ME-662 CONVECTIVE HEAT AND MASS TRANSFER

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LECTURE-39 CONV M T - REYNOLDS FLOW MODEL - 1
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1. Wet Bulb Thermometer
2. Measurement of RH - Effect of Radiation
3. Evaporation from a Porous Surface
4. Evaporation from a Lake
5. Humidification - Internal Flow

All problems require use of Psychrometry and/or steam tables
Wet Bulb Thermometer - L39(1/16)

Prob: A wet bulb thermometer records $15^0\text{C}$ when the dry bulb temperature is $27^0\text{C}$. Calculate (a) RH of air and (b) compare with Carrier’s correlation.
Assume $Le = 1$ and take $c_{p,v} = 1.88 \text{ kJ/kg-K}$ and $c_{p,a} = 1.005$.

Soln: Here, $T_w = 15$ and $T_\infty = T_{db} = 27$. Since $Le = 1$,

$$B_m = \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - \omega_{v,T}} = B_h = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{TL} + q_l/N_w} \rightarrow q_l = 0$$

Taking $T_{ref} = 0^0\text{C}$, $\lambda_{ref} = 2503 \text{ kJ/kg}$, and from Steam Tables, $\omega_{v,w} = 0.01068$, we have

$$h_{m,\infty} = 1.005 \times 27 + \{ (1.88 - 1.005) \times 27 + 2503 \} \omega_{v,\infty}$$

$$h_{m,w} = 15.075 + \{ 0.875 \times 15 + 2503 \} 0.01068 = 41.95$$

$$h_{TL} = c_{p,l} \times T_w = 4.187 \times 15 = 62.805$$
Therefore
\[ \frac{\omega_{v,\infty} - 0.01068}{0.01068 - 1} = \frac{27.135 + 2526.2 \omega_{v,\infty} - 41.95}{41.95 - 62.805} \]
Solving, \( \omega_{v,\infty} = 0.005936 \). Hence, \( W_\infty = \omega/(1 - \omega) = 0.00594 \) at 27°C. From psychrometric chart, this value corresponds to \( \text{RH} = 27 \% \) (Ans a). Also, \( B_m = B_h = 0.00479 \) (Very small).

Carrier’s correlation is
\[
\rho_{v,\infty} = \rho_{\text{sat},w} - \frac{(\rho_{\text{tot}} - \rho_{\text{sat},w})(T_{wb} - T_{db})}{1555 - T_{wb}} \rightarrow (T^0\ C)
\]
where \( \rho_{\text{tot}} = 1 \text{ bar} \), \( W_w = 0.01068/(1 - 0.01068) = 0.0108 \) and \( \rho_{\text{sat},w} = W_w \times \rho_{\text{tot}}/0.622 = 0.01736 \text{ bar} \).
Hence, substitution gives \( \rho_{v,\infty} = 0.0097 \). Therefore
\[
\omega_{v,\infty} = \frac{\rho_{v,\infty}}{1.61 \times \rho_{\text{tot}} - 0.61 \times \rho_{v,\infty}} = 0.00604 \quad (\text{Ans b})
\]
Measurement of RH - L39(3/16)

Prob: Moist air flows through a duct whose walls are maintained at 50°C. Dry and wet bulb thermometers placed in the duct record 70°C and 25°C respectively. Between thermometer bulb and air, \( h_{\text{cof}} = 17.5 \, \text{W} / \text{m}^2 \cdot \text{K}. \) Calculate RH of air (a) without radiation and (b) with radiation.

Soln: Part (a) Here, again \( W_w = 0.02, \omega_{v,w} = 0.0196, \)
\( h_{m,w} = 74.61, h_T = 105.7 \) and \( h_{m,\infty} = 70.35 + 2564.25 \omega_{v,\infty}. \)
Therefore, with \( q_l = 0, \) equating \( B_m \) and \( B_h, \)

\[
\frac{\omega_{v,\infty} - 0.0196}{0.0196 - 1} = \frac{70.35 + 2564.25 \omega_{v,\infty} - 74.61}{74.61 - 104.675}
\]

Solving \( \omega_{v,\infty} = 0.001446 \) (\( B = 0.0185 \)). Therefore, \( \rho_{v,\infty} = 0.002328 \) bar. But, from steam tables, \( \rho_{v,\text{sat}}(T_{\infty} = 70^0\text{C}) = 0.3119 \) bar.

Hence, \( \text{RH} = 0.002328 / 0.3119 = 0.746\% \) (Ans).
Soln: Part (b) Here, \( q_l = 0 \) and \( h_{TL} = h_T + q_{rad}/N_w \).

\[
B_{mh} = \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - \omega_{v,T}} = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_T} = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{TL} + q_{rad}/N_w}
\]

Now, we determine true air temperature (\( T_{a,true} \)) allowing for radiation effects. Thus

\[
h_{cof} (T_{a,true} - T_{db}) = \sigma (T_{db}^4 - T_w^4)
\]

\[
17.5 (T_{a,true} - 343) = 5.67 \times 10^{-8} (343^4 - 323^4) \quad \text{or}
\]

\[
T_{a,true} = 352.6K = 79.6^0C \quad \text{and}
\]

\[
q_{rad} = \sigma (T_w^4 - T_{wb}^4)
\]

\[
= 5.67 \times 10^{-8} (323^4 - 298^4) = 170 \frac{W}{m^2}
\]

Now, \( B_m \) and \( B_h \) are freshly evaluated (next slide).
Here, we take $T_{ref} = T_{wb} = 25$ so that $\lambda_{ref} = 2442.3$ kJ/kg.

\[
h_{m,\infty} = c_{p,a} (T_{a,true} - T_{ref}) \\
+ \left[ (c_{p,v} - c_{p,a}) (T_{a,true} - T_{ref}) + \lambda_{ref} \right] \omega_{v,\infty} \text{ kJ/kg}
\]

\[
h_{m,w} = \omega_{v,w} \lambda_{ref} = 0.0196 \times 2442.3 = 47.87 \text{ kJ/kg}
\]

\[
h_{TL} = c_{pl} (T_{wb} - T_{ref}) = 0 \text{ kJ/kg}
\]

Also $\omega_{mean} \approx 0.5 (0.0196 + 0.001446) = 0.0105$.

Therefore, $c_{p,m} = 1.041$ kJ/kg-K.

Hence, $g^* = h_{cof} / c_{pm} = 17.5 / 1041 = 0.01725$ kg/m$^2$-s.

Now, since B is expected to be small, we take $(q_{rad}/N_w) = q_{rad}/(g^* B)$, so that

\[
B = \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - \omega_{v,T}} = \frac{h_{m,\infty} - h_{m,w} - (q_{rad}/g^*)}{h_{m,w} - h_{TL}}
\]
Substitutions give \((q_{rad}/g^*) = 0.17 / 0.01725 = 9.885 \text{ kJ/kg.}\) Hence

\[ B = \frac{\omega_{v,\infty} - 0.0196}{0.0196 - 1} = \frac{54.52 + 2490 \omega_{v,\infty} - 47.87 - 9.885}{47.85 - 0} \]

This gives \(\omega_{v,\infty} = 0.001693\) and \(B = 0.01826.\)
So, our assumption of small \(B\) is verified.
Thus, \(W_{\infty} = 0.001648\) and \(\rho_{v,\infty} = 0.00264 \text{ bar.}\)
But, \(\rho_{v,sat} (T_{\infty} = 79.58^0C) = 0.4739 \text{ bar.}\)
Hence \(RH = 0.00264 / 0.4739 = 0.557 \% \) (Ans).

This shows that true RH is lower than that predicted by neglecting effect of radiation.
Evaporation - High B - L39(\(\frac{7}{16}\))

**Prob:** Air at 1 bar, 27\(^0\)C and 90 % RH flows over a porous flat surface which is kept wet by supplying water. The plate temperature is maintained at 82.5\(^0\)C by supplying heat to it from the rear side. Calculate (a) Evaporation rate and (b) Heat flux to be supplied. Given: \(U_\infty = 3.0 \text{ m/s,}\)<br>Length \(L = 0.33 \text{ m}\) and Width \(W = 1 \text{ m.}\) Take \(k = 0.025 \frac{W}{m \cdot K},\)<br>\(\rho_m = 1.37 \frac{kg}{m^3},\)<br>\(D = 0.162 \frac{m^2}{hr},\)<br>\(\alpha = 0.153 \frac{m^2}{hr}\) and \(\nu = 0.103 \frac{m^2}{hr}\)

**Soln:** In this case, 
\[
Re_L = 3 \times 0.33/(0.103/3600) = 34601.9 < 3 \times 10^5.
\]
Thus, we have a laminar BL. Also, \(Pr = 0.103 / 0.153 = 0.673\) and \(Sc = 0.103 / 0.162 = 0.636.\)

Hence, \(St = 0.664 \times Re_L^{-0.5} \times Pr^{-0.67} = 0.00465.\)

Therefore, \(g^* = h_{cof}/c_{pm} = \rho_m U_\infty St = 1.37 \times 3.0 \times 0.00465 = 0.0191 \text{ kg/m}^2\text{-s}.\)
Now, $p_{v,\infty} = 0.9 \times p_{\text{sat}}(27^0 C) = 0.9 \times 0.03567 = 0.0321$ bar and $\omega_{v,\infty} = 0.0321/(1.61 \times 1 - 0.61 \times 0.0321) = 0.02018$. Similarly, $p_{\text{sat}}(82.5^0 C) = 0.5261$ bar and $\omega_{v,w} = 0.4081$. Thus, $B = (0.02018 - 0.4081)/(0.4081 - 1) = 0.6554$, and

$$\frac{g}{g^*} = \frac{\ln (1 + B)}{B} \times \left( \frac{Pr}{Sc} \right)^{0.33} \times \left( \frac{M_{\text{mix},w}}{M_{\text{mix},\infty}} \right)^{0.667}$$

where $M_{\text{mix},w} = 24.51$ and $M_{\text{mix},\infty} = 28.76$. Therefore, substitution gives $g = g^* \times 0.7039 = 0.01344$ kg/m$^2$-s. Hence, evaporation rate is

$$\dot{m} = g \times B \times A_{\text{plate}} = 0.01344 \times 0.6554 \times 0.33 \times 1 = 0.0029\text{ kg/s, or 10.46 kg/hr (Ans a).}$$
In this case, since Le \( \approx 1 \), we have

\[
B = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{TL} + q_l/N_w} = 0.6554
\]

Taking \( T_{ref} = 0 \)

\[
h_{m,\infty} = 1.005 \times 27 + (0.875 \times 27 + 2503) \times 0.02018
\]

\[
= 78.12 \text{ kJ/kg}
\]

\[
h_{m,w} = 1.005 \times 82.5 + (0.875 \times 82.5 + 2503) \times 0.4081
\]

\[
= 1133.85 \text{ kJ/kg}
\]

\[
h_{TL} = 4.187 \times 82.5 = 345.4 \text{ kJ/kg}
\]

Hence \( (q_l/N_w) = -2400 \text{ kJ/kg} \). Negative sign indicates that heat is to be supplied. Thus,

\[
Q_{supp} = 2400 \times 0.0029 = 6.98 \text{ kW (Ans b)}.
\]
Evaporation from a Lake - L39(10/16)

Prob: A 10 kmph breeze (at 40°C and 20% RH) blows over a lake (at 30°C). The lake receives $q_{\text{solar}} = 500 \text{ W/m}^2$

Calculate the time required for the water level to drop by 1 cm. Assume turbulent boundary layer.

Take $\nu = 15 \times 10^{-6} \text{ m}^2/\text{s}$

$\text{Sc} = 0.61$ and $\text{Pr} = 0.7$.

Soln: Here

$\rho_{v,\infty} = 0.2 \times \rho_{\text{sat}} (40^0\text{C})$

$= 0.2 \times 0.07384$

$= 0.01477 \text{ bar}$. Therefore,

$\omega_{v,\infty} = \frac{0.01477}{1.61 - 0.61 \times 0.01477}$

$= 0.00919$.

Then, assuming $Le \simeq 1$

$$B = \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - 1}$$

$$= \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{\text{TL}} + q_l/N_w}$$

where $\omega_{v,w}$, $h_{m,w}$ and $q_l$ are not known.
But, \( N_w h_T + q_{rad} + q_l = N_w h_{TL} \). Hence,

\[
B_h = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_T - q_{rad}/N_w}
\]

Now, \( N_w = g \times B_h \). Hence

\[
B_h = \frac{h_{m,\infty} - h_{m,w} + q_{rad}/g}{h_{m,w} - h_T} = \frac{\omega_{V,\infty} - \omega_{V,w}}{\omega_{V,w} - 1}
\]

where taking \( T_{ref} = 0 \)

\[
h_{m,\infty} = 1.005 \times 40 + (0.875 \times 40 + 2503) \times 0.00919 = 63.3 \text{ kJ/kg}
\]

\[
h_{m,w} = 1.005 T_w + (0.875 T_w + 2503) \omega_{V,w}
\]

\[
h_T = 4.187 \times 30 = 125.4 \text{ kJ/kg}
\]
Here, $U_\infty = 10$ kmph $= 2.78$ m/s. Hence, 
$Re_L = 2.78 \times 100/15 \times 10^{-6} = 18.53 \times 10^6$
If we assume that $B \to 0$, then $g \to g^*$ and for a turbulent BL,

$$\frac{g^*}{\rho_m U_\infty} = 0.0365 \, Re_L^{-0.2} \, Pr^{-0.67} = 0.001617$$

where $\rho_m = \rho_v + \rho_a = \rho_a (1 + W_{mean})$ and 
$g^* = 0.00449 \, \rho_m$ or $\frac{q_{rad}}{g^*} = \frac{0.5}{(0.00449 \, \rho_m)} = \left(\frac{111.36}{\rho_m}\right)$ kJ/kg.

Therefore, substitution gives

$$B = \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - 1}$$

$$= 63.3 - \left[1.005 \, T_w + (0.875 \, T_w + 2503) \, \omega_{v,w}\right] + \left(\frac{111.36}{\rho_m}\right)$$

$$\left[1.005 \, T_w + (0.875 \, T_w + 2503) \, \omega_{v,w}\right] - 125.4$$

where $\omega_{v,w}$ corresponds to $T_w$ at saturation and $\rho_m$ is evaluatated from $W_{mean} = 0.5 (W_w + W_\infty)$. 
We need trial-and-error procedure as follows.

1. Assume $T_w$. Hence, determine $\rho_{v,w}$ from steam tables and evaluate $W_w = 0.622 \times \rho_{v,w}/(\rho_{tot} - \rho_{v,w})$. Hence, evaluate $W_{mean}$ and $T_{mean}$. Use gas law to evaluate, $\rho_a$ at $T_{mean}$ and, hence $\rho_m = \rho_a (1 + W_{mean})$

2. Substitute in the 2 eqns for $B$. If LHS = RHS, accept $T_w$ and $\omega_{v,w}$.

In the present case, $T_w \approx 39.5^0C$, $\omega_{v,w} = 0.043$ and $\rho_m = 1.16$ giving $B = 0.0354$ and $N_w = g^* B = 2.035 \times 10^{-4}$ kg / m$^2$-s, or 0.7324 kg / m$^2$-hr.

Now, for $\Delta h = 1$ cm drop in water level, we use mass balance

$\rho_{water} \times (\Delta h/\Delta t) = N_w$.

This gives $\Delta t = 13.65$ hrs (Ans).

Note that the lake surface temperature is very close to the free stream air temperature.
Humidification - L39(\frac{14}{16})

Prob: Moist air (\(W_{in} = 0.003\) at 24\(^0\)C) enters a tube (2.5 cm dia, 75 cm long) at the rate of 9 kg/hr. The tube-wall is maintained wet at 24\(^0\)C. Calculate (a) \(W_{exit}\) and (b) Heat to be supplied to the tube wall. Take \(D = 0.09\) \(m^2/\)hr and \(Sc = 0.6\) Assume Temp remains constant.

Soln: Part (a) In this case \(\omega_{v,w}\) is constant with \(x\).

We define
\[B_x = (\omega_{v,x} - \omega_{v,w})//(\omega_{v,w} - 1)\].
Then, \(d \omega_{v,x} = (\omega_{v,w} - 1)\ d B_x\).
For the differential element \(dx\)
\[N_{w,x} \ dx = (\dot{m}/\pi \ d) \times d \omega_{v,x}\text{ or } g B_x \ dx = (\dot{m}/\pi \ d) (\omega_{v,w} - 1) \ dB_x\text{ or, (next slide )} \]
\[ \frac{d B_x}{B_x} = \left\{ \frac{\pi d g}{\dot{m} (\omega_{v,w} - 1)} \right\} \, dx \rightarrow \frac{B_{out}}{B_{in}} = \exp \left\{ \frac{\pi d g L}{\dot{m} (\omega_{v,w} - 1)} \right\} \]

where $L$ = tube length. In this problem, corresponding to $24^0C$, $\omega_{v,w} = 0.01875$ ($W_w = 0.01632$) and corresponding to $W_{in} = 0.003$, $\omega_{v,in} = 0.002991$. Therefore $B_{in} = 0.01606$.

**Determination of $g$:** At inlet, $W_m = 0.5 (W_w + W_{in}) = 0.01088$ and at $24^0C$, $\rho_a = 1.189$. Therefore $\rho_m = \rho_a (1 + W_m) = 1.202$. Further, $\nu_m = Sc \times D = 0.054 \, m^2/hr = 1.5 \times 10^{-5} \, m^2/s$.

Also, $\dot{m} = u_{in} \times (\pi/4) \, d^2$ or $u_{in} = 5.09 \, m/s$.

Hence, $Re = u_{in} \, d/\nu_m = 8488.3$. Now, taking $Le = 1$, $Sh = g \, d/(\rho_m \, D) = 0.023 \, Re^{0.8} \, Sc^{0.4} = 26.06$ or, $g = 112.76 \, kg/m^2\cdothr$. Substitution gives $B_{out} = 0.00757$. 
Hence, $\omega_{V,\text{out}} = \omega_{V,\text{w}} + B_{\text{out}} (\omega_{V,\text{w}} - 1) = 0.01137$

or \( W_{\text{out}} = 0.01145 \text{ kg/kg of dry air (Ans)} \).

Now Energy balance over element \( dx \) gives

\[
\dot{m} \times dh_{m,x} = (N_{w,x} h_{\text{TL}} - q_{l,x}) \times \pi d \, dx.
\]

Integration gives

\[
-\bar{q}_{l} = - \frac{1}{L} \int_{0}^{L} q_{l,x} \, dx
\]

\[
= \left( \frac{\dot{m}}{\pi d} \right) \int_{0}^{L} \frac{d h_{m}}{d x} \, dx - g \, h_{\text{TL}} \int_{0}^{L} B_{x} \, dx
\]

\[
= \frac{\dot{m}}{\pi d} \left[ h_{m,\text{out}} - h_{m,\text{in}} - h_{\text{TL}} (\omega_{V,\text{out}} - \omega_{V,\text{in}}) \right]
\]

where taking \( T_{\text{ref}} = 0 \), \( h_{m,\text{out}} = 52.82 \), \( h_{m,\text{in}} = 31.664 \)

and \( h_{\text{TL}} = 4.187 \times 24 = 100.49 \text{ kJ/kg} \).

Hence \( \bar{q}_{l} = -0.646 \text{ kW/m}^2 \) (Ans).