st}}} \]

\[ \therefore \frac{g}{\delta_{st}} = 56.75 \]

\[ \therefore \delta_{st} = \frac{9.8}{(56.75)^2} \times 1000 \text{ mm} \]

\[ = 3.043 \text{ mm} \]

\[ \Omega = 1200 \text{ RPM} = \left(\frac{1200}{60}\right) (2)(\pi) = 125.66 \text{ rad/s} \]

In absence of damping

\[ Tr = \frac{1}{\left(\frac{\omega}{\omega_n}\right)^2 - 1} \]

\[ \omega_n = \sqrt{\frac{k}{m}} = \sqrt{882900/450} = 44.29 \text{ rad/s} \]

\[ Tr = 0.14 \]

\[ F = ma^2 = (0.2 \text{kg} - \text{m})(125.66)^2 \]

\[ = 3158 \text{ N} \]

\[ F_{tr} = (0.14)(3158) = 442 \text{ N} \]
9.3 The rotor of a turbine 15 kg in mass, is supported at the midspan of a shaft with bearings 0.5m apart. The rotor is known to have an unbalance of 0.2kgm. Determine the forces exerted on the bearings at a speed of 500 rpm if the diameter of the steel shaft is 20 mm.

For a simply supported shaft, at its center the stiffness is

\[ k = \frac{48EI}{L^2} \]

\[ I = \frac{\pi d^4}{64} = \frac{\pi}{64} \times (0.02)^4 \]

\[ k = \frac{48 \times 2.1 \times 10^{11} \times \frac{\pi}{64} \times (0.02)^4}{(0.5)^3} = 63345 \text{ N/m} \]

\[ \omega_n = \left( \frac{k}{m} \right)^{\frac{1}{2}} = \sqrt{\frac{63345}{15}} = 205.5 \text{ rad/s} = 1962 \text{ RPM} \]

\[ \frac{\omega}{\omega_n} = \frac{500}{1962} = 0.255 \]

\[ \gamma = \frac{\left( \frac{\omega}{\omega_n} \right)^2}{\left( 1 - \left( \frac{\omega}{\omega_n} \right)^2 \right)^2 + \left( 2 \xi \frac{\omega}{\omega_n} \right)^2} \]

Assuming negligible damping,

\[ \frac{x}{\epsilon} = \frac{(0.255)^2}{1 - (0.255)^2} = 0.069 \]

Now force due to whirling is centrifugal force due to rotating imbalance.

\[ F_{\text{cent}} = m(e + r) \omega^2 \]
$$e + r = 1.069 \times e$$

$$F_{agt} = me \times 1.069 \times \omega^2$$

$$= 1.069 \times 0.2 \times \left(\frac{500 \times 2 \times \pi}{60}\right)^2 = 586 \text{N}$$

therefore force on bearings = 586 / 2 = 293 N