1. Gas Injection - Effect of property variation and $\omega_T$ - LBL
2. Gas Injection - Effect of property variation and $\omega_T$ - TBL
3. Benzene evaporation in convective environment

Couette flow model permits effects of fluid property variations to be studied.
**Gas Injection \( (\omega_T = 1) \) - L38(1/14)**

**Prob:** Consider laminar Couette flow of air in which a gas with a specified \( \omega_{g,T} \) is injected. Develop relationship \((g/g^*) \sim B\) when the gas is \( CO_2 \), He and \( H_2 \) and study the effect of \( \omega_{g,T} \).

**Soln:** In the Couette flow model, 
\[
\partial (\rho u) / \partial x = 0 = \partial (\rho v) / \partial y.
\]

Hence, 
\[
N_w = \rho_w \nu_w = \rho \nu = \text{const.}
\]

The species transfer Eqn
\[
N_w \frac{\partial \omega_g}{\partial y} = \frac{\partial}{\partial y} (\rho_m D \frac{\partial \omega_g}{\partial y})
\]

Integrating once
\[
N_w (\omega_{g,y} - \omega_{g,w}) =
\]
\[
\rho_m D \left. \frac{\partial \omega_g}{\partial y} \right|_y - \rho_m D \left. \frac{\partial \omega_g}{\partial y} \right|_w
\]

Now, boundary condition gives (next slide)
\[ N_w = \frac{\rho_m D \partial \omega_g / \partial y}{\omega_{g,w} - \omega_{g,T}} \]

Hence

\[ N_w (\omega_{g,y} - \omega_{g,w}) = \rho_m D \left. \frac{\partial \omega_g}{\partial y} \right|_y - N_w (\omega_{g,w} - \omega_{g,T}) \text{ or} \]

\[ \rho_m D \left. \frac{\partial \omega_g}{\partial y} \right|_y = N_w (\omega_{g,y} - \omega_{g,T}) \]

where \( D = \text{const} \neq F(\omega_g) \) because \( p \) & \( T \) are const, but

\[ \rho_m = \frac{p}{R_u T} \quad M_{mix} = \frac{p}{R_u T} \left( \sum \frac{\omega_j}{M_j} \right)^{-1} \]

\[ = \frac{p}{R_u T} \left[ \frac{M_g M_a}{M_g \omega_g + M_a (1 - \omega_g)} \right] \]
Substitution and integration from $y = 0$ to $\delta$ gives

$$\int_{\omega_{g,w}}^{0} \frac{d \omega_g}{a \omega_g^2 + b \omega_g + c} = \frac{N_w R_u T \delta}{p M_g M_a D}$$

with

$$a = (M_a - M_g), \quad b = M_g - \omega_{g,T} (M_a - M_g), \quad c = -M_g \omega_{g,T}$$

where the LHS is given by

$$\text{LHS} = \frac{1}{\sqrt{b^2 - 4 a c}} \ln \left[ \frac{2 a \omega_g + b - \sqrt{b^2 - 4 a c}}{2 a \omega_g + b + \sqrt{b^2 - 4 a c}} \right]_{\omega_{g,w}}^{0}$$

$$= \frac{1}{M_g + \omega_{g,T} (M_a - M_g)} \ln \left[ 1 + B + \omega_{g,T} B \left( \frac{M_a}{M_g} - 1 \right) \right]$$

where

$$B = \frac{0 - \omega_{g,w}}{\omega_{g,w} - \omega_{g,T}} = \frac{\omega_{g,w}}{\omega_{g,T} - \omega_{g,w}}$$

and

$$\omega_{g,w} = \omega_{g,T} \times \frac{B}{1 + B}$$
Now, for the Couette flow model

\[ N_w = g B, \quad \text{and} \quad \frac{R_u T}{p M_g} = \frac{1}{\rho_g} \]

Therefore

\[ \text{RHS} = \frac{N_w R_u T \delta}{p M_g M_a D} = \frac{g B \delta}{\rho_g M_a D} \]

Equating LHS = RHS and rearranging

\[ \left( \frac{g \delta}{\rho_g D} \right) = \left( \frac{M_a}{M_g} \right) \left[ \ln \left( 1 + B^* \right) \right] \]

where

\[ B^* = B \left\{ 1 + \omega_{g,T} \left( \frac{M_a}{M_g} - 1 \right) \right\} \]

Hence

\[ \left( \frac{g}{g^*} \right)_{vp} = \frac{\ln (1 + B^*)}{B^*} \quad (\text{Ans}) \rightarrow \quad \left( \frac{g}{g^*} \right)_{cp} = \frac{\ln (1 + B)}{B} \]

where subscript ’vp’ for variable and ’cp’ for const property.
Soln - \( \left( \frac{g}{g^*} \right) \sim B \) for \( \omega_{g,T} = 1 - L38 \left( \frac{5}{14} \right) \)

<table>
<thead>
<tr>
<th>B</th>
<th>cp</th>
<th>( v_p_{CO_2} )</th>
<th>( v_p_{He} )</th>
<th>( v_p_{H_2} )</th>
<th>( \omega_{g,w} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>0.0</td>
</tr>
<tr>
<td>.25</td>
<td>.893</td>
<td>.926</td>
<td>.571</td>
<td>.422</td>
<td>.200</td>
</tr>
<tr>
<td>.50</td>
<td>.811</td>
<td>.864</td>
<td>.422</td>
<td>.291</td>
<td>.333</td>
</tr>
<tr>
<td>1.0</td>
<td>.693</td>
<td>.768</td>
<td>.291</td>
<td>.189</td>
<td>.500</td>
</tr>
<tr>
<td>1.5</td>
<td>.611</td>
<td>.695</td>
<td>.228</td>
<td>.144</td>
<td>.600</td>
</tr>
<tr>
<td>2.0</td>
<td>.549</td>
<td>.638</td>
<td>.189</td>
<td>.117</td>
<td>.667</td>
</tr>
<tr>
<td>2.5</td>
<td>.501</td>
<td>.591</td>
<td>.163</td>
<td>.0998</td>
<td>.714</td>
</tr>
<tr>
<td>3.0</td>
<td>.462</td>
<td>.552</td>
<td>.144</td>
<td>.0873</td>
<td>.750</td>
</tr>
</tbody>
</table>

1. \( \omega_{g,T} = 1 \) implies that the gas is the only transferred substance. Also, \( B^* = \frac{B M_a}{M_g} \).

2. \( \left( \frac{g}{g^*} \right)_{vp,CO_2} > \left( \frac{g}{g^*} \right)_{cp} \) because \( M_{CO_2} > M_{air} \)

3. For He and \( H_2 \), this trend reverses.

4. \( \omega_{g,w} \) increases with \( B \)
Soln - \( \left( \frac{g_{\ast}}{g} \right) \sim B \) for \( \omega_{g,T} = 0.01 \) - L38\( \frac{6}{14} \)

<table>
<thead>
<tr>
<th>B</th>
<th>( c_p )</th>
<th>( v_{p CO_2} )</th>
<th>( v_{p He} )</th>
<th>( v_{p H_2} )</th>
<th>( \omega_{g,w} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
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<tr>
<td>.25</td>
<td>.893</td>
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<td>.887</td>
<td>.888</td>
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<td>.50</td>
<td>.811</td>
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<td>.802</td>
<td>.792</td>
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</tr>
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<td>.612</td>
<td>.598</td>
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<tr>
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<td>.550</td>
<td>.536</td>
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<tr>
<td>2.5</td>
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<td>.474</td>
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<tr>
<td>3.0</td>
<td>.462</td>
<td>.463</td>
<td>.449</td>
<td>.435</td>
<td>.0075</td>
</tr>
</tbody>
</table>

1. \( \omega_{g,T} = .01 \) implies that the gas in the transferred substance is a small fraction - rest is air.

2. \( \left( \frac{g_{/g^*}}{g} \right)_{v_p,CO_2} \sim \left( \frac{g_{/g^*}}{g} \right)_{c_p} \)

3. For \( He \) and \( H_2 \), \( \left( \frac{g_{/g^*}}{g} \right)_{v_p} < \left( \frac{g_{/g^*}}{g} \right)_{c_p} \)

4. \( \omega_{g,w} \), though small, increases with B
Correlation with \( \frac{M_{\text{mix},\infty}}{M_{\text{mix},w}} \) - L38(\( \frac{7}{14} \))

Here, \( M_{\text{mix},w} = \frac{M_a M_g}{(M_a \omega_{g,w} + M_g (1 - \omega_{g,w}))} \)
and \( M_{\text{mix},\infty} = M_a \) (because \( \omega_{g,\infty} = 0 \)). Hence, from slide 4, and using \( \omega_{g,w} = \omega_{g,T} \times B/(1 + B) \)

\[
B^* = B \left\{ 1 + \omega_{g,T} \left( \frac{M_a}{M_g} - 1 \right) \right\}
\]

\[
\frac{B^*}{B} = 1 + \left( \frac{1 + B}{B} \right) \left( \frac{M_{\text{mix},\infty}}{M_{\text{mix},w}} - 1 \right)
\]

\[
\frac{(g/g^*)_{vp}}{(g/g^*)_{cp}} = \frac{\ln (1 + B^*)}{B^*} \times \frac{B}{\ln (1 + B)}
\]

This shows dependence on \( M_{\text{mix},w}/M_{\text{mix},\infty} \) and \( B \) as recommended correction from boundary layer flow model.
If \( \omega_{g,T} = 0, B^* = B \). If \( \omega_{g,T} = 1, B^* = B \left( \frac{M_a}{M_g} \right) \)
Here, the governing Eqn will be

\[ N_w (\omega_g - \omega_{g, T}) = \rho_m (D + D_t) \frac{d \omega_g}{dy} \]

where

\[ \rho_m D_t = \rho_m \frac{\nu_{t, ref}}{Sc_t} \]

But, from Van-Driest model

\[ \nu_{t, ref} = \frac{\mu_t}{\rho_{ref}} = \int_m^2 \frac{\partial u}{\partial y} \rightarrow \frac{\partial u}{\partial y} = C \]

\[ = C \left( \frac{\nu_{ref}}{u_\tau} \right)^2 \left( \kappa y^+ \right)^2 \left\{ 1 - \exp\left( - \frac{y^+}{A^+} \right) \right\}^2 \]

\[ = C \left( \frac{\nu_{ref}}{u_\tau} \right)^2 (0.08 \delta^+)^2 \text{ for } y^+ > 26 \text{ where} \]

\[ C \left( \frac{\nu_{ref}}{u_\tau} \right)^2 = C \frac{\nu_{ref}^2 \rho_{ref}}{\tau_w} = C \times \frac{\mu_{ref} \nu_{ref}}{\mu_{ref} C} = \nu_{ref} \]
Substituting for $D_t$ and $\rho_m$, we have

$$N_w (\omega_g - \omega_{g,T}) = \rho_m D \left(1 + \frac{\nu_{t,\text{ref}}}{Sc_t D}\right) \frac{d \omega_g}{dy}$$

$$= \left(\frac{D p M_a M_g}{R_u T}\right) \times \frac{u_T/\nu_{\text{ref}}}{M_a \omega_g + M_g (1 - \omega_g)}$$

$$\times F \times \frac{d \omega_g}{dy^+} \text{ where}$$

$$F = 1 + \left(\frac{Sc}{Sc_t}\right) (\kappa y^+)^2 \left\{1 - \exp\left(-\frac{y^+}{A^+}\right)\right\}^2 \text{ for } y^+ < 26$$

$$= 1 + \left(\frac{Sc}{Sc_t}\right) (0.08 \delta^+)^2 \text{ for } y^+ > 26$$
Taking $N_w = g B$, $(p M_g)/(R_u T) = \rho_g$ and $u_\tau = U_\infty \sqrt{C_{f,x}/2}$, 

\[
LHS = \left( \frac{g}{\rho_g U_\infty} \sqrt{\frac{2}{C_{f,x}}} \right) \times \text{INT} \quad \text{where} \quad \text{INT} = \int_0^{\delta^+} \frac{dy^+}{F}
\]

\[
RHS = \frac{M_a}{B} \int_{\omega_g,w}^0 \left( \omega_g - \omega_{g,T} \right) \left\{ M_a \omega_g + M_g \left( 1 - \omega_g \right) \right\} \frac{d \omega_g}{\omega_g - \omega_{g,T}}
\]

\[
= \ln \left( 1 + B^* \right) \quad \Rightarrow \quad B^* = B \left\{ 1 + \omega_{g,T} \left( \frac{M_a}{M_g} - 1 \right) \right\}
\]

Taking $A^+ = 26$ and $Sc_t = 0.9$, we have

INT = 9.62 for CO$_2$ — Air, Sc = 0.96

INT = 14.57 for H$_2$ — Air and He-Air, Sc = 0.22
Therefore

\[
\frac{g_{vp}}{\rho g U_{\infty}} \times \sqrt{\frac{2}{C_{f,x}}} \times Sc = \frac{1}{\text{INT}} \times \frac{\ln (1 + B^*)}{B^*}
\]

and

\[
\frac{(g/g^*)_{vp}}{(g/g^*)_{cp}} = \frac{\ln (1 + B^*)}{B^*} \times \frac{B}{\ln (1 + B)}
\]

This result is same as that for a Laminar boundary layer. This is because it is assumed that the value of INT is same for ’cp’ and ’vp’ conditions.

Note that \(g_{vp}\) is significantly influenced by INT (Sc).
Evaporation of \( C_6H_6 - L38(\frac{12}{14}) \)

**Prob:** \( C_6H_6 \) evaporates from the outer surface of a circular cylinder in air flowing at 6 m/s normal to the cylinder. From expts, \( h_{\text{cof},v_w=0} = 85 \ \text{W/m}^2\)-K and \( B = 0.9 \).

Allowing for property variations, estimate \( N_w \) and \( \omega_w \).

**Given:** Sc = 1.71, Pr = 0.71, \( c_{pC_6H_6} = 1.69 \ \text{kJ/kg-K} \) and \( c_{pa} = 1.01 \ \text{kJ/kg-K} \).

**Soln:** Here,

\[
B = \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - 1} = 0.9 \quad \rightarrow \quad \omega_{v,w} = 0.4737 \quad \text{(Ans)}
\]

Therefore, \( \omega_{v,m} = 0.5 \left( \omega_{v,\infty} + \omega_{v,w} \right) = 0.2368 \).

\( c_{pm} = 1.69 \times 0.2368 + 1.01 \times 0.7632 = 1.171 \ \text{kJ/kg-K} \).

Hence, \( g^* = \left( \frac{h_{\text{cof},v_w=0}}{c_{pm}} \right) = 0.0726 \ \text{kg/m}^2\text{-s} \).

Also, \( M_{\text{mix},\infty} = 29 \) and \( M_{\text{mix},w} = (0.4737/78 + 0.5263/29)^{-1} = 41.286 \).
For Flow over a cylinder\(^1\), \(N_{ucp} \propto Pr^{0.37}\).

Therefore, using the short-cut empirical formula

\[
\frac{g_{vp}}{g_{cp}^*} = \frac{\ln (1 + B)}{B} \times \left( \frac{Pr}{Sc} \right)^{0.37} \times \left( \frac{M_{mix,\infty}}{M_{mix,w}} \right)^{-0.67}
\]

\[
= \frac{\ln (1 + 0.9)}{0.9} \times \left( \frac{0.71}{1.71} \right)^{0.37} \times \left( \frac{29}{41.286} \right)^{-0.67} = 0.6525
\]

Therfore, \(g = 0.0726 \times 0.6525 = 0.0474 \text{ kg/m}^2\text{-s} \) (Ans).
Thus, the effect of property variations is to reduce \(g_{vp}\) compared to \(g_{cp}\).

If we followed the Couette flow theory, then in this case,

$$B^* = B \left\{ 1 + \omega g, T \left( \frac{M_a}{M_g} - 1 \right) \right\} = 0.3346$$

Hence

$$\left( \frac{g}{g^*} \right)_{vp} = \frac{\ln (1 + 0.3346)}{0.3346} = 0.8626$$

But, for variable properties, $h_{cof, vp} = h_{cof, cp} \times Pr^{0.25}$. Therefore, $g_{vp} = g_{cp}^* \times (0.71)^{0.25} \times 0.8626 = 0.0575 \text{ kg/m}^2\text{-s}$. This value is greater than that obtained from the empirical formula. Thus, Couette flow theory provides an approximate answer due to linear velocity profile assumption.